Intermediate Microeconomics

PROFIT MAXIMIZATION

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Profit

Profit (Π) : The amount by which a firm's revenues exceed its costs.

Revenue: (TR) The amount that the firm receives for the sale of its output.

$$\Pi = TR - TC$$

Both TR and TC depend on the quantity (q) of output the firm produces.

- The firm chooses an output level such that Π is maximized.

Marginal revenue

A TR function maps each value of *q* to a unique quantity of revenue.

Marginal Revenue: (MR) The extra revenue a firm receives when it sells one more unit of output.

- The first derivative of the TR function.

The exact form of MR depends on the market structure, so we will discuss this more later.

Profit maximization

Firms can potentially have increasing returns to scale over a range of output.

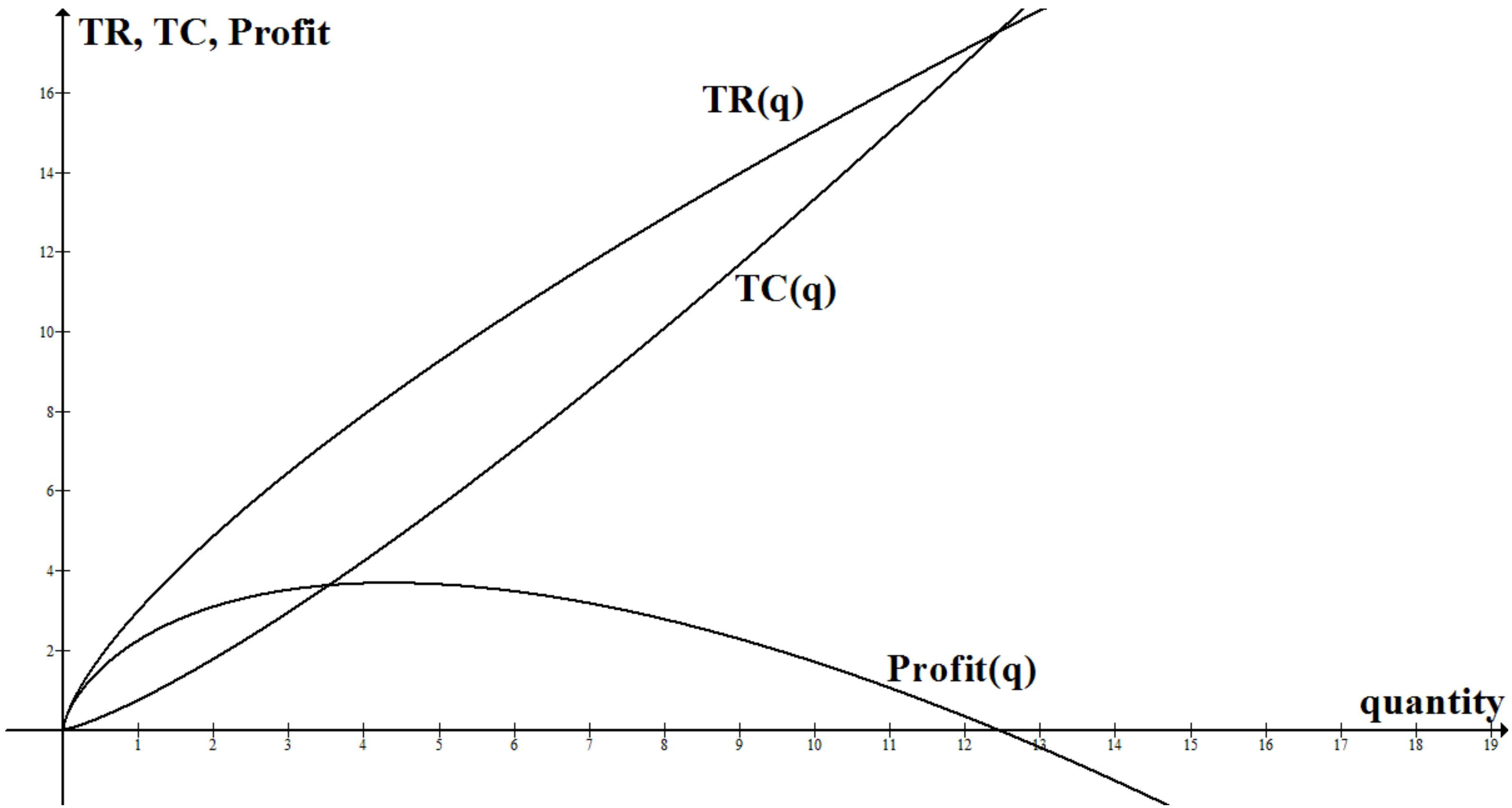
- Think of a very small firm that doubles it scale and takes advantage of additional specialization by its workers and tools by more than doubling output.

But eventually as firms get large, they experience decreasing returns to scale.

- And, hence, increasing marginal costs.

The firm will choose L and K in such a way as to minimize TC for any given level of output.

TC, TR, Π (graphically)



Observations about the previous graph

The Profit curve hits zero exactly where TC equals (intersects) TR.

The Profit curve reaches its maximum where there is the largest “gap” between TC and TR.

The slope of each curve has a marginal interpretation:

Slope of TC → Marginal Cost (MC)

Slope of TR → Marginal Revenue (MR)

Slope of Π → Marginal Profit

The MC and MR are equal at maximum profit.

MR = MC

Considering what you know about calculus, this should come as no big surprise.

The profit maximizing output level can be identified by the level at which *marginal profit equals zero*, i.e.,

- the slope of the profit function is flat.

Since the profit function is just comprised of the Revenue and Cost functions, its derivative is the MR minus the MC:

$$\frac{\partial \Pi}{\partial q} = \frac{\partial TR}{\partial q} - \frac{\partial TC}{\partial q} = MR - MC.$$

When you set this to zero to get the optimal q, you get:

$$MR - MC = 0 \rightarrow MR = MC.$$

Example

Say that Revenue and Costs are given by:

$$TR = 9q \text{ and } TC = 3q^2.$$

The Profit function, then, is:

$$\Pi = TR - TC = 9q - 3q^2.$$

Differentiating it with respect to q:

- Marginal Profit $= 9 - 6q$.

Setting it equal to zero to solve for optimal q:

$$9 - 6q = 0 \rightarrow q^* = \frac{9}{6} = 1.5 \text{ units.}$$

Intuition

If the firm produces a quantity less than q*, the MR would exceed the MC.

- The firm could increase profit by producing more units.

If the firm produces a quantity more than q*, the MC would exceed the MR.

- The firm could increase profit by producing less units.

As long as either of these is true, the firm is not maximizing its profit.

Profit level

Finally it is possible to substitute the optimal quantity back into the profit function to solve for the firm's profit level.

$$\Pi(q^*) = 9(1.5) – 3(1.5)^2 = \$6.75.$$

This is the amount of profit the firm makes if it chooses its output level optimally.

Summary

The firm's objective is to maximize profits.

Profit is defined as Revenue minus Cost.

Profit is maximized when the firm chooses an output level at which marginal revenue equals marginal cost.

Conclusion

We have answered the question of how the firm maximizes profit for a general case.

Next we explore the results when firms sell their output in a goods market.

This creates the firm's revenue, so the price that the output fetches in the market is of crucial importance.

You will see that the relationship between output and its price varies across markets, so we will examine several important features of output markets in the next lesson.