Intermediate Microeconomics

IMPERFECT COMPETITION

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**Oligopoly:** A market with few firms but more than one.

**Duopoly:** A market with two firms.

**Cartel:** Several firms collectively acting like a monopolist by charging a common price and dividing the monopoly profit among them.

**Imperfect Competition:** A market in which firms compete but do not erode all profits.
Perfect competition with only 2 firms

Assume the following model:

- 2 firms that produce the same good (call the firms “Kemp’s Milk” and “Dean’s Milk”).
- They both have the same marginal cost (c), and it is constant (so constant AC, too).
- The Demand for their good is downward-sloping, but does not have to have a particular shape.
- The firms simultaneously choose their prices ($P_K$ and $P_D$).
- The firm with the lower price captures the whole market, and the other firm gets nothing.
- If both firms set the same price, they split the market demand evenly.
Bertrand competition

The profit for each firm is given by:

\[ \Pi_D = Q(P_D, P_K)[P_D - c] \] and

\[ \Pi_K = Q(P_D, P_K)[P_K - c]. \]

This model is called the Bertrand model of Duopoly—after 19th Century economist, Joseph Louis Francois Bertrand.

To find the equilibrium, however, we’re not going to use calculus—just reason.
Equilibrium in the Bertrand model

Say that you’re Kemp’s dairy. If Dean’s is charging any price that is greater than your own $c$, then you can choose from among three intervals for your own price:

- $P_K > P_D$,
- $P_K = P_D$,
- $c \leq P_K < P_D$.

If you go with the first one, you get no sales because Dean’s undercut your price.

If you go with the second one, you get:

$$\frac{1}{2} [Q(P_D, P_K)(P_K - c)].$$
Equilibrium in the Bertrand model

If you go with the last one \((c \leq P_K < P_D)\), and charge \(\varepsilon\) less than Dean’s, you get double the sales while only cutting the small value of \(\varepsilon\) from the price.

- This gives you roughly twice the profit of setting a price equal to the competition.

For any price that the competitor hypothetically sets, it is better for your firm to undercut it.

- This is true for both firms in the duopoly.
Bertrand equilibrium

As long as their “price war” doesn’t drop below the marginal cost, both firms can alternately cut their price in an attempt to capture the market.

◦ They will continue doing this mentally until they both declare a price exactly equal to $c$—their marginal cost.

This will give both firms zero profit—the same outcome that results from perfect competition.

$\Pi_D$ and $\Pi_K$ are both zero and $P_D = P_K = c$ in this Bertrand duopoly equilibrium.

Under this set of assumptions, you get the competitive outcome with only 2 firms.
Monopoly outcome with 2 firms

Multiple firms can always attempt to act like a monopoly collectively by figuring out how much a monopolist would produce.

- And dividing that quantity among the members of a cartel.

The problem with this arrangement, as we will demonstrate, is that it is unstable.

All firms in a cartel have the incentive to “cheat” by producing more output than they are supposed to.
Cournot competition

A model with an intermediate result—between perfect competition and monopoly—comes from another 19th Century economist:

- Antoine Augustin Cournot.

The model has the following assumptions:

- 2 firms produce the same good.
- The Market Demand for their good has negative slope.
- Each firm chooses a quantity to maximize its own profits. The market price is determined by the sum of the two firms’ quantities, i.e., $P(q_1, q_2)$.
- Each firm takes the behavior of the other firm(s) as given.
Example

Call the 2 firms, “Johnsonville” and “Klements”.

They produce bratwurst, and the demand for bratwurst is given by: \( Q = 200 - 10P \) with inverse demand:

\[
P = 20 - 0.1Q.
\]

\( Q \) is measured in 1000s of pound of bratwurst.

The market quantity (\( Q \)) is the sum of the two firms quantities: \( Q \equiv q_J + q_K \).

Both firms have the same marginal cost of producing bratwurst of $2 per pound.

\( \text{Marginal cost is constant, i.e., } TC = 2q. \)
Profit functions in Cournot competition

Each firm has a profit function:

\[ \Pi_J = P(q_J, q_K)q_J - 2q_J \] and
\[ \Pi_K = P(q_J, q_K)q_K - 2q_K. \]

The calculus of maximizing each firm’s production function is pretty familiar to you by now; set MR = MC and solve for each firm’s optimal \( q \).

The new aspect of this problem is the **jointly determined price** term in both firms’ profit functions.
Market demand

P ($/unit)

Demand

Q (1000s lbs)
Solving the Cournot model

We basically have to figure out how the market price depends on the quantities chosen by the two firms in the duopoly.

For this we use the inverse demand function and the definition of $Q$:

$$P = 20 - 0.1Q = 20 - 0.1(q_J + q_K).$$

Substitute this into each firm’s profit:

$$\Pi_J = \left(20 - 0.1(q_J + q_K)\right)q_J - 2q_J \text{ and }$$
$$\Pi_K = \left(20 - 0.1(q_J + q_K)\right)q_K - 2q_K.$$
Cournot profit maximization

Each firm maximizes profits by differentiating its profit function and setting the “marginal profit” equal to zero:

\[
\Pi_J = \left(20 - 0.1(q_J + q_K)\right)q_J - 2q_J \quad \text{and} \quad \Pi_K = \left(20 - 0.1(q_J + q_K)\right)q_K - 2q_K.
\]

\[
\frac{\partial \Pi_J}{\partial q_J} = 20 - 0.2q_J - 0.1q_K - 2
\]

\[
\frac{\partial \Pi_K}{\partial q_K} = 20 - 0.2q_K - 0.1q_J - 2
\]

\[
\frac{\partial \Pi_J}{\partial q_J} = 0 \rightarrow 0.2q_J = 18 - 0.1q_K
\]

\[
\frac{\partial \Pi_K}{\partial q_K} = 0 \rightarrow 0.2q_K = 18 - 0.1q_J
\]
Cournot firms’ reaction functions

Each firm now has a function that tells him his own optimal quantity based on the quantity chosen by his competitor.

- Solving for the q’s:

\[ q_J = 90 - 0.5q_K \]
\[ q_K = 90 - 0.5q_J \]

These are called the firms’ reaction functions.
Reaction functions graphed

Guess where equilibrium occurs . . .
Solving for equilibrium

Substitute Klements’ reaction function into Johnsonville’s:

\[ q_J = 90 - 0.5q_K = 90 - 0.5(90 - 0.5q_J) \]

Solving for \( q_J \):

\[ q_J = 90 - 45 + 0.25q_J \rightarrow 0.75q_J = 45 \ldots q_J = 60. \]

Substitute this back into Klements’ reaction function to get \( q_K \):

\[ q_K = 90 - 0.5q_J = 90 - 0.5(60) \rightarrow q_K = 60. \]

- As you may have surmised already, this equilibrium (\( q_K = q_L = 60 \)) corresponds to the intersection of the reaction functions on the graph.
Evaluating equilibrium

One comparison is what would happen under a monopoly (or successful cartel).

- A monopolist would face the whole market demand, therefore having $TR = Q(20 - 0.1Q)$ and $MR = 20 - 0.2Q$.
- Setting $MR = MC$, we have $20 - 0.2Q = 2$
- Solving for $Q$ we have $Q_M = 90$.
- Substituting the solution into the inverse demand function, we would have $P = 20 - \left(\frac{90}{10}\right) = $11.

In the Cournot equilibrium, the quantity in the market is $(60+60)120$, and the price is $P = $8.

- So Cournot competition leads to a lower price with more of the good being produced than a monopolized market.
Comparison with perfect competition

If the market were perfectly competitive, we would expect marginal cost pricing, so $P = MC = $2$ with a quantity of 180 in the market.

- The Cournot quantity is smaller and the price is higher than in perfect competition.

So the Cournot equilibrium is an intermediate between perfect competition and monopoly.
The cartel profit is \( \frac{1}{2} \) the monopoly profit of \( [(11-2)\times90] \).
Comparing Cournot competition to being in a cartel

Assuming both firms stick to the cartel arrangement, the previous table indicates that being in a cartel earns the firms more profit than competing.

- But imagine yourself as one of the firms in the cartel, and recall the reaction function derived earlier. If you expect the other firm to produce the quantity chosen by the cartel, what should you do?
- If you are Klements and you expect Johnsonville to produce 45 units (as decided by your cartel arrangement), your reaction function dictates that you should produce \( q_K = 90 - 0.5(45) \) 67.5 units to maximize your profit.

Producing 67.5 is a betrayal of the cartel, but both firms have the incentive to do this if they expect their competitor to stick to the agreement.
The cartel game

The cartel decision can be formulated as a game in which both “players” decide whether to obey the cartel or to compete “Cournot style”.

Each has a set of strategies that can be formalized as: $q = \{45, 60\}$, with 45 indicating loyalty to the cartel and 60 indicating Cournot.

The combination of strategies chosen by the two firms “players” determines the market price and consequently each firm’s profit.
Cartel game

We have already determined the outcomes when both firms choose the same strategy.

- When only one firm is loyal to the cartel, the market quantity is $(45+60) = 105$, so the price is $9.50.
- The profit for the firm producing 45 is $337.50, and the profit for the firm producing 60 is $450.

All the combinations of strategies and profits can be combined into a table called a payoff matrix.

- For this game the payoff matrix is on the next slide.
## Cartel game payoff matrix

<table>
<thead>
<tr>
<th></th>
<th>Klements Cartel</th>
<th>Klements Cournot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Johnsonville</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cartel</td>
<td>$405, $405</td>
<td>$337.50, $450</td>
</tr>
<tr>
<td>Johnsonville</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cournot</td>
<td>$450, $337.50</td>
<td>$360, $360</td>
</tr>
</tbody>
</table>

Each cell is structured as follows: \{Johnsonville payoff, Klements payoff\}. 
Dominant strategy

**Dominant Strategy:** A strategy that is a player’s best response to all combinations of strategies by other players.

Both firms have a dominant strategy in the Cartel game, and it can be verified by examining the payoff matrix on the previous slide.

Regardless of what the other player does, the best response is to play “Cournot”.
The underlined payoffs indicate each firm’s best action, given what the other player does. Note that the lower right cell has both payoffs underlined.
Nash Equilibrium

Nash Equilibrium: a simultaneous best response by all players.

- Named after economist and mathematician, John Nash—the author of the proof of its existence.

In the Cartel game, the only simultaneous best response by both players consists of both playing “Cournot”.

- This is the lower right cell in the payoff matrix.

This result is supposed to indicate that cartels are inherently unstable.
Repeated cartel games

So cartels seem dead in light of these results, right?

Well, not so fast. We will talk about *uncertainty* and choices over *multiple time periods* later, but consider this a brief introduction.

If the game is repeated an indefinite number of times, it is possible that collusion could still occur if:

- Firms care about their future profits, and
- The final period of the game is uncertain.
Model for repeated cartel game

Say that the game will be repeated for an unknown number of periods.
  ◦ Think of this as the time before the firms’ product becomes obsolete.

Each future period, the probability of the game continuing for another “round” is given by probability (g).

Firms choose strategies consisting of actions in all periods instead of only the present.

What if both players adopt a strategy at the beginning that says, “obey the cartel as long as the other player does, but if the other player cheats, cheat for all remaining periods”? 
  ◦ Is this a Nash equilibrium?
First of all, we ask what the expected payoff for this strategy is when it is played by both players.

- Each player gets $405 this period (the one-period cartel profit), plus $g$ times $405$ in the next period . . . plus $g^2$ times $405$ in the period after that. And so on for an infinite number of potential future periods.

- The expected payoff next period is $g \times 405 + (1 - g) \times 0$, since there is $(1 - g)$ probability of the game ending.

- The further into the future you go with the expectations, the larger is the probability of the game having ended.

The expected payoff is: $\sum_{t=0}^{\infty} (405 \times g^t)$. 
Geometric series

This sum of expected future payoffs has a simpler expression when you find what the series converges to.

It can be shown that the expected payoff when both players play the “trigger strategy” outlined before is:

$$E(\text{payoff}) = \frac{405}{1 - g}.$$ 

If the player compares this payoff to what he would receive by cheating, he can determine if the “trigger strategy” is his best response.
Collusion possible?

The payoff for playing Cournot (and setting off the opponent’s trigger) in the first period is:

\[ E(\text{payoff}_2) = 450 + \sum_{t=1}^{\infty} (337.50 \times g_t) . \]

The 2\textsuperscript{nd} term is another series that converges to:

\[ E(\text{payoff}_2) = 450 + \left( \frac{g}{1 - g} \right) \times 337.50. \]

We can set the two payoffs equal to one another to find the value of \( g \) that will make the players indifferent to these two strategies.

- This will give us a critical value of \( g \) above which collusion is possible in the repeated Cartel game.
Collusion possible

\[ E(\text{payoff}) = E(\text{payoff}_2) \rightarrow 450 + \left[ \frac{g}{1-g} \right] \times 337.50 = \frac{405}{1-g} \]

\[ 450(1-g) + 337.50g = 405 \]

\[ 450 - 450g + 337.50g = 405 \rightarrow 45 = 112.5g \rightarrow g^* = 0.4. \]

If \( g \geq 0.4 \), the payoff of playing the “trigger strategy” is higher than the alternative of cheating on the cartel.

So under the right conditions, the cartel is sustainable.
Summary

It is theoretically possible to get any outcome ranging from monopoly to perfect competition in an imperfectly competitive market.

- The spectrum of Bertrand, Cournot, and Cartel models bears this out.

In single period “games” it is likely that cartels will dissolve as a result of the firms’ individual profit motives.

Under the right conditions, however, a repeated cartel game can result in a sustained cartel.

Imperfect competition models are part of the most active areas of research in Economics.
Conclusion

This is the last lecture that explicitly discusses market structure.

The preceding discussion of market structures and the “theory of the firm” should provide some practical insights into the operation of firms in the economy.

It also complements the “consumer theory” covered earlier and gives a cohesive picture of “supply and demand”.

The remaining topics we will cover are extensions of these basic theories of production, market allocation, and consumption.