1. In all the following questions \( \{Z_t\} \) is a purely random process with mean \( \mathbb{E}[Z_t] = 0 \), variance \( \text{Var}(Z_t) = \sigma^2 \), and successive values of \( Z_t \) are independent so that \( \text{Cov}(Z_t, Z_{t+k}) = 0, k \neq 0 \).

(a) Derive the mean function of the process
\[
X_t = Z_t + 0.7Z_{t-1} - 0.2Z_{t-2}.
\]
Show that the autocorrelation function \( \rho(t) \) of \( \{X_t\} \) is given by
\[
\rho(t) = \begin{cases} 
1 & k = 0 \\
0.37 & k = \pm 1 \\
-0.13 & k = \pm 2 \\
0 & \text{otherwise}.
\end{cases}
\] (1)

How many parameters does \( \{X_t\} \) have?

(b) Derive the mean and the autocorrelation functions of the process
\[
X_t = \sum_{k=0}^{m} (m+1)^{-1}Z_{t-k}.
\]
How many parameters does \( \{X_t\} \) have?

(c) Consider the infinite-order process defined by
\[
X_t = Z_t + c(Z_{t-1} + Z_{t-2} + \cdots),
\]
where \( c \) is a constant. Show that the process is not covariance-stationary. Also show that the series of first differences defined by
\[
Y_t = X_t - X_{t-1}
\]
is covariance-stationary. Find the autocorrelation function \( \rho(t) \) of \( \{Y_t\} \). How many parameters does \( \{Y_t\} \) have?

(d) Find the mean function \( \mu(t) \) and the autocorrelation function \( \rho(t) \) of the process
\[
X_t - \mu = 0.7(X_{t-1} - \mu) + Z_t.
\]
Plot \( \rho(k) \) for \( k = -6, -5, \ldots, -1, 0, +1, \ldots, +6 \). How many parameters does \( \{X_t\} \) have?

2. Do problems 3.2, 3.3, 3.4, and 3.6 in the textbook.