Time

So far we have modeled consumers’ choices between multiple goods in the same time period: a “static” optimization problem.

Consumers also make choices about consumption in different time periods.

- In order to model this, we abstract away from the multiple goods model,
- instead considering “baskets” of consumption in different periods as the goods that give consumers utility.

In this model, time is indexed into discrete units, and we consider a two-period model that enables us to graph the utility function in 2 dimensional space.

- Some intuition behind this is to think of the first period as the consumer’s working years and the second period as retirement.
Consumers’ utility over time

In this “dynamic” model, the consumer has a utility function like the following:

\[ U(C_t) = \log c_0 + \log c_1. \]

- Question: Is this consumer risk averse? Why?

Keeping with the “working, retirement” characterization of the model, say that the consumer earns all his income in the first period \((t = 0)\). Denote this income \((Y)\).

- “t” is the index for time. When used as a subscript, it denotes the period in which the consumption occurs.

The consumer may spend as much of his income as he likes in period zero, but he must keep in mind the need to save some for period 1.
Good news

Firms are willing to pay this person in exchange for letting them use some of his income to purchase or rent capital.

- I.e., firms are willing to borrow from consumers and pay interest on the loans.
- The rate of interest is competitive. Firms will not borrow if the rate of interest is more than the marginal product of capital (they would incur losses by doing so).

Remember the rental rate of capital ("v" from our discussion of firms’ costs)? That’s the same rental rate we refer to here. This is the real interest rate that consumers get on their savings.

- Real refers to “adjusted for inflation”. For more on this, take ECON 352 or ECON 380.
Savings

First a slight change in notation: I know we used “v” for rental rate in the costs lesson, but for the consumer’s return on savings, the letter “r” is used.

- Sorry if this is confusing, but they basically refer to two different sides of the same coin.

When a consumer spends $C_0$ of his income in period zero, he saves the remainder,

$$(Y - C_0).$$

He gets to keep this amount for the next period plus he earns $[r(Y - C_0)]$ in real interest.

So the amount the consumer has at his disposal in the next period is: $(Y - C_0) + r(Y - C_0)$. 
Lifetime budget constraint

Since there is no point in saving income beyond period 1 (the individual will “die” and can’t take wealth with him), he will consume all his remaining income and interest in the last period of his life.

- His $C_1 = (Y - C_0)(1 + r)$.
- The consumer is constrained by this equation. When you rearrange terms to solve for $Y$, it appears the same as a regular budget constraint:

$$Y(1 + r) = C_1 + C_0(1 + r) \rightarrow Y = \frac{C_1}{1 + r} + C_0.$$  

- When $C_0 = 0$, the consumer saves all income, so $C_1$ is $Y(1 + r)$.
- When $C_1 = 0$, he saves nothing, so $C_0 = Y$. 

$C_0 = 0$ and $C_1 = 0$ give us the intercepts for the budget constraint.
Slope of budget constraint

Just like in a static problem, the slope of the budget line is:
\[
\frac{\Delta C_1}{\Delta C_0}.
\]

In this case the “price ratio” is \( \frac{1+r}{1} \) and the slope of the constraint is (negative) \((1 + r)\).

As long as \( r \) is positive (it is never negative), the “price” of \( C_0 \) is always greater than the “price” of \( C_1 \) because of the foregone interest.

- The quantity \( \frac{1+r}{1} \) is greater than 1 if \( r > 0 \).
Intertemporal optimization

All we need to show the optimal lifetime consumption profile is an indifference curve from the utility function.

- Again the consumer selects a combination of $C_0$ and $C_1$ that gets him on the “highest” (furthest from the origin) indifference curve possible without exceeding his budget.
- At this optimum, the marginal rate of intertemporal substitution will equal the price ratio, $(1 + r)$.

MRS is the ratio of marginal utilities $\frac{MU(C_0)}{MU(C_1)}$.

The tangency of these two is shown on the next slide.
Optimal consumption
Temporal preferences

Would you expect a consumer to value consumption today and tomorrow . . .

- equally?
- tomorrow more?
- today more?

Generally people prefer to consume sooner rather than later.

- This is sometimes called the “primacy of the present”.
- It means that if you could choose between having $10 today and $10 next week, you prefer today.
Temporal discounting

How much would we have to offer you next week to make you indifferent to the choice between today and next week?

- The answer to this question determines your temporal discount rate.
- If you would be indifferent to $10 today and $11 next week, your temporal discount rate ("d") is 0.1. "d" is the rate that makes the following relationship hold with equality:

\[ C_0 = \frac{C_1}{1 + d} \]

If \( C_0 = 10 \) and \( C_1 = 11 \), this implies \( (1 + d) = \frac{11}{10} \), so \( d = 0.1 \).
Temporal discounting

Individuals exercise preferences over their intertemporal consumption by valuing utility in future periods less than utility in the present period.

- Their utility function might look like the following:
  \[ U(C_t) = \log C_0 + \left[ \frac{1}{1 + d} \right] \log C_1. \]

- We say that utility in period 1 is “discounted” by the factor \( \frac{1}{1+d} \) for one period.

Basically an individual’s “d” is a measure of how impatient he is about consumption.
Discounting over “t” periods

This is only a two period model, so nothing really get discounted or compounded for more than 1 period.

On a longer time horizon, the procedure still works; you just have to discount multiple times.

◦ If your temporal discount rate is 0.1, what is $20 2 periods from now worth to you today?

\[
V_{today} = 20 \left[ \frac{1}{1 + 0.1} \right] \left[ \frac{1}{1 + 0.1} \right] = 20 \left[ \frac{1}{1 + 0.1} \right]^2.
\]

◦ This is \( \approx \$16.53 \).
Discounting over “t” periods

Generally, \( V_{today} = V_{(today+t)} \left[ \frac{1}{1+d} \right]^t \).

Notice that it goes the other way, too.

- If you have $1 today and it earns a rate of interest “r” each period (and you don’t withdraw from the account), the value of the account \( t \) periods from now is:
  \[
  V_{(today+t)} = 1(1 + r)^t.
  \]
- Instead of “discounting”, this is called “compounding” when you go forward in time.
Temporal preferences and rationality

Temporal discounting can explain many rational behaviors that seem irrational when considered from the standpoint of temporal “indifference”.

Example

Some would characterize *tobacco use* as irrational when the user has been abundantly informed of its negative health consequences.

- Performing the “marginal benefit > marginal cost” analysis, it seems unthinkable that the benefit could outweigh these health costs.
- But when you consider a person’s temporal discounting, maybe it makes sense after all. The health “costs” are almost always borne in the distant future.
- If the consumer doesn’t care very much about the future (i.e., discounts it at a high rate), it is rational for him to use tobacco.

Another example is consumers’ failures to save enough and propensities to borrow too much.

- They’re not doing it because they’re *irrational*, they might just be *impatient*. 
Effect of changing $r$

What happens when you arbitrarily raise the return ($r$) that consumers receive on their savings?

- Just like in our other consumer problems, there is an **income** and a **substitution effect** on the consumer’s optimal consumption.
- Consumption in period zero is now relatively more “expensive” because you are foregoing a higher rate of interest in period 1. So the substitution effect is negative on $C_0$ (positive on $C_1$).
- But the higher income conceptually increases the lifetime income, and this effect is positive on both $C_0$ and $C_1$ since both are **normal goods**.
An increase in \( r \)

\( C_0 \) decreases when the interest rate increases, so savings increase.
Saving and investment

When consumers decrease their present consumption and save more, it is effectively an increase in the supply of loans.

- Each increase in $r$ increases saving and increases the amount consumers are willing to loan.
- This is the origin of the Supply Curve in the market for loans.
- It can also be thought of as the Supply of Capital or Gross Investment (as a function of $r$).
Supply of capital
Demand for loans

When consumers save, their savings get turned into investment (capital) for firms to use in production.

- There is usually some Financial Intermediation that occurs by a bank or brokerage in between, but ultimately the savings of households end up as capital for firms.

Firms are demanders of capital in this framework. Their Demand for Loans depends on their decisions to maximize profits.
Derived demand for capital

Let’s return briefly to the Cobb-Douglas production function used earlier:

\[ q(K, L) = K^{2/3}L^{1/3}. \]

And the profit function:

\[ \Pi = Pq - (\text{wage})L - (\text{rental rate})K. \]

Substitute the production function into the profit function:

\[ \Pi = PK^{2/3}L^{1/3} - wL - rK. \]

Choosing K and L such that the marginal profit equals zero:

\[ \frac{\partial \Pi}{\partial L} = 0 \rightarrow P \left( \frac{1}{3} \right) \left( \frac{K}{L} \right)^{2/3} = w, \text{ and} \]

\[ \frac{\partial \Pi}{\partial K} = 0 \rightarrow P \left( \frac{2}{3} \right) \left( \frac{K}{L} \right)^{-1/3} = r. \]
Derived demand for capital (2)

Re-writing the profit maximization conditions in terms of $K$ and $L$:

\[
\left( \frac{P}{3w} \right)^2 K^3 = L^3, \text{ and }
\left( \frac{2P}{3r} \right)^{\frac{1}{3}} L^3 = K^\frac{1}{3}.
\]

\[
L = K \left( \frac{P}{3w} \right)^{\frac{3}{2}} \text{ and } K = L \left( \frac{2P}{3r} \right)^{\frac{3}{2}}.
\]

This is the factor demand for Labor and Capital.
Derived demand for capital (3)

Notice that the partial derivative,

\[
\frac{\partial K}{\partial r} = -3L \left( \frac{2P}{3r} \right)^3 \left( \frac{1}{r} \right) = -L \left( \frac{2P}{r} \right)^3 \left( \frac{1}{r} \right),
\]

is negative.

- The Quantity of Capital Demanded for capital is negatively related to the price of capital ("r").
Demand for capital
Equilibrium

The equilibrium quantity and rental rate are determined by the intersection of these supply and demand curves.

- A similar characterization can be made of the Labor market by deriving Labor Demand from firms and Labor Supply from the Consumer’s trade-off between Leisure and Consumption.
Summary

Consumers’ choices can be modeled as intertemporal by writing their utility functions as sums of utility in different time periods.

- Their choices of consumption in each period determine the amount they will save out of their incomes.
- The consumers’ savings turn into capital when they are made into loans that firms use to acquire capital.
- The rental rate of capital becomes the consumer’s real rate of return on savings.
Summary

Consumers typically exercise intertemporal preferences over their consumption by discounting future utility.

- Intertemporal preferences explain some decisions that would otherwise seem irrational.
- Increasing $r$ has a substitution effect on consumers’ choices, leading them to save more. This creates the upward-sloping supply curve in the capital market.
- Firms’ demands for capital depend on the marginal product of capital (which is diminishing), so it explains their downward-sloping demand for capital.
Conclusion

Intertemporal choice is a convenient place to conclude, considering the next class in the sequence at Purdue. ECON 352 is Intermediate Macroeconomics.

One of the subjects you ought to study in that class is the process by which consumers’ choices lead to investment and economic growth.

- Such Macroeconomic models are said to have “Micro Foundations” because they rely on optimization by individuals.

Microeconomics is instrumental in all the applied fields in Economics—including Macro—so the lessons learned in this class should be extremely valuable to your success in future classes.