Intermediate Microeconomics

DEMAND

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Demand

Demand Function: A representation of how quantity demanded depends on prices, income, and preferences.

Our objective in this chapter is to derive a demand function from the consumer’s maximization problem.

- It will have the form:

  \[ Q(P_j, M) \text{ where } P_j \text{ are the relevant prices and } M \text{ is income.} \]
Utility maximization

In ECON 340 we have already discussed how consumers maximize utility subject to a budget constraint.

- They get on the “highest” indifference curve possible and consume a bundle of goods where their Marginal Rate of Substitution equals the Price Ratio of the two goods.

Deriving the consumer’s demand curve for a good is only a small step beyond this principle.
The consumer’s problem

Recall that the consumer maximizes a utility function, such as the following:

\[ U(X,Y) = X^\alpha Y^{1-\alpha}. \]

subject to a budget constraint such as:

\[ M = XP_X + YP_Y. \]

The consumer has marginal utilities:

\[ MU_X = \alpha X^{\alpha-1} Y^{1-\alpha} \quad \text{and} \quad MU_Y = (1 - \alpha)X^\alpha Y^{-\alpha}. \]

So his MRS is:

\[ \frac{MU_X}{MU_Y} = \left[ \frac{\alpha}{1 - \alpha} \right] \cdot X^{\alpha-1} Y^{1-\alpha} \div X^\alpha Y^{-\alpha}. \]
The consumer’s problem

Recall that when you divide by a term like $X^\alpha$, it’s the same thing as subtracting the exponents:

$$\frac{MU_X}{MU_Y} = \left[ \frac{\alpha}{1 - \alpha} \right] \cdot \frac{(X^{\alpha-1}Y^{1-\alpha})}{X^\alpha Y^{1-\alpha}} = \left[ \frac{\alpha}{1 - \alpha} \right] \cdot \frac{(X^{-1}Y^1)}{1}$$

$$= \left[ \frac{\alpha}{1 - \alpha} \right] \cdot \frac{Y}{X}.$$
MRS = price ratio

Setting \( \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y} \):

\[
\left[ \frac{\alpha}{1 - \alpha} \right] \frac{Y}{X} = \frac{P_X}{P_Y}
\]

- A little piece of trivia. If you want an intuitive way of understanding this, the utility maximization condition can be written,

\[
\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}.
\]

By this expression it’s like saying that the consumer’s marginal utility per dollar is the same for all goods.
The optimal quantity of Y

If you write the expression from the previous slide with “Y” by itself on one side, you get:

\[ Y = \frac{(1 - \alpha)XP_X}{\alpha P_Y}. \]

This tells you the utility-maximizing level of Y the agent should consume as a function of the price ratio and how much X is consumed.

Substituting this into the budget constraint will enable you to write the constraint as a function only of X.
Constraint with optimal Y

From the budget constraint we have:

\[ M = XP_X + YP_Y. \]

With optimal Y:

\[ Y = \frac{(1 - \alpha)XP_X}{\alpha P_Y}. \]

Combining the two, we get:

\[ M = XP_X + \frac{(1 - \alpha)XP_X}{\alpha P_Y} P_Y \]

Simplifying and collecting like terms:

\[ M = XP_X \left(1 + \frac{(1 - \alpha)}{\alpha}\right) = XP_X \left(\frac{1}{\alpha}\right) \]
Demand function

Solving for \( X = \frac{\alpha M}{P_X} \)

is called the Marshallian Demand Function for good X. As promised it delivers quantity demanded of the good as a function of prices, preferences, and income. You can even verify that it is downward-sloping as you would expect from the Law of Demand:

\[
\frac{\partial X}{\partial P_X} = -\frac{\alpha M}{P_X^2} < 0.
\]
Note that the function on the previous slide gives quantity as a function of price—which makes sense considering how we think people make decisions. For example, you might go to the grocery store after checking your bank account balance, look at the relative price of applesauce and decide how much to buy on the basis of its price and your budget.

Since your first week in ECON 251, however, you have seen Demand curves graphed with price on the vertical axis and quantity on the horizontal axis.

Considering that the independent variable is supposed to go on the horizontal axis and not t’other way around, why does Economics always graph supply and demand this way?
A typical demand curve

What we are used to . . . but shouldn’t the axes be reversed?
Answer

Because Alfred Marshall graphed it this way a long time ago, and old habits die hard.

If you want to graph demand on Marshall’s axes, you have to take its inverse, which means you have to change it into a function that gives price as a function of quantity.

For example instead of \( X = \frac{\alpha M}{p_X} \), you would re-write it as:

\[
P_X = \frac{\alpha M}{X},
\]

which conceptually is downright silly, but that’s just the way things are done.
Significance of $\alpha$

Note that the Marshallian Demand function can be written:

$$XP_X = \alpha M.$$ 

So the total expenditure on good X equals $\alpha M$. Since M is income, $\alpha$ is the proportion of income that the consumer spends on good X.

Note that $\alpha$ is a constant. This means that the consumer spends a fixed proportion of income on good X.

Exercise: derive the demand function for good Y and verify that the consumer spends the remaining fraction of income $(1 - \alpha)$ on good Y.
Demand and utility relationship

The form of the demand curve depends highly on the form of the utility function.

◦ The utility function that produced the demand function \( X = \alpha M/P_x \) was \( U = X^{\alpha}Y^{1-\alpha} \).

This form is called a **Cobb-Douglas** utility function.

◦ It is part of a larger category called **Constant Elasticity of Substitution (CES)** utility functions.

◦ Recall from 103 that **Elasticity** is the ratio of two variables’ percentage change. E.g., price elasticity of demand is \( \% \Delta Q \div \% \Delta P \).

◦ **Elasticity of substitution**, denoted \( \sigma \), is \( \% \Delta Y \div \% \Delta X \) or, \( \Delta Y/Y \div \Delta X/X \).
Elasticity of substitution

Elasticity is the ratio of two variables’ % changes.
E.g., price elasticity of demand is \( \% \Delta Q \div \% \Delta P \).

Elasticity of substitution, denoted \( \sigma \), is \( \frac{\% \Delta \left( \frac{X}{Y} \right)}{\% \Delta MRS} \) or,

\[
\sigma = \frac{\delta \left( \frac{X}{Y} \right) MRS}{\delta MRS \left( \frac{X}{Y} \right)}.
\]

- It’s about measuring how fast the slope of the indifference curve diminishes as you move down it (the ratio of \( \frac{X}{Y} \) increases).
- Higher \( \sigma \) means MRS doesn’t diminish and the goods are more substitutable.
“Constant elasticity of substitution”

Why is the utility function called this?

Recall that the MRS for the example in this lecture was:

\[ MRS = \left[ \frac{\alpha}{1-\alpha} \right] \left( \frac{Y}{X} \right). \]

The elasticity of substitution is:

\[
\frac{1 - \alpha}{\alpha} \left[ \frac{\alpha}{1-\alpha} \right] \left( \frac{Y}{X} \right) = 1.
\]

1 is obviously a constant for Cobb-Douglas utility, but it’s also a constant for anything ranging from:

- perfect complements \( \left( \frac{X}{Y} \right) \) never changes so \( \sigma = 0 \) to
- Perfect substitutes \( (MRS \) never changes so \( \sigma \to \infty \)).
Cobb-Douglas is one of the easiest CES utility functions to work with.

- Others involve more complicated calculus that we will not discuss in an undergraduate class.

For the Cobb-Douglas utility example here, the price elasticity of demand is also constant:

\[
\frac{\Delta X}{\Delta P_X} = -\frac{\alpha M}{P_X^2} \ \text{times} \ \frac{P_X}{X}
\]

is:

\[
\varepsilon_P = -\frac{\alpha M}{XP_X}.
\]

\[
\varepsilon_P = -1 \text{ (substituting } X = \frac{\alpha M}{P_X} \text{ from the demand function),}
\]

so the price elasticity is constant (-1).
Income and substitution effects

Graphically the demand curve is depicted beginning with the indifference curve map with which we are already familiar (see next slide).

For a consumer to maximize his utility, he finds a consumption bundle where the indifference curve is tangent to the budget constraint.

We want to analyze the effects of a price change beginning from this state.
Optimal consumption

Utility = $U_0$
Variation in price of JBeans

Recall that the horizontal intercept of the budget constraint is

\[ X = \frac{M}{P_X}. \]

If the consumer’s income is $24, consider the different budget constraints that correspond to prices of $2, $3, $4, and $6.

They will all have the same vertical intercept, but will have 4 different horizontal intercepts.
Budget constraints with different prices
Optimal response to price changes

Each time the price of JBeans changes, the consumer realizes a different optimal consumption and different utility level.

The quantity of JBeans that the consumer buys increases as the price of them goes down as Law of Demand predicts.

But the change in JBeans can be decomposed into a substitution effect and an income effect.
Optimal consumption bundles

The budget constraint rotates outward as $P_X$ decreases.
Demand decomposition

For graphical simplicity, let’s just examine the change in price from $6 to $2.

The substitution effect is the change in X that results from moving along the original indifference curve to the new price ratio.

The income effect is the change in X that results from moving consumption outward to the new budget constraint.
Substitution effect
Income effect
Substitution and income effects together
Normal and inferior Goods

**Normal Good**: A good that is bought in greater quantities as income increases.

**Inferior Good**: A good that is bought in smaller quantities as income increases.

- **Examples**: Ramen noodles, public transportation, Roundy’s brand whiskey, et al.

Whether a good is normal or not is measured by its **income elasticity of demand** \((\varepsilon_M)\).

\[ \varepsilon_M > 0 \] indicates a normal good; \[ \varepsilon_M < 0 \] indicates an inferior good.
Substitutes and complements

**Substitutes:** Two goods such that, if the price of one increases, the quantity demanded of the other good rises.

**Complements:** Two goods such that, if the price of one increases, the quantity demanded of the other good decreases.

Complementarity and Substitutability are measured by the cross price elasticity of demand ($\varepsilon_{\text{Price of other Good}}$).

- If it is positive, the two goods are substitutes. If it is negative, the two goods are complements.
Consumer’s surplus

When consumers participate in markets, they are “price takers” in the sense that their individual choices do not affect the market price of goods.
  ◦ The market determines a price—which the individual takes as given.

The consumer’s welfare is measured by the area below his demand curve and above the market price.
Consumer’s surplus
Market demand

After deriving an individual consumer’s demand function, it is only a small step to aggregate their demands.

The market demand is merely the summation of the individual consumers’ demand functions.

Example: there are 3 consumers with demand functions:

1. \( X_1 = \frac{3}{P_X} \)
2. \( X_2 = \frac{2}{P_X} \)
3. \( X_3 = \frac{4}{P_X} \)

The market demand \((X)\) is \( X_1 + X_2 + X_3 \).

\[
X = \frac{3 + 2 + 4}{P_X} = \frac{9}{P_X}.
\]
Summary

The consumer’s utility maximization problem is the origin of the demand function.

- A demand function associates the price of a good, the consumer’s income, and his preferences to the quantity of the good he consumes.

The shape of the demand curve depends on the utility function.
Summary

The price elasticity of demand measures the responsiveness of quantity demanded to a change in the good’s relative price.

The effect of a price change on quantity demanded can be decomposed into a substitution effect and an income effect.

A consumer’s welfare can be measured by his consumer’s surplus—the area below his demand curve and above the equilibrium price.

The market demand is the sum of the individual consumers’ demands.
Applying utility and prices

To illustrate the basic functioning of an economy, the next topic we discuss is the simplest conceivable economy:

- 2 Goods, 2 Consumers
- The “Desert Island” exchange economy.

We will analyze this economy by using an “Edgeworth Box”, preferences, and the price ratio.

- Our goal is illustrate an equilibrium and the concept of economic efficiency.