Intermediate Microeconomics

PRODUCTION

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Definitions

Production: turning inputs into outputs.

Production Function: A mathematical function that relates inputs to outputs.

- e.g., $q = f(K, L)$, is a production function that maps the quantities of inputs capital (K) and labor (L) to a unique quantity of output.

Firm: An entity that transforms inputs into outputs, i.e., engages in production.

Marginal Product: The additional output that can be produced by adding one more unit of a particular input while holding all other inputs constant.

- e.g., $\frac{\partial q}{\partial K}$, is the marginal product of capital.

Two input production

Graphing a function of more than 2 variables is impossible on a 2-dimensional surface.

- Even graphing a function of 2 variables presents challenges—which led us to hold one variable constant.

In reality we expect the firm's production function to have <u>many</u> inputs (arguments). Recall, if you have ever done so, doing inventory at the place you work: lots of things to count.

- Accounting for all of this complexity in an Economic model would be impossible and impossible to make sense of.

So the inputs are grouped into two categories: human (labor) and non-human (capital). This enables us to use the Cartesian plane to graph production functions.

Graphing a 2 input production function

There are two ways of graphing a production function of two inputs.

1. Holding one input fixed and putting quantity on the vertical axis. E.g.,

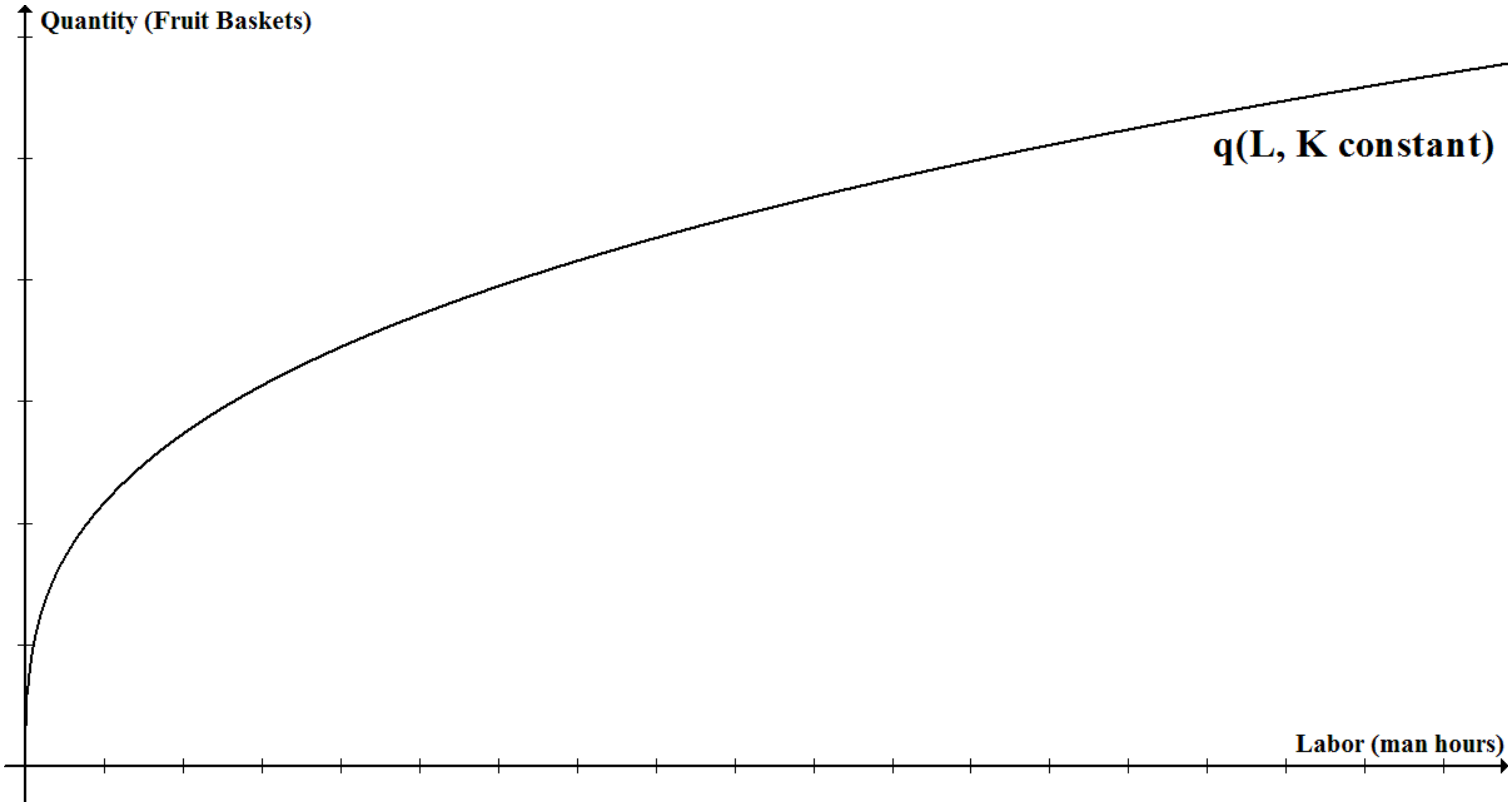
$$q = f(L, K\ constant).$$

2. Allowing both inputs to vary and showing quantity as a level set.
   - This is analogous to an indifference curve from utility theory.

Consider the production function,

$$q(L, K) = K^{\frac{2}{3}}L^{\frac{1}{3}}.$$

First way



The short run

Graphing the production function with capital fixed is a characteristic of the firm's problem in the *short run*

- a scenario in which they only have to choose how much labor to employ when choosing an output level.
- The rationale for this is that it takes time to adjust the amount of capital the firm rents, i.e., its factory, office, and machinery.
- The idea is that it is comparatively easy to change the number of workers the firm employs; think of calling up a temp agency to add staff.

We sometimes say that, "in the short run, the firm takes its level of capital as given," i.e., constant.

Marginal product of labor

Look at the graph of the short run production function again. Its slope, given the labels of the axes, would have the form,

$$\frac{\Delta quantity}{\Delta labor}$$

Mathematically we could find this using differentiation:
For a given level of capital, say 8 units,

$$q = 8^{\frac{2}{3}} L^{\frac{1}{3}}.$$

Taking the partial derivative with respect to labor:

$$\frac{\partial q}{\partial L} = \left(\frac{1}{3}\right) 8^{\frac{2}{3}} L^{-\frac{2}{3}}.$$

This is the marginal product of labor (MP_L).

Diminishing marginal product

If $MP_L = \left(\frac{1}{3}\right)\, 8^{\frac{2}{3}} L^{-\frac{2}{3}}$ (from previous slide), and you choose values L_0 and L_1 such that $L_1 > L_0$, the marginal product at L_1 should be less than the marginal product at L_0.

If $L_0 = 2$ and $L_1 = 7$ and noting that $8^{\frac{2}{3}} = 4$,

$$MP_L(2) = \left(\frac{4}{3}\right) 2^{-\frac{2}{3}} = 0.84 \text{ and}$$

$$MP_L\,(7) = \left(\frac{4}{3}\right) 7^{-\frac{2}{3}} = 0.36.$$

This confirms our suspicion of diminishing marginal product of labor.

Exercise: find the marginal product of capital and confirm that it has the same property.

Graphical confirmation

Looking carefully at the graph of the short run production function, you should be able to tell that the slope of the curve gets flatter (diminishes) as you trace from left to right.

This should further confirm that the production has diminishing returns to its inputs.

Some intuition

You can verify mathematically or graphically that the <u>marginal product</u> of any input is <u>never negative</u>, <u>but</u> as you employ more and more if it, the marginal product <u>diminishes toward zero</u>.

Remember that as you increase, say, labor, you're making the workers use the same amount of tools and machines as before.

Gradually the work space becomes crowded and the workers are basically "maxing out" the capacity of the fruit basket "assembly line". Adding more workers will not enable them to make more. This is why MP diminishes to zero.

The 2nd way

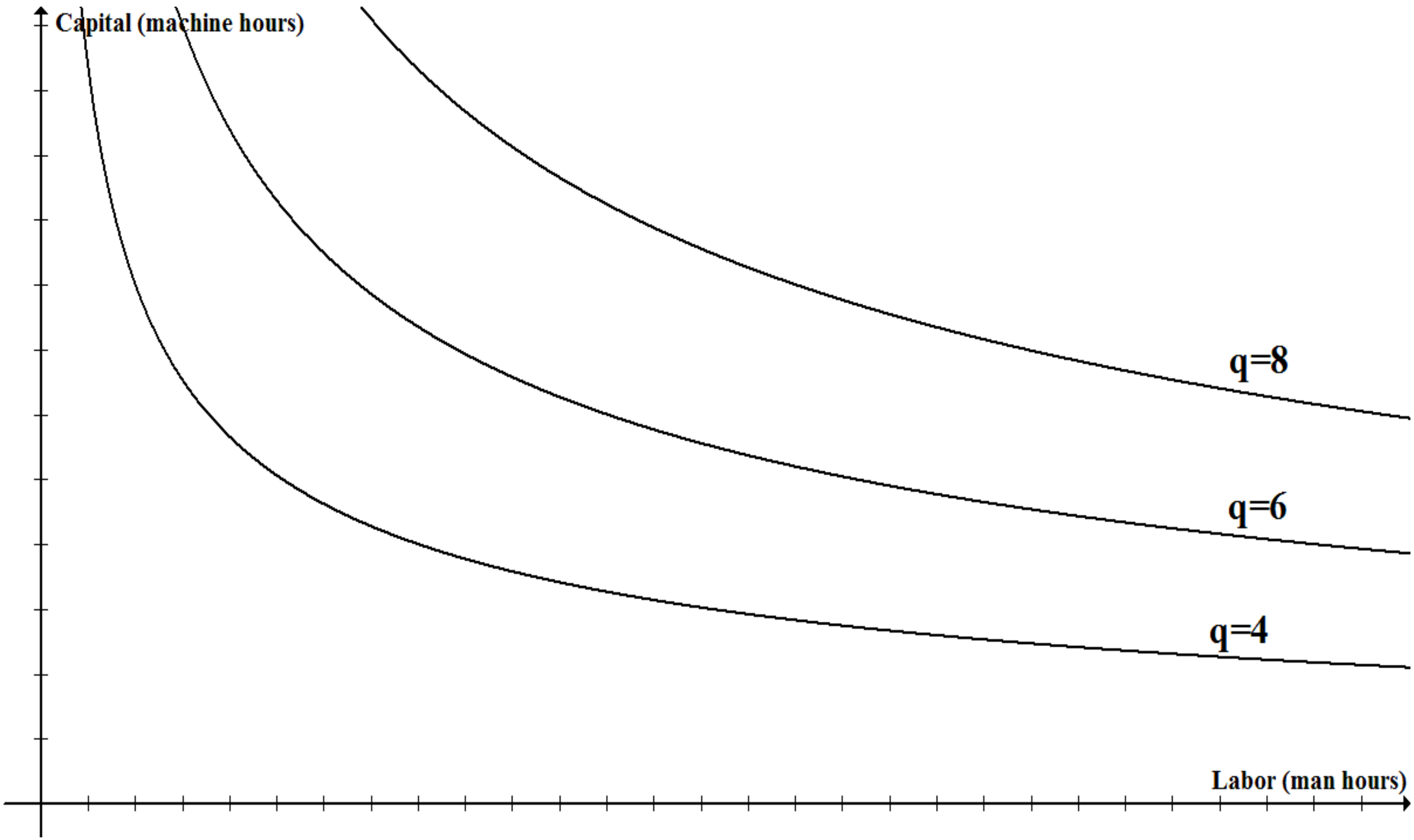
To graph the production function in the long run (where both inputs are variable), we must conceptually hold quantity fixed.

The next slide shows labor on the horizontal axis with capital on the vertical.

The curves are called isoquants, meaning sets of input combinations that produce the same quantity of output.

Specifically I have graphed $q \in \{4,6,8\}$.

An isoquant map



Significance of slope of an isoquant

The slope of any function graphed in L,K space will have a form, $\frac{\Delta K}{\Delta L}$.

The slope of an isoquant is the ratio of the marginal product of labor to the marginal product of capital.

This ratio is called the Rate of Technical Substitution (RTS).

Diminishing RTS

Arbitrarily choose two points on the same isoquant: (L_0, K_0) and (L_1, K_1), such that $L_1 > L_0$.

- Which point will have higher RTS?

At (L_0, K_0) the firm is using a relatively low amount of labor and relatively high amount of capital.

At (L_1, K_1) the firm is using a relatively high amount of labor and relatively low amount of capital.

Recall that the RTS is the ratio, $MP_L \div MP_K$ and that each factor has diminishing marginal product.

Diminishing RTS

For the input combination with a lot of capital and small amount of labor, the MP_L will be large and the MP_K will be small. So the ratio $(MP_L : MP_K)$ will be large and RTS will be large.

For the input combination with a lot of labor and small amount of capital, the MP_L will be small and the MP_K will be large. So the ratio $(MP_L : MP_K)$ will be small and RTS will be large.

- $RTS(L_0, K_0) > RTS(L_1, K_1)$.

You can confirm graphically that the slope of the isoquant gets flatter as you move from left to right, indicating that the RTS diminishes.

The property of homogeneity

If you increase all the inputs in a production function proportionally by a common factor ("t"), what happens to the quantity of output?

In our example, what is $f(tL, tK)$?

$$f(tL, tK) = (tK)^{\frac{2}{3}}(tL)^{\frac{1}{3}}.$$

This is equivalent to, $t^{\frac{2}{3}}K^{\frac{2}{3}}t^{\frac{1}{3}}L^{\frac{1}{3}}$.

Finally combining the "t" terms,

$$f(tL, tK) = t^{\frac{2}{3}+\frac{1}{3}}K^{\frac{2}{3}}L^{\frac{1}{3}} = t^{1}K^{\frac{2}{3}}L^{\frac{1}{3}} = t^{1} * f(K, L).$$

Homogeneity defined

So the production function increases by a factor of (t^1) when we multiply all its arguments by t.

- Consequently, this production function is homogeneous of degree 1.

A function "f" is homogeneous of degree "n" if,

$$t^n f(x_0, x_1 \ldots x_i) = f(tx_0, tx_1, tx_i).$$

Note: most of the utility and production functions we use in this class will be homogeneous, but in general many functions are not.

Significance of homogeneity

Economically when a production function is homogeneous of degree 1, it has a property called <u>constant returns to scale</u>.

This means, succinctly, that: if you double all the inputs, you double the amount of output produced.

If the degree of homogeneity is less than 1, the function has <u>decreasing returns to scale</u>.

If the degree of homogeneity is more than 1, the function has <u>increasing returns to scale</u>.

Different degrees of substitutability

Similar to utility functions, the arguments in a production function can exhibit various degrees of substitutability and complementarity.

- Cobb-Douglas is the example we've been using so far:

$$q = f(K, L) = K^{\alpha} L^{\beta}.$$

The inputs might be perfect substitutes in the production:

$$q = f(K, L) = \alpha K + \beta L.$$

The inputs might have to be used in fixed proportions:

$$q = f(K, L) = \min\{\alpha K \,, \beta L\}.$$

Summary

Firms combine inputs into outputs.

A production function maps the quantity of inputs used to a quantity of output.

Inputs have diminishing marginal products.

The Rate of Technical Substitution is the ratio of the inputs' marginal products.

RTS is diminishing because of diminishing marginal product.

Summary

When both inputs in a two-input production function are variable, the production function is graphed as an isoquant map.

An isoquant is the set of all input combinations that produce the same level of output.

Production functions can have varying returns to scale, i.e., to proportionally increasing all their inputs.

Returns to scale is measured by the property of homogeneity.

Conclusion

The firm's objective is not to maximize production—in fact, we have said nothing so far about its objective yet.

We have merely examined the common methods for modeling production functions in Economics.

We will see in the next lecture that firms choose their inputs to accomplish the objective of minimizing costs for any level of output.