

The Slope of a Line

For EAPS 10000 001 *Planet Earth*, EAPS 10000 Y01 *Planet Earth online*, EAPS 19100 Y01 *Planet Earth Laboratory online* – L. Braile, April, 2013, revised January, 2017.

Homework and Lab assignments in these courses include graphing exercises that involve a best fit straight line approximation (“**least squares fit**” – calculated or estimated) to a set of data points. From past experience we have found that some students need a refresher lesson on the **slope of a line**. So, here is a short discussion (and some additional resources) for the slope of a line.

For the Homework 2 or 3 (for EAPS 10000 001, Hw 2 or 4 for EAPS 10000 Y01, or several of the EAPS 19100 Y01 labs) assignment, the “best fit” (or approximate-least squares) line through (*for example*), the **age** (**x** axis in millions of years, **m.y.**) versus **distance** (**y** axis in kilometers, **km**) data points approximately fits the linear equation

$$y = a + bx,$$

where, for the EAPS Hw assignments, **y = distance in km**, **x = age in millions of years**, and **a** and **b** are coefficients of the equation (**a** = the *y*-intercept and **b** = the slope of the line). (This equation is equivalent to the other form that you might remember, **y = mx + b**. *In this form, b is the y-intercept and m is the slope.*)

To calculate the **slope (b)**, from the graphed line (for the **y = a + bx** equation), use

$$\text{Slope (b)} = dy/dx \text{ (“rise over run”)} = (y_2 - y_1)/(x_2 - x_1),$$

where (**x₁**, **y₁**) and (**x₂**, **y₂**) are coordinates of **two points on the line**. To obtain the most accurate **slope (b)**, one should use two points **on the line** that are **far apart** from each other. Use the axes numbers and tick marks to determine the coordinates of the points **on the line** and calculate **dy/dx** (**don’t just “count squares” for dy/dx, and don’t use the data point values**). Also, note the units of the slope (from the **dy/dx** calculation – the units of the **y** and **x** axes). In this case the **y axis** is in **km** and the **x axis** is in **m.y.** (millions of years) so the slope (**dy/dx**) has units of **km/m.y.** which is a speed or velocity. The **slope** and the **y** intercept can be negative or positive.

Units of slope are units of **y axis** divided by units of **x axis**.

Example and method to check to see if your equation is correct: For the example shown in the Figure below, the **y = a + bx** equation is **y = 0.7 + 1.33x** (see Figure). **We can check that this is the correct equation** by substituting values of **x** into this equation and seeing if the **x-y** points fall on the least squares line. For example, if **x = 0**, then **y = 0.7** (the **y**-intercept); for **x = 1**, then **y = 2.03**; and for **x = 3**, then **y = 4.69**. It is easy to see that all of these points fall on the least squares line.

The least squares line concisely describes the mathematical relationship between the **x-y** properties measurements or data (in the example below, between relative stride length and dimensionless speed). We can use it to represent the **x-y** data and to calculate an estimate of **y** for any given **x**, or an estimate of **x** for any given **y**.

This predictive capability is powerful. In the Figure below, data are presented for dimensionless (normalized) speed versus relative stride length for measurements made for **several different kinds of animals** (two legged and four legged) **walking or running**. Because the data display a distinct relationship between the **x-y** measurements, and appear to be valid for many animals, we can use it to predict walking and running speed for other animals, such as **dinosaurs** for which are extinct, so we have no other observable

measurements of speed. The specific calculations require information about the dinosaurs such as **leg length** and **stride length** which we do have from fossil bones and fossil footprints. So the straight line relationship, involving the slope of a line, is very useful! (More information on dinosaur tracks from the famous Glen Rose, Texas site can be found at: <http://www.utexas.edu/tmm/exhibits/trackway/>, and http://en.wikipedia.org/wiki/Paluxy_River.)

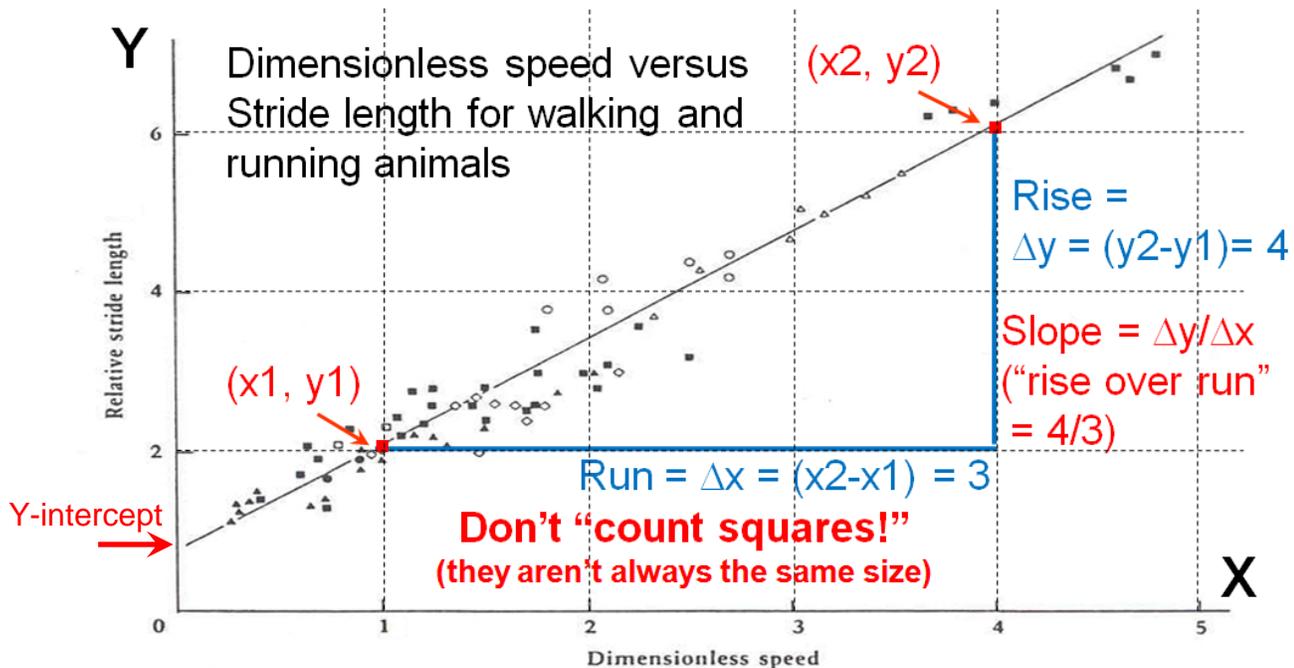


FIGURE 3.10. A graph of relative stride length against dimensionless speed for ostrich: (●), humans (○), dogs (■), elephant (□), rhinoceros (◇), sheep (△), and camels (▲). Data from Alexander 1976 and Alexander and Jayes 1983.

The linear equation ($y = a + bx$; or equivalent forms) and the use of the slope coefficient are very common in applications of science (including social sciences), engineering, and business, so it is useful to understand and review these concepts. Some resources are provided below.

Slope of a line resources (these are generally good, but **many** others just help promote the **misconception** of "counting squares" to determine the slope of a line!):

<http://www.mathopenref.com/coordslope.html> (slope of a line)

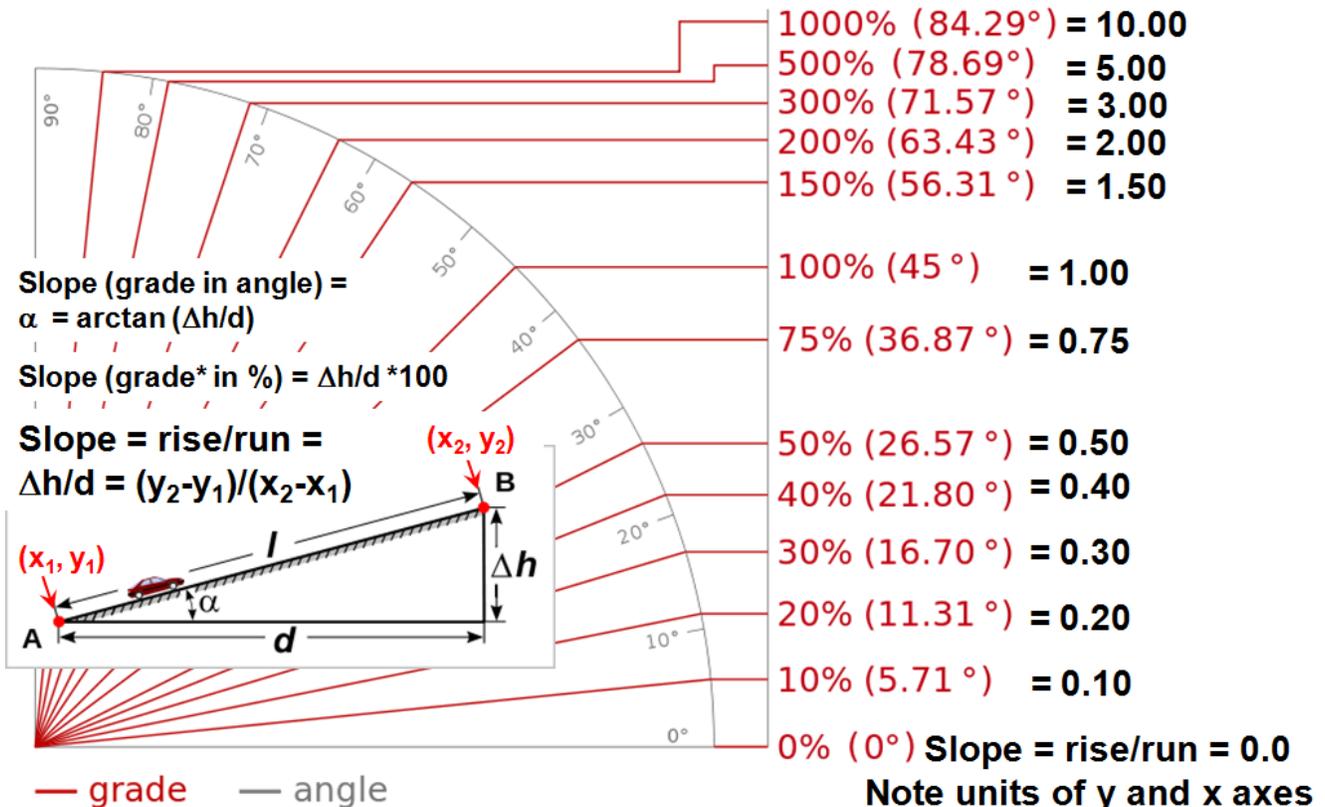
http://math.kendallhunt.com/documents/da2-ca/DA2CA_CL/DA2CA_CL_951_04.pdf

<http://labwrite.ncsu.edu/res/gt/gt-reg-home.html> (with Excel)

http://www.stat.ncsu.edu/people/reiland/courses/st101/chap8_least%20squares.pdf (linear regression/least squared fit)

Relationship between **slope**, **angle of slope**, **slope as a percentage**, **grade**, and **gradient**:

Slope expressed as rise/run, percent, and angle



* Grade is sometimes called **gradient**, such as the gradient of a river and can be expressed as **X** meters/km meaning that the river elevation decreases by **X** meters for each km of distance along a segment of the river. The grade or gradient of a river or a highway usually corresponds to a very small slope (< 0.10 or 5.71 degrees). Adapted from: http://en.wikipedia.org/wiki/Grade_%28slope%29



Example of a highway road sign warning of a “steep” slope ahead. Note that the slope is given as a percent slope. Also note that the slope shown in the illustration (the black right triangle) is exaggerated, as the upper side of the triangle actually shows a slope that is much larger than 10% (see 10% slope angle in diagram above). Most highways have slopes that are 6% or less (grade or slope in percent) which would correspond to a dimensionless slope of 0.06 or less.