Scaffolding Student's Conceptions of Proportional Size and Scale Cognition with Analogies and Metaphors

Abstract

The American Association for the Advancement of Science identifies scale as one of the four powerful common themes that transcend disciplinary boundaries and levels. Engineering is one of these disciplines that requires a strong spatial ability involving scale, as well as the ability to reason proportionally when using scale models. In addition, advancing nanosciences is opening new opportunities for engineers to pursue opportunities for designing nanotechnologies. However, today's middle school students do not demonstrate an adequate understanding of concepts of scale and size on the micro and the nano level. Students are unable to identify the relative sizes between micrometer-sized and nanometer-sized objects. The focus of this study is the role of proportional reasoning as one of the cognitive processes behind qualitative and quantitative proportional scale cognition. Proportional reasoning is the cognitive process that supports our ability to compare two rational expressions; it has also been recognized that proportional reasoning deals with one of the most common forms of structural similarity. Analogical reasoning involves a process of structural alignment and mapping between mental representations. Therefore, this study seeks to answer the following research question: Will analogies and metaphors scaffold proportional conceptions of size and scale? Participants for the initial study included 150 seventh graders from a science class of a Midwestern middle school. For identifying student's conceptions of the logical and numerical proportional scale cognition a mixed method procedure was designed. Data was analyzed by comparing among student's logical and mathematical proportional scale cognition as well as contrasted with similar results from the literature. This research was conducted to better understand the cognition associated with these skills and to design instructional methods to effectively develop these abilities in learners. We believe that the results of this study will inform the design of curricula that effectively convey scaling related concepts. Future work related to this area will also be discussed.

Introduction

Notions of scale have been identified as one of the four powerful common themes that transcend disciplinary boundaries and levels helping learners structuring knowledge (American Association for the Advancement of Science [AAAS] 2006)[1]. Scaling related concepts are directly applied to the study of phenomena in the micro, nano, and atomic level. As new tools, techniques, and instrumentations that allow study of phenomena in the micro and nano world are developed, new and complex technologies rise resulting in new fields of science and technology. As a consequence, new global markets emerge as well as the need of a workforce to meet them. Engineering is one of these disciplines that requires a strong spatial ability involving scale, as well as the ability to reason proportionally when using scale models. In addition, advancing nanosciences is opening new opportunities for engineers to pursue opportunities for designing nanotechnologies.

Therefore, it has increased the importance for young learners to become scale-literate in order to possess the structured knowledge required to understand complex phenomena. However, by analyzing summative evaluation reports and surveys conducted in informal educational settings, it was found that the general public have limited knowledge and consistent misconceptions about atoms, molecules, DNA, cells, and other things the interviewees cannot see (Edu.Inc, n.d.)[2]; misconceptions such as the smallest thing that they can think of is something they can actually see (early elementary students) or objects at the microscopic scale (Holladay, 2005)[3]. Another common naive conception is the fact that most of the interviewees had no working concept of one billion and did not understand 10^{-9} (Edu.Inc)[2]. Tretter, et al. (2006)[4] report that today's middle school students do not demonstrate an adequate understanding of concepts of scale and size on the micro and the nano level. Moreover, students are unable to identify the relative sizes between micrometer-sized and nanometer-sized objects (Edu.Inc, n.d.[2], 2004[5], 2005[6]; Holladay 2005[3]; Jones et al. 2004[7]; Waldron, 2006[8]).

The focus of this study is on the role of proportional reasoning as one of the cognitive processes supporting scale cognition, and at the same time serving as the bridge between the qualitative understanding of size into a quantitative understanding of scale. Several of our driving questions are – How do young learners identify how much an object is bigger than another? In which terms do they express this relationship? What skills are required to answer these questions? How do these skills develop? How can children reason in a similar way when the objects of comparison cannot be seen? We believe that analogical reasoning plays an important role in answering these questions. Therefore, this study seeks to answer the following research question: Will analogies and metaphors serve as a vehicle to scaffold proportional conceptions of size and scale? We anticipate that the results of this study will inform the design of curricula that effectively convey scaling related concepts.

Defining proportional size and scale

Size and scale are related to each other. While size refers to a qualitative semantic property of an object, scale refers to a quantitative formal property of an object. The transition from one to the other is referred to as scaling. Rozeboom (1966)[9] explain this relationship as follows:

> "Just as every formal variable has an uscaled (natural) counterpart, so is it that to every natural variable there corresponds not merely one but a multitude of num-valued formal variables, and conceptually replacing a natural variable with one of its formal equivalents is basically what is meant by scaling" (p. 178).

Where "'num' will be a systematically ambiguous term, like an unspecified parameter in an algebraic equation, which is to be heuristically understood as 'number'…" (Rozeboom, p.177)[9]. We suggest proportional reasoning as the cognitive process required to bridge a qualitative conception of size; namely the logical; to a quantitative conception of scale; namely the mathematical. Person, Berenson and Greenspon (2004)[10], have emphasized that research demonstrates that proportional reasoning is at the core of mathematics curriculum; and have emphasized it as a good indicator of higher mathematical achievement.

Inhelder and Piaget (1958)[11] describe the proportional schema as composed of two aspects; the logical and the mathematical. "In its general logical form, a proportion is the equivalence of the relations connecting two terms α and β to the relations connecting two other terms γ and σ." (p. 314). The authors also argue that the logical aspect leads to the mathematical one; once the learner has acquired the schemata of the logical one, later on, at any point the numerical values could be inserted. To exemplify the proportional conceptions of size and scale let's consider some differences in relative sizes among these objects: DNA double strand, bacteria, ant, and human. For the logical proportional conception of size an example would be: the difference in sizes between the height of a human and the length of an ant is approximately the same proportion as the difference in sizes between a bacteria and the diameter of a DNA double strand. This relationship was made in qualitative terms, considering the objects' size. On the other hand, an example illustrating the case of the mathematical proportional conception of scale would be: the difference in length of an ant compared to the height of a human is that the height of a human is a thousand times bigger than the length of an ant as the difference in size between a bacteria and the diameter of a DNA double strand is that the length of a bacteria is about a thousand times bigger than the diameter of a DNA double strand. In contrast, this relationship was made in quantitative terms involving a scaling process; where the ant is the unit of measurement mapped then to the DNA double strand.

The mathematical proportional conception of scale has also been identified in other studies such as Delgado, Stevens, Shin, Yunker, and Krajcik (2007)[12], as the quantitative relative conception of size, which basically refers to "how many times larger or smaller one object is than another" (2007, p.9). Related to this conception of scale is the student's ability to make comparisons between objects; a process that has been identified as a prerequisite to measurement (Delgado et al., 2007[12]; Wiedtke, 1990[13]). According to Campbell, measurement "... is the process of assigning numbers to represent qualities" (p.223 as cited in Rozeboom, 1966[9]); but "in the tough sense of the word, 'measurement' is assessment of quantity" (Rozeboom, p. 224[9]). Therefore, the only difference between the absolute and the quantitative proportional sizes is what Delgado et al.[12], describe as the process when the unit used is another object (for the case of the quantitative proportional) rather than a conventional measurement unit (for the case of the absolute size).

Proportional Reasoning: the Cognitive Process behind the Proportional Conceptions of Size and Scale

According to Lesh, Post, and Behr, (1988)[14], proportional reasoning is a form of mathematical reasoning involving multiple comparisons, inference and prediction, as well as both qualitative and quantitative methods of thought. In their work, Lesh et al.[14] examined it from the perspective of proportional reasoning as a capstone of elementary arithmetic, number, and measurement concepts. Proportional reasoning is the cognitive process behind the ability to reason about the relationship between two rational expressions. Therefore, our first inference is that proportional reasoning is the required cognitive process in order to attain the proportional size and scale cognition. We have identified that scale cognition is composed by the logical proportional and numerical proportional conceptions of size and scale; these conceptions and the cognitive processes behind them are explained below.

For describing the logical proportional conception of size we referred to the work conducted by Tretter et al. (2006)[4]. Tretter et al.[4] describe unitizing as the first required cognitive process in order to attain scale cognition. Unitizing refers to the process of creating new meaningful units from the existing objects. Tretter et al.[4] suggest that students must create scale conceptions focusing on relative sizes and not much on exact size information. These conceptions could be based on everyday uses of size with relations based on experience, organized into categories and containing prototypes as exemplars of categories; such as well known landmarks and reference points. For example, a well known unit for a novice learner can be the relationship in sizes between two objects from the macro scale that will serve as a landmark or reference size for objects in the microscale, atomic scale and nanoscale. (E.g. the difference in sizes between the height of a human and the length of an ant is approximately the same–proportional as the difference in sizes between a bacteria and the diameter of a DNA double strand).

The numerical proportional conception of scale, on the other hand, point us to consider the mathematical relationships with pairs of rational expressions; such as ratios, proportions, rates, quotients, and fractions. For the case of ratios and proportions, Hart, (1988)[15] and Behr, Lesh, Post, and Silver, (1983)[16] agree that a ratio is a comparative index between two entities that conveys the notion of relative magnitude. On the other hand, a proportion refers to the equivalence of two ratios, "When two ratios are equal they are said to be in proportion to one another. A proportion is simply a statement equating two ratios." (Behr et al., p.95[16]). For the case of rates, and as explained by Lesh et al. (1988)[14], it refers to a single quantity (e.g. 30 miles/hour), while ratios involve two quantities. Fractions include percentages, decimal expressions, and operations or points on the number line (Person, Berenson, and Greenspon, 2004[10]) and finally, a quotient is simply an operation of indicated division (Karplus, Pulos, and Stage, 1983[17]).

The proportional mathematical conception of scale would be considered then as the comparison of two equivalent ratios; namely a proportion (e.g. the difference in length of an ant compared to the height of a human is that the length of an ant is about a thousand times smaller than the height of a human; therefore, the difference in size between a bacteria and the diameter of a DNA double strand is that the diameter of a DNA double strand is about a thousand times smaller than the size of a bacteria).

To better understand proportional reasoning and at the same time infer instructional strategies that will address this cognitive process, research related to this area was analyzed. For this purpose three authors and their research were considered: perspectives proposed by Piaget, Karplus, and Lesh. Piaget's perspective was selected because he and his colleagues conducted the first attempts to measure proportional reasoning in their experiments for clarification of young people's development of the logico-mathematical concept (Lesh et al., 1988[14]). But because the designs of their tasks were not designed specifically to illustrate proportional reasoning, more specialized studies were consulted. Robert Karplus, in contrast, focused on proportional reasoning by trying to minimize the need for knowledge of physical principles (Lesh et al., 1988[14]). Finally, the research conducted by Lesh was also considered because of his attempts to contrast and compare prior research in this area.

Piaget (1968, as cited in Lesh et al., 1988[14]) describes the development of adolescent's proportional reasoning in three stages. The first one is the global compensatory strategy which focuses on additive strategies, the second is based on a multiplicative strategy, and the third one is based on a formulation of a law of proportions. In contrast, Karplus focused on categorizing the responses of children as demonstrative of a level of understanding of proportion (Hart, 1988[15]). Karplus et al., (1983)[17] argue that Piagetian tasks were not selected exclusively for characterizing subjects' proportional reasoning, leaving unanswered many questions related to how proportional reasoning is applied in problem solving in different contexts and numerical relationships. Instead Karplus et al. and the research group at the Lawrence Hall of Science at Berkeley, focused on assessing children's proportional reasoning using tasks in which knowledge of the physical principles was minimized. They found that "academically upper-track or upper middle-class students used proportional reasoning increasing after about age 12 years, only a small fraction of urban low-income and academically lower track-students used proportions at age 14 or even 17 years" (Karplus, 1981 as cited in Karplus et al., 1983, p. 47[17]).

Compared to the results obtained by Karplus et al., as well as Inhelder and Piaget; Lesh et al.[14] concludes that in mathematics education research, proportional reasoning is characterized by a gradual increase in local competence; not by a global ability related to a cognitive structure. These results point us to consider that environmental interactions are a very influential factor for the development of proportional reasoning.

Singer-Freeman and Goswami (2001)[18] also investigated children's intuitive approach to proportions, arguing that their intuitions of proportional reasoning were based on their ability to recognize relational similarity. This idea is consistent with Lesh et al. (1988)[14] remark that "proportional reasoning deals with one of the most common forms of structural similarity" (p.95). This recognition of structural similarity is what points us to consider analogies and metaphors as a way to scaffold proportional reasoning in young learners for the following reasons. First, although similarity and analogy are not the same, Gentner and Markman (1997)[19] suggest that the process of carrying out a comparison is the same in both cases, concluding that similarity is like analogy involving a process of "structural alignment and mapping between mental representations" (Gentner and Markman, 1997, p.45[19]).

Second, classical or conventional analogies take the form of A:B::C:D (English, 2004[20]), where the A and B can be termed as the base or source, and C and D can be termed as the target (Gentner, Holyoak, & Kokinov, 2001[21]). These analogies are basically proportional or relational problems (English[20]; Gentner and Markman[19]). In our context, an example of a classical conventional analogy can take the form of: the difference in sizes between the height of a human (A) and the length of an ant (B) is approximately the same--proportional as the difference in sizes between a bacteria (C) and the diameter of a DNA double strand (D); where the height of a human is mapped with the size of the bacteria, and the length of an ant with the diameter of a DNA double strand.

Finally, we rely on the fact that it has been identified that one reasonable way to convey proportions is through means of existing conceptual knowledge as a basis for teaching (Singer-

Freeman & Goswami, 2001[18]), as well as on the fact that children's ability to recognize relational similarity, namely the capacity for analogical thinking, may be present as early as infancy; (Goswami, 2001[22]; Holyoak and Thagard, 1997[23]).

Analogies and Metaphors for Attaining Proportional Scale Cognition

Analogies are a fundamental cognitive mechanism that people use to map processes by identifying relevant information from a more familiar domain to a less familiar one (Mason, 2004). Since early times it has been well recognized the powerful role of analogies in enabling people to communicate, explore, and infer about novel phenomena, as well as to transfer learning across subject domains (English, 2004[20]; Gentner and Markman, 1997[19]; Goswami, 2001[22]; Richland, Morrison, and Holyoak, 2006[25]).

While analogy is a sophisticated process used in creative discovery, similarity is, as described by Gentner and Markman, (1997)[19], a brute perceptual process. Gentner (1983) define an analogy as "a device for conveying that two situations or domains share relational structure despite arbitrary degrees of difference in the objects that make up the domains. This promoting of relations over objects makes analogy a useful cognitive device, for physical objects are normally highly salient in human processing" (as cited in Gentner and Markman, 1997, p.46[19]). They emphasize the importance of common relations to analogy and not common objects. In contrast, English, (1997)[26] describes metaphors as characterized by cross domain mappings. She explains that reasoning by metaphor implies conceptualizing the phenomena in terms of another mental domain by finding a mapping between the target domain and the source domain. "Like analogical reasoning, metaphorical reasoning can generate new inferences and lead to the construction of mental models based on the relational structure shared by the source and target" (English, p.7[26]).

Although Piaget concludes that reason by analogy must depend on the development of categorization skills and therefore developed in the formal operations stage; recent research provides insights that the ability to reason by analogy has been shown by young children (Vosniadou, 1995[27]). Vosniadou argues that in the Piagetian perspective the focus was on "how the development of analogical reasoning could be explained in the context of his [Piaget's] theory of intellectual development, rather than how analogical reasoning might contribute to intellectual development" (p.300)[27]. Supporting this idea Goswami (1992)[28] and her colleagues found no support for the claims made by Piaget in his theory of analogical development, including the idea that reasoning about relational similarity requires the ability to reason about proportional equivalence (as cited in Vosniadou, 1995[27]). Goswami also concludes that "domain knowledge is the primary constraint in children's analogical reasoning" (p.250, as cited in Richland et al., 2006[25]).

We conclude that due to the facts that a) analogical reasoning does not depend on the ability to reason about proportional equivalence, b) young children have the ability to reason by analogy, and c) analogies and metaphors are powerful tools in enabling people to communicate, explore, infer about novel phenomena, and to transfer learning across subject domains; we therefore suggest that proportional analogies may be a sense-making way to unitize and at the same time serve as a scaffold that will emphasize relative sizes of objects in different scales. These analogs, as explained by Tretter et al. (2006)[4] could be based on everyday uses and conceptions of size with categorical relations, and these conceptions of scaling must be based on experience containing prototypes as exemplars of categories; such as landmarks and reference points.

Method

Participants

The subjects for the study included the entire population of seventh graders from the science classes of a Midwestern middle school. According to SchoolMatters (2006)[29] the student population of the school in 2005 was composed by 89% White, 6.7% Hispanic, 4% African American and 0.3% Asian/Pacific Islander.

From the eight classes of seventh graders, six groups and about 110 students were exposed to instruction based on analogies and metaphors for conveying proportional scale cognition, and the two remaining groups formed by approximately 40 students were taught by means of the traditional science and math curriculum. In addition, this last group was exposed to an informal learning environment (i.e. a museum exhibit) focused on nanotechnology related concepts.

Instrument

The metaphor presented to the students was a logarithmic scale that represents powers of ten, that is, powers of ten were considered as points on a line (see Figure 1). This scale was used to display number sequences and scales: nanoscale, microscale, and macroscale.

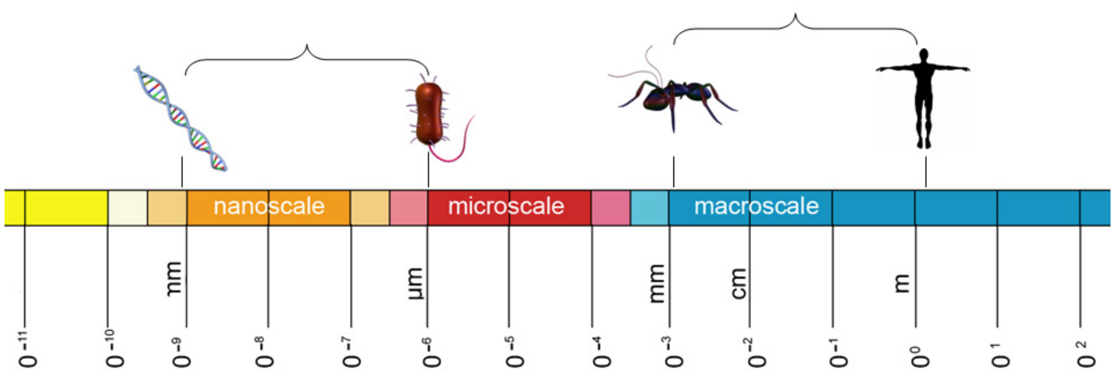


Figure 1. Scale metaphor and proportional analogy

Together with this scale, a proportional analogy was used as a way to explicitly compare two things in which their relational structure; namely their proportional sizes were emphasized. The analogy presented was intended to create new meaningful units at the micro and nano level from the existing objects at the macro level, emphasizing their proportional relationship (e.g. the difference in sizes between the height of a human and the length of an ant is approximately the same proportion as the difference in sizes between a bacteria and the diameter of a DNA double strand).

For evaluating the learning strategy, a mixed method procedure was designed (See Appendix A). The quantitative component was composed by a questionnaire which required the students to locate the diameter of a DNA strand given that its scale is 1 x 10−9. Student's understanding of the scale metaphor was considered correct if he/she was able to locate correctly the object of the first task. For the second task, they were given the following analogy: the difference in size between the ant and the human (displayed on the scale) was similar to the difference in size between the diameter of a DNA strand and bacteria. Then it was required that given the analogy, they should locate the bacteria on the scale. If students were able to identify the location of the second object on the scale given the analogy at the macroscale (E.g. the proportional relationship between the human and the ant to the DNA strand and the bacteria), it was considered that they were able to identify the logical proportional relationship between both pairs of objects.

The third task consisted of asking the students about the difference in the length of an ant compared to the height of a human. The fourth task required them to report the difference in diameter of a DNA strand compared to the size of bacteria. The mathematical proportional relationship was assessed by asking the students the approximate difference between the two pairs of objects. This means that the solution to the third task, the difference in length of an ant compared to the height of a human, was equal to the solution of the fourth task, the difference in diameter of a DNA strand compared to the size of bacteria. The qualitative part of the study was presented in the fifth task. It requested the students to explain the strategy they used in finding the difference between the sizes of the objects. The aim of the fifth task, an explanation of their strategy in finding the difference between the sizes of the objects, was used as an attempt to identify their thinking process.

Design and procedure

The middle school in which the materials were tested was identified by the school corporation who provided all the permissions to conduct the study.

Two science teachers participated in the study. One teacher delivered the instructional materials based on analogies and metaphors. A total of 110 students were exposed to these materials. The instruction was delivered during two days during consecutive one hour classes. The first day of instruction was conducted by the teacher explaining the concepts and the students participated in the discussion. Four analogies were presented together with the scale metaphor. The teacher scaffolded each of the four analogies and students participated by answering the verbal cues the teacher presented. With each and every analogy the teacher emphasized the relative sizes of the objects and students were elicited to say how many times an object was smaller or bigger than the other. For the second day of the instruction students worked in small group teams of approximately 5 students each in creating their own analogies. While working in teams, the instructor would join their conversations to provide feedback if required. After interacting for about 20 minutes, the full class reassembled and each team shared their work with the rest of the class. They presented the results by each member of the team pointing in the scale projected on the blackboard, different positions of their set of objects that composed their proportional analogy. Immediately after that, the instrument was administered. The researchers carried out the corresponding analysis and evaluation.

The other science class did not received additional formal training other than that delivered as part of the common instruction based on measurements and powers of ten. In addition, these 44 students were exposed to the informal learning environment; namely a museum exhibit focused on teaching nanotechnology related concepts.

Analysis and Results

Descriptive statistics were used to describe and summarize the data. In addition, patterns were used to identify students' strategies for solving the given tasks. The results of the study are summarized in Figure 2:

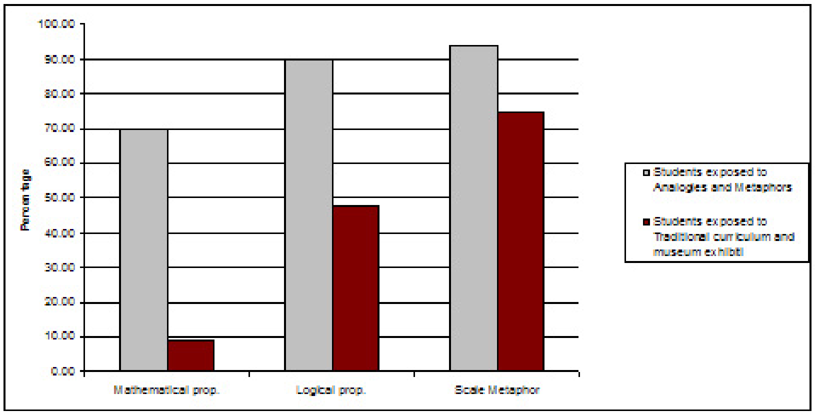


Figure 2. Summarized results

Scale metaphor

From the group of students who were not exposed to the instructional materials based on analogies and metaphors, 75% were able to read and understand the metaphor of the logarithmic scale, and were able to successfully locate the object in the correct position. In comparison, 94% of the students exposed to instruction based on analogies and metaphors were able to correctly read the scale and locate the object successfully.

From these results we suggest that a logarithmic scale may serve as a tool to convey scale related concepts. For this study, students did not have to devote cognitive load in order to read and use the scale.

Logical proportional conception of size

From the group of students who were exposed to the traditional curriculum and the museum exhibit, 48% of the students were able to identify the proportional logical relationship between the human and ant and the bacteria and the DNA double strand. This 48% was further divided into those who were able to correctly map the size of the human as the size of the bacteria and the size of the ant as size of the DNA double strand (75%), and those who although were able to identify the proportional relationship of sizes between the objects, were not able to make the correct mapping (23%). In contrast, 90% of the students exposed to the instructional materials based on analogies and metaphors were able to correctly identify the proportional relationship between the pairs of objects and were able to correctly map the size of the human as the size of the bacteria and the size of the ant as size of the DNA double strand.

Mathematical proportional conception of scale

For this item to be considered correct, students had to recognize that the difference in sizes between the pairs of objects was approximately 1000 times bigger or smaller accordingly. From the group of students who were not exposed to analogies and metaphors we can see that only 9% of the students were able to identify the mathematical proportional relationships between the two pairs of objects. In comparison, 70% of students who were exposed to the instruction in analogies and metaphors correctly identified this relationship.

Results suggest that there is a clear level of difficulty between the logical and the mathematical proportional relationship of size and scale among all students, but with an adequate scaffold, in this case the analogies and metaphors; we can leverage student's understanding of these abstract concepts.

Conclusions and Recommendations

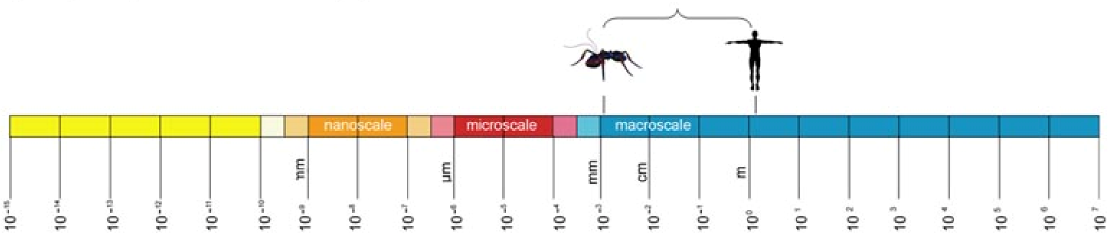
In the final analysis, we conclude that scaling related concepts have been identified as one of the important unifying topics in science, engineering, and technology education. We have also identified that today's middle school students do not demonstrate the adequate understanding of concepts of scale and size on the micro and the nano level.

We characterized the proportional scale cognition into two components; logical proportional, and mathematical proportional. We have also suggested that proportional reasoning is the cognitive process supporting the proportional conceptions of size and scale. We identified the process of unitizing as the first required cognitive process in order to attain the logical proportional scale cognition and mathematical relationships of ratios as a comparative process for attaining the mathematical proportional conception of scale.

We suggested that classical or conventional analogies facilitated the logical aspect of the proportional schema of scale cognition. These same analogies, together with a scale metaphor, also facilitated the mathematical aspect of the proportional schema of scale cognition. We conclude that analogies and metaphors may serve as a scaffold for the learners. Proportional analogies in which two pairs of objects that students were familiar with and visible to the naked eye served as the source, and two other objects that are in the micro, nano, and/or atomic scale served as the target by students mapping the structural similarity of the objects; namely their difference in sizes.

Appendix A

By using the figure answer the following questions:



1. If the diameter of a DNA strand is 1×10^{-9}, where would you locate it on the scale? (Mark the DNA on the scale above)

2. If the difference in size between the ant and the human is similar to the difference in size between the diameter of a DNA strand and a bacteria, where would you locate the bacteria on the scale? (Mark the bacteria on the scale above)

3. What is the difference in length of an ant compared to the height of a human?

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4. What is the difference in diameter of a DNA strand compared to the size of a bacteria?

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5. How did you find the difference between the sizes of these objects?

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References

1. AAAS. (2006). American association for the advancement of science.project 2061. *benchmarks on-line*. Retrieved on September 5, 2006 from: http://www.project2061.org/publications/bsl/online/bolintro.htm.
2. Edu.Inc. (n.d.). *Evaluating museum visitors' readiness for and interest in learning new science.* Retrieved on May 8, 2005 from http://www.nanozone.org/museum.htm.
3. Holladay, C. (2005, July). *A year in review: Internship in nanotechnology museum exhibit design.* Retrieved on May 12, 2006 from http://www.mrsec.wisc.edu/Edetc/IPSE exhibits/about/share.html.
4. Tretter, T., Jones, M., Andre, T., Negishi, A., & Minogue, J. (2006). Conceptual boundaries and distances: students' and experts' concepts of the scale of scientific phenomena. *Journal of research in science teaching*, *43* (3), 282-319.
5. Edu.Inc. (2004, June). *It's a nano world. a study of use. findings from a summative study.* Retrieved on May 10, 2006 from http://eduinc.org/.
6. Edu.Inc. (2005). *Promoting public awareness of research in nanotechnology through informal science education in a science museum exhibition.* Retrieved on May 8, 2005 from http://www.nanozone.org/museum.htm.
7. Jones, M., Andre, T., Kubasko, D., Bokinsky, A., Tretter, T., Negishi, A., et al. (2004). Remote atomic force microscopy of microscopic organisms: Technological innovations for hands-on science with middle and high school students. *Science Education*, *88* (1), 55-71.
8. Waldron, A. (2006, May). Nanotechnology in public. *Nano Today. Retrieved on June 23, 2006 from http://www.nanotoday.com/pdfs nanotoday 02 2006/Opinion-Waldron.pdf*, *1*(2), 56.
9. Rozeboom, W. W. (1966). Scaling theory and the nature of measurement. *Synthese*, 16 (2), 170-233.
10. Person, A. C., Berenson, S. B., & Greenspon, P. J. (2004). The role of number in proportional reasoning: a prospective teacher's understanding. In (Vol. 4, p. 17-24). Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education.
11. Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence.* New York: Basic Books.

12. Delgado, C., Stevens, S. Y., Shin, N., Yunker, M. L., & Krajcik, J. S. (2007, April). The development of students' conceptions of size. In J. S. Krajcik (Ed.), *(chair),emerging science in the classroom: The case of nanoscience and nanotechnology. narst annual conference 2007.*
13. Wiedtke, W. (1990). Measurement. In J. Payne (Ed.), *Mathematics for the young child* (p. 559-588). Reston, VA: NCTM.
14. Lesh, R., Post, T., & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (Vol. 2, p. 93-118). Lawrence Elrbaum Associates. National Council of Teachers of Mathematics.
15. Hart, K. (1988). Ratio and proportion. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (Vol. 2, p. 198-219). Lawrence Elrbaum Associates. National Council of Teachers of Mathematics.
16. Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). Rational-number concepts. In R. Lesh & L. Marsha (Eds.), *Acquisition of mathematics concepts and processes* (p. 91-126). Academic Press.
17. Karplus, R., Pulos, S., & Stage, E. K. (1983). Proportional reasoning of early adolescents. In R. Lesh & L. Marsha (Eds.), *Acquisition of mathematics concepts and processes* (p. 45-90). Academic Press.
18. Singer-Freeman, K. E., & Goswami, U. (2001). Does half a pizza equal half a box of chocolates? proportional matching in an analogy task. *Cognitive Development*, *16* (3), 811-829.
19. Gentner, D., & Markman, A. (1997). Structure mapping in analogy and similarity. *American Psychologist.*, *52* (1), 45-56.
20. English, L. (2004). *Mathematical and analogical reasoning of young learners.* New Jersey: Lawrence Erlbaum and Associates.
21. Gentner, D., Holyoak, K. J., & Kokinov, B. N. (2001). *The analogical mind. perspectives from cognitive science*. Cambridge, Massachussetts. London, England: A Bradford Book The MIT Press.
22. Goswami, U. (2001). Analogical reasoning in children. In D. Gentner, K. J. Holyoak, & B. N. Kokinov (Eds.), *The analogical mind. perspectives from cognitive science* (p. 437-470). Cambridge, Massachussetts. London, England: A Bradford Book The MIT Press.
23. Holyoak, K. J., & Thagard, P. (1997). The analogical mind. *American Psychologist*, 52 (1), 35-44.
24. Mason, L. (2004). Fostering understanding by structural alignment as a route to analogical learning. *Instructional Science*, *32*, 293318.
25. Richland, L. E., Morrison, R. G., & Holyoak, K. J. (2006). Childrens development of analogical reasoning: Insights from scene analogy problems. *Journal of Experimental Child Psychology*, *94*, 249273.
26. English, L. D. (1997). Analogies, metaphors, and images: vehicles for mathematical reasoning. In L. D. English (Ed.), Mathematical reasoning analogies, metaphors, and images.
27. Vosniadou, S. (1995). Analogical reasoning in cognitive development. *Metaphor and symbilic activity*, *10* (4), 297-308.
28. Goswami, U. (1992). *Analogical Reasoning in Children*. Psychology Press
29. SchoolMatters. (2006). *Schoolmatters a service of standard and poor's, a division of the mcgraw-hil l companies, inc.* Retrieved on August 26, 2007 from http://www.schoolmatters.com/app/location/q/stid=15/llid=118/stllid=212/locid=1035261/catid=-1/secid=-1/compid=-1/site=pes.