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RESEARCH ARTICLE

General equilibrium methodology applied to the design, implementation and performance evaluation of large, multi-market and multi-unit policy constrained auctions

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Abstract

The paper reports on the methodology, experiments, design and outcome of a large auction with multiple, interdependent markets constructed from principles of general equilibrium as opposed to game theoretic auction theory. The auction distributed 18,788 entitlements to operate electronic gaming machines in 176 interconnected markets to 363 potential buyers representing gaming establishments subject to multiple policy constraints on the allocation. The multi-round auction, conducted in one day, produced over $600M in revenue. All policy constraints were satisfied. Revealed dynamics of interim allocations and new statistical tests provide evidence of multiple market convergence hypothesized by classical principles and theories of general equilibrium. Results support the use of computer supported, "tâtonnement–like" market adjustments as a reliable empirical processes and not as purely theoretical constructs.

Keywords Auction design · Testbed experiments · Stability

JEL Classification C60 · D44 · D47

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1 Introduction

The paper reports on the design and field implementation of a new, large, multiple-market and policy-constrained auction based on principles from general equilibrium theory as opposed to a traditional theory of auctions. The auction involved the sale of 18,788 ten-year entitlements for the use of electronic gaming machines in Victoria Australia, in May, 2010. Policy goals dictated the operation of 176 interconnected markets to allocate sales of these licenses to 363 potential buyers representing licensed gaming establishments. The auction outcomes satisfied all policy constraints, was conducted in one day, and produced over $600M in revenue. The auction architecture rested on principles of an exchange economy in which bidders are assumed to have well-formed preferences and make choices similar to those guided by the classical tâtonnement model of market adjustment. The multiple market interdependencies created by policy constraints on the allocation, the size of the problem, and policy imposed limitations on timing challenged any obvious application of traditional forms of auctions.[1] Competitive market principles inform the theoretical framework used to interpret the results, which demonstrate convergence to an equilibrium with many features predicted by the classical theory. The paper reviews the policy background, the theoretical architecture, some key features of the laboratory experimental testbedding, and results and dynamic performance, providing the first field demonstration of a tâtonnement-like adjustment based on classical principles of general equilibrium as an empirical model of price formation.

We introduce and expand two broad, overriding questions used to evaluate the mechanism's performance relative to policy-focused market designs.[2] The first question is a form of proof of principle evaluating a basic question about the policy's implementation. (1) "Was the implementation successful in satisfying the policy goals? Or, equivalently, did the implementation do what it was supposed to do?" The second question investigates consistency in relation to the underlying theory. (2) "Were the models used to guide the design successful in the sense that the observed market behaviors are consistent with the principles used in the design? Or, equivalently, did it do what it did for the right reasons?"

In the light of those questions three results stand out. First, the auction outcomes satisfied all the complex policy constraints. Second, the auction provides the first substantial field support for basic principles of classical general equilibrium and competitive economic theory including Walrasian price discovery, inherent randomness, and tâtonnement convergence dynamics. The mechanism was not narrowly Walrasian because bidders submitted value functions and did not simply report quantities demanded at announced prices. The value functions were used to compute surplus maximizing allocations and determine equilibrium supporting prices that served as

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[1] Methodology and strategies for using alternative auction formats are open questions beyond the scope of our analysis. We present an auction that addressed the given allocation problem. In doing so, we demonstrate the successful application of the theoretical principles underlying the auction's design and empirical properties of price formation and general equilibration across multiple markets.

[2] These questions were first posed by Plott (1994) as a methodology for capturing the relationship between experimental testbeds and policy implementation. Subsequent literatures addressing design methodology (see Milgrom (2000) or Roth (2008)) played no role in the design.

the basis for additional bidding. Nevertheless, the process was very “tâtonnement-like” as it employed an algorithm for round-to-round dynamic adjustments of prices that responded to revealed excess demand at previous prices. This process continued until it “approximately” converged. The assignment process was also “tâtonnement-like” because no trading occurred until prices reached the approximate equilibrium. Moreover, the data show how the market reached equilibrium as opposed to simply assuming that the outcomes correspond to an equilibrium outcome. Third, both the field data and experimental data support the application of the auction architecture that guided the gaming machines design.

The auction was the result of the Victorian government’s efforts to change policies regulating gambling operations in the state. In 2008, the government initiated a reorganization of this industry by changing the method of allocating the entitlements to operate electronic gaming machines (e.g. poker and slot machines) and the method of finance. Historically, two large corporations managed the distribution of gaming machines. The machines were allocated to businesses consistent with local policies governing their use. Finance had been based on the revenue produced by the machines with roughly a third going to the local establishment, a third to the managing company, and a third to the government.

Governmental concerns with the historical policy reflected a desire to better control gambling and concomitant social problems, gambling-related government public finance, and a desire for conformity with frameworks used for economic regulation. Auction-style mechanisms emerged as possible tools with the aggregate supply quantity linked to historical levels. Authorities wished to create minimal economic dislocations and a climate where future regulatory efforts could be based on principles of decentralized competition and operator profits. The auction was also implemented to allow for possible entry by new operators and shifting entitlements from past use to reflect underlying economic value rather than historical administrative practices.

The resulting auction mechanism and its implementation present a remarkable success for the decades of abstract theorizing about general equilibrium in classical economics. It shows that tâtonnement can be a useful empirical concept in market design with properties that can be estimated statistically. As demonstrated in Sect. 2’s presentation of the formal, multiple-market structure, general equilibrium theory proved quite useful in practice when defining the mechanism. Further, Sect. 3 and the associated Appendix A demonstrate the auction procedures and illustrate how lessons learned from laboratory testbed experiments provided insights for its field implementation.[3] Partial equilibrium analysis of individual markets in Sect. 4 and Appendix C illustrates the operation of competitive market principles supporting an efficient allocation of licenses within each market and demonstrates that bidders act as price-takers in these markets.

Empirically evaluating the efficiency of the overall allocation of licenses across markets requires verifying the auction followed the principles used in the theory of its design to reach an efficient general equilibrium. To approach these questions, we focus

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[3] These testbed experiments are designed to develop intuition about bidder behavior and support judgements about auction procedures. The exploratory nature of the experiments stands in contrast to traditional applications of experimental economics to evaluate a fully articulated theory against alternative hypotheses using large sample sizes to provide power for classical statistical testing.

on the dynamics of equilibration as driven by excess demand dynamics interpreted through an important principle, “excess demand revealed at the margin.” This excess demand, readily measured from observed bidding behavior and interim allocations, was first observed in the testbed experiments. Section 5 demonstrates that this excess demand becomes exhausted through the auction mechanism’s bid revision process. Subsequent sections focus on coordinated equilibration of multiple markets. Section 6 describes the dynamic behavior of the auction mechanism, characterizing the total revenues and surplus generated as bidding rounds progressed. Section 7 investigates price dynamics across markets in relation to the revealed excess demand in all other markets. This analysis verifies conditions for stability that lead to an efficient multi-market allocation and general equilibrium across all market segments given the policy constraints. Taking advantage of the rich data available, these novel statistical tests present the first empirical verification of equilibrating dynamics based on the principles of tâtonnement. Section 8 concludes with a summary of the findings from the implementation of an economic mechanism to address the allocation problem at the heart of a complex government policy project.

2 Auction structure

2.1 Policy constraints

The auction design problem was to sell entitlements simultaneously, subject to many overlapping policy constraints. Policy constraints were focused on the nature of the businesses that were allowed to participate in the auction. Half of the 27,500 entitlements were to be sold to businesses classified as “Hotels.” The other half was to be sold to smaller venues called “Clubs” that cater to local populations. This reflected differences between the economic environment and social purposes of these venues and differing political bases in various Victorian communities.

Legislation divided Victoria into 88 geographic regions and assigned each region a maximum number of entitlements based on area population or other regulations. These constraints placed limits on the saturation of machines relative to population and were motivated, in part, by social, community, and health issues related to gambling. Additional policy concerns regarding the geographic distribution of entitlements resulted in the creation of a single set of geographic regions designated as metropolitan and maximum number of entitlements that could be allocated to the set.[4]

Accounting for these constraints, and neglecting the metropolitan designation, each entitlement has two characteristics: the type of venue (Club or Hotel) and the geographical region (88 distinct areas) in which that establishment is allowed to operate. This required 176 simultaneous markets. While the underlying resource in all markets is an “entitlement,” the policy restrictions differ across clubs, hotels, and areas. So, from an economic and modeling perspective, the items sold in these 176 markets are completely different commodities even though all are “entitlements.”

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[4] In particular, geographical areas designated as “Metropolitan” were limited to obtain no more than 80 percent of all entitlements. Since this constraint did not bind at any point during the auction, we do not discuss the features implemented to accommodate the constraint should it be binding.

The existing economics literature on auctions reports several auction designs that were considered but seemed inapplicable given the structure of multiple policy constraints, the scale required, and the overriding government goal of efficiency. Obvious auction types included sealed bid auctions similar to U.S. Treasury auctions and the sequential forms of auctions such as the simultaneous, multi-round, ascending prices auction (SMRA) used to allocate the electromagnetic spectrum (Milgrom 2000; Bichler and Goeree 2017). Both types of auctions have the capacity to accommodate large numbers of bidders competing for large numbers of items. However, the governmental policy constraints limited the allocations of available units to different subsets of markets, creating structural complexities not addressed in traditional auction applications. For example, in the traditional, multi-market (SMRA) auction the number of units for sale in each market is fixed. The auction analyzed here introduces a market institution that facilitates efficiency-improving movement of resources and arbitrage among markets, typical of general equilibrium. Time allowed for the auction presents a further challenge, as the Victorian machine auction was permitted only one day to operate. In contrast, simultaneous ascending price auctions can require months from start to finish and it is unclear how to conduct such auctions more quickly without bidder errors and compromised efficiency.

Given the auction scale (numbers of units and markets), human information processing, decision speeds and error correction delays, open questions exist regarding how to make traditional auction systems work while guaranteeing that all governmental constraints would be satisfied. We were (and still are) unaware of appropriate modifications, so both classes of auction architectures were ruled out by scale and policy constraints. Of course, in the absence of an impossibility theorem, it is conceivable that modified versions of the traditional auction architectures might be developed and used. Currently, none exist.

2.2 Determining allocations and prices in a continuous model

The basic auction design can best be understood in the context of a continuous model that seeks an efficient allocation given the values revealed by bidders. This analysis assumes away complexities created by the discrete integers required for the allocation, which will be addressed in the later sections that analyze the data from the auction.

The model assumes that a constrained optimization problem can identify the efficient allocation and supporting prices. It begins with a classical economic postulate. Bidders are assumed to have well-formed preferences and objectives that form the foundation of economic efficiency and are revealed only through actions taken in the market. Assume that each bidder submits a continuous valuation schedule $V_i(x)$ reporting their total willingness to pay for an allocation of x entitlements. Assume further that $D_i(x) = \frac{\partial V_i(x)}{\partial x}$, representing the bidder's marginal willingness to pay, is non-negative, monotonic, and (weakly) decreasing, so that the representation of bidders' demand schedules for licenses is continuous and monotonically weakly decreasing. While bids are observable, the underlying preferences that might produce a valuation function are not observable. All inquiries that might have yielded information about the existence and form of the preferences of bidders were prevented by government probity policies, which also strictly prohibited all communications between policy makers or policy implementers and bidders.

Regulations regarding bidding establishments depend upon the type of establishment, a Club or a Hotel and the area in which the establishment is located. Let $i \in I$ index each establishment and let $a_i \in A$ denote the area in which the establishment seeks to obtain licenses.[5] The indicator variable $h_i \in \{0, 1\}$ identifies the i^{th} establishment type, equaling 1 if establishment i is a hotel. The system allocates entitlements to bidders to maximize the total cumulative reported value of the allocation consistent with all policy constraints. An efficient allocation is one that optimizes total value subject to constraints. Define X as a vector of allocations with the i^{th} entry x_i representing the allocation to establishment i. The total market value, $V(X)$, is the aggregate value of bidders' willingness to pay for their given allocations:

$$V(X) = \sum_{i \in I} V_i(x_i)$$

Allocations must satisfy the set of policy constraints facilitated by defining the total allocations to each area by venue type:

$$x_{aC} = \sum_{i \in I} x_i (1 - h_i) 1\{a_i = a\}$$

$$x_{aH} = \sum_{i \in I} x_i h_i 1\{a_i = a\} \quad x_a = x_{aC} + x_{aH}$$

Imposing these definitions as equality constraints on the maximization problem identifies the shadow costs for allocating a marginal license to each area and bidder type. The total allocation to an area, x_a, must satisfy the constraint defined by the Victoria government, denoted $\bar{x}_a$: $x_a \leq \bar{x}_a, \forall a \in A$.

Administering Victoria-wide constraints on the total allocations to Hotels and Clubs, respectively denoted $\bar{x}_H \, and \, \bar{x}_C$, is facilitated by similar constraints:

$$x_H = \sum_{a \in A} x_{aH} \quad x_C = \sum\nolimits_{a \in A} x_{aC}$$

Each of these aggregated allocations must satisfy government-imposed inequality constraints, $x_H \leq \bar{x}_H$ and $x_C \leq \bar{x}_C$.

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[5] Each establishment corresponds to a policy-defined "venue" meaning the location where the machines would be housed and operated. Bids are submitted by the venue and the entitlement is issued to the venue where the machine must be located and counts against the area constraints. A business might own more than one venue and employ a representative bidder authorized to submit bids for more than one venue that the business might own. Auction rules were designed to minimize coordination between bidders. All bidders were located in a large convention hall with cubicles from which other bidders could not be viewed (see Figure 8). External communication devices (e.g., mobile phones) were prohibited and monitors ensured no unauthorized communication occurred amongst the bidders. While a representative bidder might tender bids for all of the venues from the same terminal, each bid is attached to a specific venue and recorded as made by that venue. As such, even though a single business might own more than one venue even in one particular area, the system records and treats each venue as a separate entity. Special rules and monitoring were imposed for multiple venues operating under the same ownership.

The Lagrangian for the constrained allocation problem is:

$$\begin{aligned}
\mathcal{L}(X) = {} & V(X) \\
& \left.\begin{aligned}
& -\textstyle\sum_{a\in A}\lambda_{aC}\left(x_{aC}-\sum_{i\in I}x_i\left(1-h_i\right)1\left\{a_i=a\right\}\right) \\
& -\textstyle\sum_{a\in A}\lambda_{aH}\left(x_{aH}-\sum_{i\in I}x_i h_i 1\left\{a_i=a\right\}\right) \\
& -\textstyle\sum_{a\in A}\lambda_a\left(x_a-x_{aC}-x_{aH}\right) \\
& -\textstyle\sum_{a\in A}\mu_a\left(\bar{x}_a-x_a\right)
\end{aligned}\right\}\text{Constraints on area Club and Hotel allocations} \\
& \left.\begin{aligned}
& -\lambda_H\left(x_H-\textstyle\sum_{a\in A}x_{aH}\right)-\lambda_C\left(x_C-\textstyle\sum_{a\in A}x_{aC}\right) \\
& -\mu_H\left(\bar{x}_H-x_H\right)-\mu_C\left(\bar{x}_C-x_C\right) \\
& -\mu_O\left(\bar{x}-x_H-x_C\right)
\end{aligned}\right\}\text{Constraints on aggregate Club and Hotel allocations}
\end{aligned} \tag{1}$$

The shadow costs denoted by λ impose binding equality constraints for aggregating allocations within different market segments, and Kuhn-Tucker shadow costs denoted by μ correspond to non-negative inequality constraints that may or may not bind on the final allocation. The shadow costs, μ_a, μ_C, μ_H, and μ_O translate into the prices for licenses associated with different areas, clubs and hotels.

Given sufficient regularity conditions, the optimization problem (1) solves for a Pareto efficient allocation of licenses and the second welfare theorem states that this allocation can be supported as the competitive equilibrium of a market mechanism with associated prices. Those prices are approximated by the shadow costs for the binding constraints.[6]

3 Auction procedures to determine allocations and prices

Figure 1 provides an overview of key features of the auction structure. The auction proceeded as a series of rounds. Bidders submitted bids in each round, and provisional prices and allocations were computed. At the end of each round, individual bidders were privately informed of their provisionally winning bids and prices and the minimum number of rounds remaining in the auction were publicly announced. Bidders could then revise bids upward, submit new bids and cancel bids that were not provisional winners. If bidding activity was sufficiently active, the auction stayed opened for additional round(s). If the auction closed the provisional allocation became the final allocation.

In practice, bidders submit schedules reporting their willingness to pay for different allocations through a bidding mechanism described in Sect. 5. Here, we describe the submitted bid schedules under the simplifying assumption that reported bid schedules reveal bidders' true valuations, while recognizing that bidders' reported willingness to pay and valuations may diverge in practice and that the auction need not have a "preference revelation" property.

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[6] In practice, the constraint on aggregate Club allocations was not binding whereas the constraint for aggregate Hotel allocations did bind. Further, not all areas' allocation constraints were binding, so these constraints only affected the prices paid for licenses within those areas facing binding constraints.

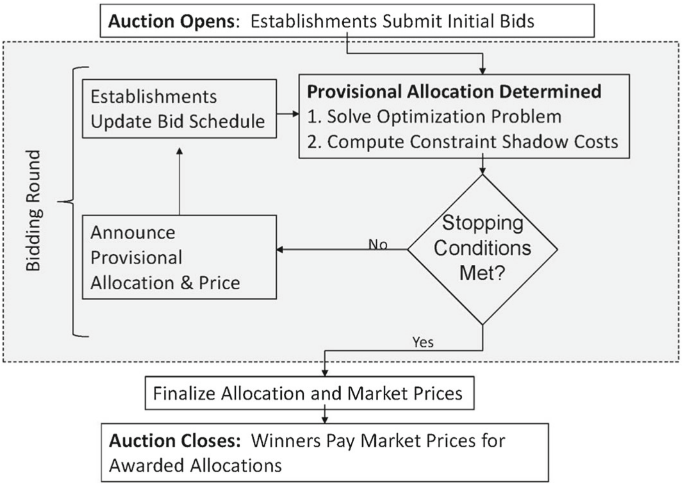

Fig. 1 Overview of auction timing and structure

3.1 Reported bid schedules and accumulated bid functions

Each establishment submits a bid schedule containing L_i entries specifying its willingness to pay for each marginal unit. The lists' entries are sorted by descending bid and the entry at the l^{th} level in the bid schedule is denoted $B_{il} = (b_{il}, x_{il})$. The bid price b_{il} reports the price the bidder is willing to pay and the quantity x_{il} reports the number of marginal units the bidder demands at that price in addition to the units they would receive from any higher-priced bids. Bidder i's cumulative bid schedule, denoted $X_i(p)$, reports the total quantity of bids with reported value weakly greater than p, computed by summing $X_i(p) = \sum_{l=1}^{L_i} x_{il} 1\{b_{il} \geq p\}$.

Let $\hat{V}_i(x)$ denote bidder i's cumulative reported valuation for an allocation of x licenses, calculated by summing the area under the bidder's reported bid function up to the quantity of x.[7] Since $\hat{V}_i(x)$ can be evaluated at any quantity, it can also be stated as a function of price evaluated at bidder i's cumulative bid schedule. Denoted $\hat{V}_i(X_i(p))$, this represents the total valuation bidder i assigns to the licenses they would bid for if the price were p.

Table 1 provides a hypothetical example of an individual bid schedule and its translation into cumulative bids and reported valuations. Panel A presents a schedule with four entries at four different price points for an establishment that bids for up to 25 entitlements if the price is no greater than 80. The Cumulative Bid Schedule in Panel B demonstrates how different prices translate into total quantity desired by that bidder at each price.

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[7] The formula for $\hat{V}_i(x)$ is a bit convoluted, due to the discrete nature of bids, but can be calculated as: $\hat{V}_i(x) = \sum_{l=1}^{L_{il}} b_{il}\left[x_{il} 1\left\{\sum_{j=1}^{l} x_{ij} < x\right\} + \left(x - \sum_{j=1}^{l-1} x_{ij}\right) 1\left\{\sum_{j=1}^{l} x_{ij} \geq x\right\} 1\left\{\sum_{j=1}^{l-1} x_{ij} < x\right\}\right].$

Table 1 Sample individual and cumulative bid schedules

Panel A: Reported Bid Schedule			*Panel B: Cumulative Bid Schedule*		
List Entry [l]	Bid [b_{il}]	Bid Quantity [x_{il}]	Price [p]	Cumulative Bid [$X_i(p)$]	Cumulative Value [$\hat{V}_i\ (X_i(p))$]
1	100	5	100	5	500
2	95	5	96	5	500
3	90	10	95	10	975
4	80	5	90	20	1,875
			80	25	2,275

3.2 Implemented allocation rule

The auction allocation system approximates the continuous model presented in Sect. 2.2 using the elicited valuations. The system determines the allocation by maximizing the "measured" total welfare by the aggregated reported license valuations, $\hat{V}\ (X) = \sum_{i \in I} \hat{V}_i\ (x_i)$, maintaining all the relevant constraints indicated in (1).

The discrete nature of the problem complicates this optimization problem. These are well known features of integer programming optimization, requiring tie-breaking rules, resolving non-uniqueness of shadow costs, and potential for multiple solutions due to overlapping constraints. We resolved tied bids through a first come, first serve rule, so bids are rationed at the market price (if needed) according to their arrival time. The market clearing prices need not be unique if the quantity demanded at a price exactly equals the supply to that market. Adding a very small quantity to every bid (essentially picking the best price for the seller) allowed the quantity demanded at a price to always slightly exceed the integer parts.[8] Given the adjustments to the optimization problem necessary due to these practical considerations, the shadow costs from the optimization only approximate the shadow costs from the continuous Lagrangian in equation (1). Still, these approximations do not induce disequilibrium in any individual market's allocation.

3.3 Preview of dynamic auction features and bidding revisions

The system arrived at its final allocation after progressing through a series of 63 bidding rounds. Each round began with establishments submitting provisional bid functions. The auction algorithm computes the allocation and prices based on these bid functions. These provisional prices and allocations are then announced at the conclusion of the round. Thus, bidders observe the quantity of entitlements that they would purchase, and the price they would pay per unit, if the auction were to stop in that round.

Given this information, bidders are aware of the prices revised bids need to meet or beat to obtain additional licenses. Before the next round began, bidders could use this

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[8] Although multiple constraints could bind and thus create multiple price solutions, this could only arise from relationships between metropolitan and area constraints. Fortunately, this problem never surfaced because the metropolitan constraint was never binding.

information to revise their submitted bid functions, subject to the restriction that they increase their original bid by at least a specified minimal increment. This restriction induces an ascending auction format in markets defined by constraints, with excluded bid pricing as bidding progresses from round to round. Importantly, the total quantity demanded at the announced price is revealed to the auction system but not revealed to participants who only know their own allocation.

Even though individual bidders could not decrease leading bids, prices in some markets could go down while the prices in other markets go up due to the interdependence of the markets, the role of "consumer surplus," and shifting supplies.[9] Price decreases could occur in other circumstances that involve multiple market coordination and derived demand. Unlike traditional forms of auctions, the system response need not be related to revealed excess demands at existing prices. These potential price decreases reflect movement of items across markets to achieve overall efficiency, illustrating a fundamental departure from traditional ascending price auctions with a fixed number of items available for sale in each market.

The system initiates a two-clock ending process based on the number of significant revisions (attempts to acquire more units) in individually submitted bid schedules (clock one) and the resulting patterns (numbers) of market price changes (clock two). At the end of each round, bidders are notified about the minimum number of potential remaining rounds. This process terminates with an announcement that the market may close in the subsequent round and bidders are given a final opportunity to revise their bid schedules. Absent any significant additional revisions, the auction closes and the bidders pay the announced price for their market for each entitlement awarded.

3.4 Design decisions and testbed methods

The auction design was guided by testbed experiments outlined in Appendix A, which focused on evaluating selected behavioral features as well as expanded scale. The objectives of testbed experiments are to inform judgements regarding behavior and possible outcomes in a wide range of environments where theory is incomplete (or absent) and to stress-test those predictions in very unlikely or extreme events.

While the policy objectives of the machines auction were clearly stated, the operational design and the process itself faced many challenges aside from the size and technical complexity of the allocation problem. First, probity concerns about infor-

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[9] A simple example illustrates this unintuitive feature. Suppose there are three units to be allocated in two markets, {commodity 1 in market 1 and commodity 2 in market 2}and six agents, {a,b,c,d,e,f}. Each wants only one unit and can place bids only in that market. Suppose agents a,b,c have values (4, 0.05, 0.05), respectively, for commodity 1 and agents d,e,f have values (1,1,1) for commodity 2 and all agents reveal their value in their bids. Prices, which equal the bid of the last accepted unit in a market, are (4,1) for market 1 and market 2 and volumes are (1,2). Total revenue is 4+2 = 6 and (maximized) total "buyer surplus" is 6 = 4+1+1. Now, suppose agent b increases her value from 0.05 to 2 and raises her bid to 2. As a result of this demand increase, the surplus maximizing feature of the process automatically shifts an entitlement unit from the supply in market 2 to the supply in market 1. Prices as determined by the marginal bid, decrease to 2 in market 1 and remain 1 in market 2 while volume increases to 2 in market 1 and decreases to 1 in market 2. Total revenue is 5 = 2+2+1 and total buyer surplus is 7 = 4+2+1. Notice that prices and total revenue have both decreased but buyer surplus has increased. Bids increased but the prices decreased. Such instances are shown later when in the time-series of market prices and area price premia in Fig. 9.

mation advantages among bidders prevented all inquiry, feedback, discussions with or exercises involving potential bidders or the circumstances they faced, both during the design phase and while the auction was open. Second, the time of the auction was limited to one day. Third, no comparable auction processes existed in practice or in theory to provide any useful history of applications.

The design exercise began with a theoretical sketch of an auction with a structure where: (i) bidder preferences were limited to a single establishment; (ii) agents submit "truthful" demand functions; (iii) the auction winners are chosen by maximizing the revealed value of the allocation; and (iv) policy limits constrain all final allocations across multiple markets. This sketch suggested the theory of general equilibrium as a possible source of principles. The appendix discusses subsequent and evolving design judgements.

Laboratory experimental studies of multiple markets lend strong support to the basic principle that market dynamics are driven by excess demands, which guide the system to a general equilibrium. In spite of theoretical discussions that raise doubts about applicability of general equilibrium, such as Ackerman (2002), the Sonnenshein-Mantel-Debreu Theorem (Mas-Colell et al. 1995), papers in Bridel (2011) and additional general equilibrium critics listed (and challenged) in Mukherji (2019), the design decisions rested on the fact that the convergence principles are evident in a wide range of experiments. The excess demand driven convergence is found in the multiple markets of international trade in Noussair et al. (1995), disequilibrium dynamics in Gillen et al. (2021), Anderson et al. (2004), and Hatfield et al. (2016), and finance in Asparouhova et al. (2003). Importantly, the fundamental role of excess demand in driving price adjustments exists when bidding is expressed as demand functions (Goeree and Lindsey 2016). Convergence is also seen in call markets (Plott and Pogorelskiy 2017), and complex auctions exhibit the same tendency. A prime example is the simultaneous, ascending price auction used for the auction of the broadcast spectrum, with convergence properties clearly demonstrated in experiments (Plott and Salmon 2004). The principle is observed in complex networks such as power grids (Chao and Plott 2008) and combinatorial auctions (Lee et al. 2014). However, experiments have demonstrated the unreliability of the pure tâtonnement institution with no trading at announced prices (Plott, 1988; 2001) and served as a warning about the incomplete nature of theory as a model of actual behavior. Still, in other experiments exploring institutional variations a key behavioral feature, demand revelation at the margin, had been identified in an unpublished working paper on fuel efficiency under the CAFE constraint (Katz and Plott 2008).

The auction uses repeated rounds as a design foundation, as opposed to a one shot, sealed bid computation, is based on findings from general equilibrium related experiments that consistently reflect increasing efficiency, dynamic equilibration, and convergence over repeated rounds of bidding. Even in more simplified environments with established incentives to report true valuations, such as second-price sealed bid auctions for a single unit, dynamic (English ascending price) versions lead to increasingly truthful revelations (Kagel and Levin 2015). The scale of the gaming machines auction and nature of the multi-unit demand for licenses required analyzing functions (inverse demand functions) as opposed to separate bids on (thousands of) individual units. Competitive theory applied to smooth demand functions identified theoretical and technical relationships among bids, prices (as Lagrangian multipliers), equilibrium (as a competitive equilibrium), allocations and efficiency. The actual auction required solving an integer-constrained, linear program for the allocation problem each round.

The scale of the design problem derives from the size of markets and the number of markets, units, and bidders. The testbed exercises outlined in Appendix A established the economic performance and technical control of the system in relation to scale. Two performance measures were useful tools to refine the rules of the auction. The first was market efficiency measured as consumer surplus as developed by Plott and Smith (1978). Since the cost of the entitlements is ignored in this government allocation problem, this measure is simply the sum of observed willingness to pay divided by the maximum sum of willingness to pay given experimentally induced preferences. The second was speed of convergence measured in terms of number of rounds required for equilibration.

The testbed experiments started small and were scaled up. Some of the over 40 different experiments were repeated to explore problems they exposed. The largest testbed employing human subject participants operated with 50 markets and 160 participants at a subject payment cost of $8,866. Larger scales were simulated with multiple computers programmed to place bids to test network configurations, processing speeds, and computation reliability.

Experiments revealed the auction could coordinate convergence for prices and allocations across multiple markets, equilibrating derived demand with the available supply. Allocations and prices typically ended near the predictions of the general competitive equilibrium and thus efficiency tended to be in the high 90% levels and often near 100%. Such high performance occurred at all tested levels of scale.

While each experiment examined multiple dimensions of performance, many focused on two specific areas. The first concerned real time control of price movement and procedures for ending the auction. Previous experimental work revealed that bidding incentives and stopping rules are important for performance. The second broad area included market performance, efficiency and reliability in both software and bidding behavior.

The timing of the auction rounds needed adjustments to account for the reaction speed of bidders. An increment requirement defining the minimal allowable increase had the obvious role of ensuring revisions were economically meaningful. We adopted a two-clock methodology to encourage serious and aggressive bids.[10] Requiring only a single bid revision for auction continuation is not practical because randomness in bidding and bid timing. The question becomes "how many" bids or price changes in a round justifies keeping markets open for additional rounds. Testing in experiments with different controls led to a decision to use the number of bidders that attempted to increase their allocation as the controlling measurement for the first clock and number of markets that changed prices as the controlling measure for the second clock. Time

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[10] In continuous time auctions one clock counts down in seconds and resets with the submission of a new bid in any market. In the absence of additional bids, the clock counts down to zero and the auction ends. A second clock is employed in auctions with complex bids, such as bid functions, because new bids need not result in price changes and can become cheap talk that simply keeps the auction open. We avoid such possibilities by using a second clock that counts down and resets if a bid results in new winners, thus exerting pressure to place bids that affect prices.

was measured in number of rounds required before a change in these thresholds. New bid increment requirements were announced as a percentage over existing prices and these were enforced beginning in a specified future round.

Experiments provided substantial experience with how the auction would respond to the chosen parameters. The testbed experiments provided the primary source of information about the likely auction ending time, which was important given the government's decision to limit the auction to one day. Experiments demonstrated that bidding followed a principle of revelation at the margin, discussed below in Sect. 5. Announced prices were accompanied by the increment requirement. All new bids or changed bids had to be no less than the existing price plus increments. Unchanged bids remained in the system. The upcoming price was not known and price often remain unchanged in many individual markets. New bids were automatically integrated with the bidder's existing bids to form a revised bid function.

The new bid function is a type of "revealed demand" but it is not fully revealing. The revealed demand function always falls short of the limit values (demand prices) of infra-marginal units but an important element – demand at the margin – is accurately revealed. As illustrated in the experimental background from Appendix A, the quantity demanded at the announced price was typically very close to the quantity demanded according to the induced preferences.[11]

4 Partial equilibrium properties of the final allocation

A prominent feature of the theory of general equilibrium is a suggestion that a complex economic system can be viewed as a collection of separate, identifiable markets each of which rests at its own equilibrium given that all other markets are in equilibrium. For the machines auction such a pattern is described by market equilibrium equations (1), where the constellation of partial equilibrium relationships is modeled as having emerged from a pattern of binding policy constraints. The theoretical equations describe two large markets and a set of smaller markets. One large market consists of "unconstrained" clubs. A second consists of "unconstrained" hotels and the others are smaller market segments associated with the several constrained areas commanding a local price premium. The broad question posed for testing is whether the data suggests the allocations in each of these separate markets are consistent with partial equilibrium theory coordinated by the theory of general equilibrium. In some respects, that question is at the heart of microeconomic theory.

We define and aggegate the derived demand for establishments competing to obtain licenses based on their reported bid schedules. Throughout this analysis, we assume three stylized facts for all allocations in the system. First, we partition the set of areas $A = A_C \cup A_U$ into constrained areas A_C (where $x_a = \bar{x}_a, \forall a \in A_C$) and

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[11] Given the rules, the bidder could adjust the bid price to ensure the purchase of the marginal unit given the announced price and do so without directly influencing that price. To express a preference for an additional unit at the stated price the bidder merely needed to express a willingness to pay for it by tendering a bid price for the unit above the stated market price. Thus, the value of the marginal demand is revealed. The demand function becomes traced out as price moves up following the required bid increments. In particular, the slope is useful information revealing the state of demand relative to prices and thus when the auction is near a competitive equilibrium.

unconstrained areas A_U (where $x_a < \bar{x}_a, \forall a \in A_U$). Second, hotel establishments are allocated the maximum number possible given the regulations ($x_H = \bar{x}_H$) while club establishments' allocation do not meet this maximum ($x_C < \bar{x}_C$). Third, all licenses available to the system are allocated (i.e., $\bar{x} = x_H + x_C$).

Within each of the market segments, the derived supply is price-inelastic at a fixed quantity.[12] We approximate the price equating demand and derived supply in each market using the price premia for different types of licenses. As such, the final allocation and prices in each market segment are consistent with an ex-post partial equilibrium given bidders' unwillingness to submit revisions to their reported bid schedules.

4.1 Partial equilibrium in the market for club licenses in unconstrained areas

Consider the market for club licenses in unconstrained areas under an allocation in which $x_C < \bar{x}_C$. Bidders in these markets compete with each other for the pool of licenses that are not allocated to hotels or to any of the constrained areas. Collectively, the aggregated bid schedules for bidders in these markets identify the *Derived Demand* for Unconstrained Clubs calculated as:

$$D_{UC}(p) = \sum_{a \in A_U} \sum_{i \in I} X_i(p)(1 - h_i)1\{a_i = a\}.$$

The supply available to these bidders is determined after all constrained area markets for both clubs and hotels, and the unconstrained hotel market have already cleared:

$$S_{UC}^t(p) = \bar{x} - x_H - \sum_{a \in A_C} x_{aC}.$$

Finally, the "Club Base Price," p_{UC}^*, is the market clearing price where $D_{UC}\left(p_{UC}^*\right) = S_{UC}\left(p_{UC}^*\right)$.

Figure 2 presents the derived demand and supply for unconstrained clubs in the final round of the auction relative to the minimum permissible club price of \$5,500. The residual supply available to this market consisted of 3,693 units, which matched Derived Demand to set the Club Base Price of \$5,500. Due to a large mass of demand at exactly the minimum price of \$5,500, the quantity demanded at this price exceeded the available supply. The proportion of demand met at this price was determined by priority based on when establishments submitted their bids. The mass at this price point has two important implications for the system. First, any establishment that did not receive an allocation at the \$5,500 price could have increased their bid to receive additional licenses with no impact on price. Given the quantity supplied at this margin, an establishment could have obtained up to 61 additional licenses without having any impact on price, an amount that exceeded the number acquired by any individual

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[12] The derived quantity supplied may depend on allocations to other market segments, though this feature of the market system does not impact the partial equilibrium analysis in this section. We return to discuss the general equilibrium properties of the market system below when analyzing stability of excess demand functions and a model of convergence to an equilibrium.

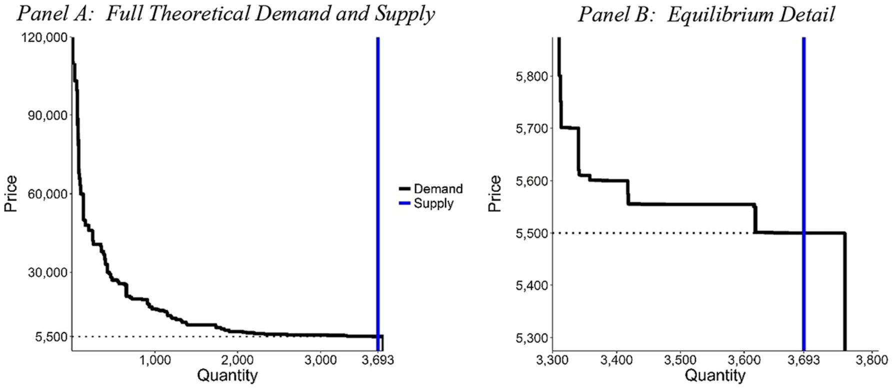


Fig. 2 Derived demand and supply for licenses for clubs in unconstrained areas

club. Second, the unmet demand of 64 units at the external margin suggests that no establishment would have any power to reduce prices by lowering their bids while still receiving the same allocation. If any bidder were to reduce their stated value for licenses allocated at this external margin, they would lose those licenses to the unmet demand.

Finally, we connect the price for this market segment to the optimization problem (1). The allocation of all other license types were at their constrained maxima, either due to individual area constraints or the aggregate hotel maximum license constraint. Consequently, any additional licenses made available to the market in total (i.e., an increase in $\bar{x}$) would be sold in the unconstrained club market at price p_{UC}^{*}. As the potential increase in the value function from relaxing the total license quantity constraint, this price must then equal the shadow cost of the constraint so that $p_{UC}^{*} = \mu_O = 5{,}500$.

This demonstrates that the market for unconstrained club licenses cleared in a classical sense where revealed bid schedules are interpreted as demand functions. Bidders for unconstrained club licenses could unilaterally revise their bid schedules to increase their provisional allocation without changing the prices they would pay for the allocated licenses. Bidders could cancel bids that were not provisional winners and avoid the possibility that the bids would be filled on subsequent rounds. Bidders chose not to take advantage of such revision opportunities, demonstrating that the allocation of licenses among bidders for unconstrained club licenses represents an ex-post equilibrium as summarized in Result 1.

Result 1: **Partial Equilibrium Model in the Market for Unconstrained Club Licenses**

a. The **Derived Demand Curve** model for Club licenses in unconstrained areas presented in Figure 2 aggregates the bid schedules for participants seeking club licenses in all unconstrained areas at any given price:

$$D_{UC}\left(p\right)=\sum_{a\in A_U}\sum_{i\in I}X_i\left(p\right)\left(1-h_i\right)1\left\{a_i=a\right\}$$

b. The **Derived Supply Curve** model for Club licenses in unconstrained areas presented in Figure 2 is inelastic and determined by the quantity of licenses that are not allocated to hotels or to clubs in constrained areas.

$$S^t_{UC}\left(p\right)=\bar{x}-x_H-\sum_{a\in A_C}x_{aC}$$

c. The **Club Base Price**, $p^*_{UC}=\$5{,}500$, is the market clearing price:

$$D_{UC}\left(p^*_{UC}\right)=3{,}693=S_{UC}\left(p^*_{UC}\right)$$

d. Bidders act as competitive price takers. Any bidder could have obtained up to 61 additional licenses by increasing bids without affecting the market clearing price and would have lost at least 64 units by lowering bids without affecting the price paid.

4.2 Partial equilibrium in the market for hotel licenses in unconstrained areas

We now extend this analysis to licenses for hotel establishments competing in unconstrained regions. Similar to the market for unconstrained clubs, these bidders compete solely with each other for the pool of hotel licenses that are not allocated to any of the constrained areas. Again, the expressed bid schedules are interpreted as demand functions. Define the t^{th} round Derived Demand and Supply, respectively, for Unconstrained Hotels as:

$$D_{UH}\left(p\right)=\sum_{a\in A_U}\sum_{i\in I}D_i\left(p\right)h_i1\left\{a_i=a\right\},\ and,\ S_{UH}\left(p\right)=\bar{x}_H-\sum_{a\in A_C}x_{aH}.$$

The "Hotel Base Price" represents the market clearing price, p^*_{UH}, that balances derived demand and supply so that $D_{UH}\left(p^*_{UH}\right)=S_{UH}\left(p^*_{UH}\right)$.

Figure 3 presents the derived demand and supply for unconstrained hotels in the final round. The 10,356 units of supply available in this market matched Derived Demand to set the Hotel Base Price of \$33,350. This derived demand appears relatively elastic compared to demand for unconstrained club licenses, likely due to greater heterogeneity in values for establishments in this market segment. Still, market power for individual bidders is quite limited, with fourteen units allocated out of twenty demanded at the Hotel Base Price. Consequently, a bidder in the unconstrained hotel market would be able to obtain an additional fourteen licenses by stating a higher willingness to pay without impacting their actual price paid. Bidders could also reduce their stated willingness to pay for the non provisional winning bids and thus avoid acquiring units in subsequent rounds.

We refer to the difference between the Base Hotel Cost and Base Club Cost, $\mu_H=27{,}850$, as the Hotel Price Premium. As in the market for unconstrained

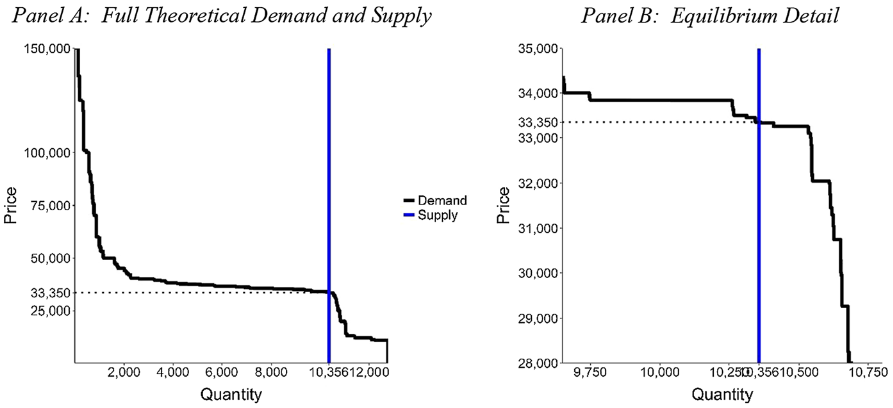


Fig. 3 Derived demand and supply for licenses for hotels in unconstrained areas

club licenses, the market for unconstrained hotel licenses cleared in a classical sense. Bidders in this market retain the unilateral ability to revise their bid schedules and alter their provisional allocation without changing the prices paid and thus supports the interpretation of the bid schedules as demand functions. Consequently, bidders' revealed unwillingness to make such revisions demonstrates the ex-post equilibrium nature of the allocation.

Result 2: **Partial Equilibrium Model in the Market for Unconstrained Hotel Licenses**

a. The **Derived Demand Curve** model for Hotel licenses in unconstrained areas presented in Figure 3 aggregates the bid schedules interpreted as demand functions for participants seeking hotel licenses in all unconstrained areas at any given price:

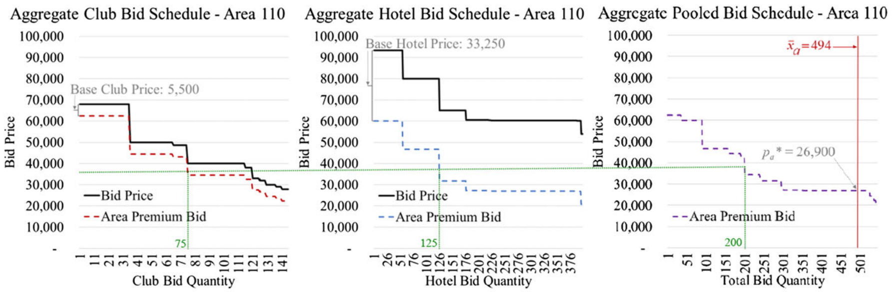
$$D_{UH}(p) = \sum_{a \in A_U} \sum_{i \in I} D_i(p) h_i 1\{a_i = a\}$$

b. The **Derived Supply Curve** model for Hotel licenses in unconstrained areas presented in Figure 3 is inelastic and determined by the quantity of hotel licenses available that are not allocated to hotels in constrained areas.

$$S_{UH}(p) = \bar{x}_H - \sum_{a \in A_C} x_{aH}$$

c. The Hotel Price Premium,$\mu_H = 27{,}850$, is the shadow cost on the constraint for hotels and identifies the price paid for hotel licenses in excess of the Club Base Price. The **Hotel Base Price**, $p^*_{UH} = 33{,}350$, is the market clearing price where:

$$D_{UH}\left(p^*_{UH}\right) = 10{,}356 = S_{UH}\left(p^*_{UH}\right)$$



Panel A: Club Bid Schedules Panel B: Hotel Bid Schedules Panel C: Pooled Bid Schedules

Fig. 4 Pooling bid schedules for licenses allocated to constrained areas

d. Bidders act as price takers, as any bidder could have obtained up to 5 additional licenses by increasing their bids without affecting the market clearing price and would have lost at least 14 units by lowering their bids before affecting the price.

4.3 Partial equilibrium and derived demand and supply in areas with constrained allocations

Complicating the analysis in geographic areas where the quantity of licenses is constrained, Clubs and Hotels compete with each other to determine the Area Price Premium. Pooling different types of bidders requires normalizing the submitted bid schedules in terms of an area price premium to be paid in addition to the base prices for clubs and hotels as defined in Sects. 4.1 and 4.2. After accounting for this complexity, bid schedules can be interpreted as the demand functions found in the general theory.

The derived demand for licenses in these areas aggregates the demand schedules for bidders in the area in excess of the base price determined by the unconstrained markets for each venue type:

$$D_a(p) = \sum_{i \in I} \left[X_i \left(p + p_{UH}^* \right) h_i + X_i \left(p + p_{UC}^* \right) (1 - h_i) \right] 1\{a_i = a\}.$$

Impounding the venue base price into the demand schedule translates each establishment's bid in terms of the premium realized by not allocating the license to a bidder in an unconstrained market. In constrained markets, the inelastic derived supply is fixed at the maximum constraint so $S_a(p) = \bar{x}_a$. The "Area Price Premium," denoted p_a^*, represents the price that clears the market, so that $D_a\left(p_a^*\right) = S_a\left(p_a^*\right)$.

Figure 4 illustrates the partial equilibrium model as it applies to a single area constrained to distribute $\bar{x}_a = 494$ licenses. The aggregated bids reported by hotels to obtain entitlements in the area are represented in Panel A and the aggregated bids reported by Clubs are in Panel B.

Derived demand for specific area entitlements are the marginal values of hotel and club entitlements for operating in the area minus the market price of acquiring the entitlements from areas where allocations are not constrained. Thus, the marginal

Table 2 Closing prices and allocations in constrained areas

Area identifier	Price Premium	Licenses Allocated	License Demand at Closing Price Premium		
			Satisfied	Unsatisfied	Total
Area 105	71,100	77	11	18	29
Area 106	16,190	465	5	15	20
Area 107	30,006	19	1	0	1
Area 110	26,900	494	141	19	160
Area 112	110,221	73	8	7	15
Area 118	97,150	92	8	12	20
Area 121	27,425	702	180	0	180
Area 123	47,310	189	6	59	65
Area 134	21,650	398	28	22	50
Area 159	72,611	93	23	3	26
Area 167	53,653	29	18	2	20
Area 171	9,652	908	57	6	63
Area 176	120,039	38	1	0	1
Area 178	50,900	332	52	39	91
Area 185	36,650	191	35	30	65
Area 186	4,234	639	2	11	13

demand value of an entitlement for the area reflects the profitability of operating in that area minus the market price of the entitlement as illustrated in Panels A and B. The equilibrium model treats location as a factor of production and yields the market demand for the factor as the classical sum of demands of enterprises that use the factor. Demand from hotels (Panel A) and the demand from clubs (Panel B) are normalized and summed to acquire the market demand for the area in terms of the area price premium as illustrated in Panel C. The competitive equilibrium price equates market demand with supply. The normalized prices reflect the values of entitlements set by other markets as predicted by the partial equilibrium model extended across markets.

Area prices reveal the complementary roles of partial equilibrium theory and general equilibrium theory. The roles are often obscured by the complexity that shapes equilibration in multi-market systems. Unconstrained areas operate as a single market for entitlements with a common price for hotel entitlements and a separate, common price for club entitlements. These prices impact the separate areas differently, with constrained club and hotel markets within an area differing. In these markets, the derived demand for the area component of the entitlement becomes visible as a resource that commands a value determined jointly by the demands for club and hotel entitlements in the area. The model of this joint adjustment process as predicted by theory of general equilibrium is illustrated in Fig. 4. Result 3 summarizes the emergence of the subtle, interdependent, theoretical relationships in the data.

Table 2 demonstrates the partial equilibrium properties of the final allocation to each of the constrained areas, with the aggregated demand and supply for each area appearing in Appendix C. It reports the total number of licenses allocated to each market segment, along with that market segment's closing price premium. It also reports the licenses demanded at the closing price, including the satisfied demand and unsatisfied demand. The atom of demand at the closing price demonstrates the lack of bidders' ability to alter prices in the auction mechanism. In each of the markets, a bidder would have been able to obtain at least one more license without impacting prices. A bidder would have been able to decrease the price of the licenses in only three markets, but auction rules prevented bidders from decreasing their bid schedules.

Result 3: Competitive Equilibrium in Areas with Constrained Allocations

a. Each area featuring constrained allocations has a well-defined Derived Demand Curve pooling demands from Hotel and Club Establishments and an inelastic Derived Supply Curve in terms of the Area Price Premium.
b. The Competitive Equilibrium Price where these curves intersect corresponds to the closing price premium for obtaining constrained licenses in the auction mechanism.
c. Bidders act as competitive price takers. Any bidder could have obtained additional licenses by increasing bids without affecting the market clearing price and marginal winners would have lost units by lowering bids without affecting the price paid.

5 Bidding dynamics, excess demand revelation, and equilibrium convergence

This section presents the auction's dynamic features across rounds.[13] We define the demand revealed at the margin between rounds as a measure of unobserved excess demand for licenses suggests prices adjusted following a tâtonnement-like process. Using the rich bidding data, we estimate the parameters of this price adjustment process to verify empirically that it satisfies well-known stability conditions of general equilibrium theory. The theoretical and empirically verified properties of the system demonstrate the final allocation satisfied the conditions of general equilibrium and implemented an efficient allocation across markets.

5.1 Demand revelation and incentives at the margin

To track revisions in bid schedules across rounds, we superscript bids, prices, and demand calculations with their associated round. To illustrate, $B_{il}^t = \left(b_{il}^t, x_{il}^t\right)$represents the l^{th} entry from the i^{th} establishment's bid schedule submitted in the t^{th} round of the mechanism with associated cumulative bid schedule $X_i^t\,(p)$. Similarly, let x_i^t denote the t^{th} round's provisional allocation to establishment i, with p_{aH}^t and p_{aC}^t identi-

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[13] Recall that, at the conclusion of each bidding round, bidders learn the provisional prices for each market so they are aware of the prices they need to meet or beat to obtain a different allocation of entitlements. The integer restrictions on allocations sometimes induced a continuum of equilibrium prices. This was resolved by adopting last-accepted bid rules for the uniform price, rather than first-rejected bid rules. Although these uniform price rules do not make value revelation incentive compatible, as described earlier, theoretically they nevertheless encourage value revelation on the margin as the market price rises through successive bidding rounds.

fying the market clearing prices for this allocation. Given the uniform pricing rule, these market clearing prices identify the point in the demand schedule at which a bidder's incentives for truthful demand revelation have a binding property in the theoretical sense that the individual would choose the quantity demanded as measured by the demand function.[14] For bid schedules with prices above or below this margin, a bidder could respectively inflate or deflate their stated willingness to pay without changing either their allocation or the payment required to receive that allocation.

Round t opens after announcement of provisional prices and quantities allocated based on the bid functions submitted by all bidders in round t-1. As the solution to the optimization problem, these provisional allocations and prices each satisfy the partial equilibrium conditions balancing derived supply and reported demand demonstrated in Sect. 4. Bidders respond by submitting round t's bid schedules. If the bidder is satisfied with their t-1 round allocation at the t-1 round ending price, they have no incentive to revise their reported bid schedule. If all bidders are satisfied with their t-1 round allocation at the t-1 round's ending price, then no bidders would revise their reported bid schedules and the market would close due to inactivity, determining the final allocations and prices.[15] Given their potential as final auction outcomes, the provisional allocations and prices offer incentives for further revelation due to threat of closing.

If a bidder wanted to increase their t-1 round allocation, they could do so by increasing the bid price for some entries in their schedule to be above the t-1 round ending price. Bid revisions arise when bidders decide they want a greater allocation at the announced price, with the system prompting a bidder to consider whether they want more licenses at the announced price. As experimentally demonstrated in Appendix A, the system incentivizes bidders to report the quantity they wish to buy at (or slightly above) the publicly announced prices for the market. At each newly announced price, bidders reveal the maximum quantity they want at that price, a process we refer to as "demand revelation at the margin."

Through demand revelation at the margin, changing prices across auction rounds "trace out" points on the market's aggregate demand curve as illustrated in Fig. 5. The market opens with an initially announced price of P_1, leading bidders to reveal (approximately) the actual quantity demanded at this price in the subsequent round, but with potential under-revelation of demand at higher prices as illustrated in aggregated bid function B_1. The reported bids generating bid function B_1 lead to an announced

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[14] As demonstrated empirically in Sect. 4 and Appendix C no individual bidder has the power to influence prices, so truthfully reporting their demand ensures they obtain exactly the quantity they want regardless of the final price. Such a result is more likely in large-scale auctions such as the one reported here, or auctions with a smaller number of interconnected markets. As advice to bidders, market designers called attention to this possibility when submitting bids. While some bidders followed this strategy, the pilot experiments and testbed studies demonstrated most bidders responded to announced prices and provisional allocations in a marginal manner.

[15] When there was an insufficient amount of bidding activity in a round, the auctioneer would publicly announce that the auction will close if there is insufficient activity in one more round of bidding. If that tentatively "last" round of bidding featured significant revision activity, the auction would again proceed until revision activity ceased again. The auctioneer would then repeat their announcement and the process would continue until there is insufficient revision activity in that "last" round of bidding. In practice, the auctioneer announced only two tentative ends to the bidding process, with the second corresponding to the close of the auction.

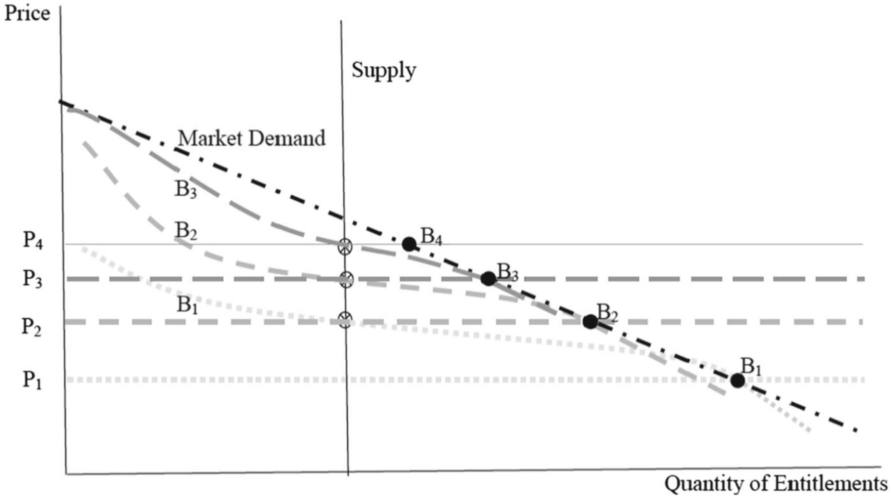

Fig. 5 Illustrating dynamics of demand revelation

price of P_2. The round 2 bid revisions generate a new revealed demand curve, with the point B_2 indicating the true quantity desired at this price. The reported bids generating aggregated schedule B_2 lead to a market clearing price of P_3, which then becomes the operative point of demand revelation for the round 3 bid revisions. As rounds progress and prices continue to rise, demand revelation at the margin will trace out more points very close to the true demand curve until the system converges.

Figure 6 contains the time path of revealed demand for the hotel establishments operating in unconstrained regions. The total number of units allocated to constrained hotels changes very little over periods so the supply of entitlements to unconstrained hotels remains relatively constant (as illustrated in the figure). Shown for each period is the aggregate demand revealed at the market price of for unconstrained hotels. Total demand revealed at the margin appears somewhat inelastic and with the difference between total excess demand at the margin and supply slowly shrinking to zero where the auction terminates. The inset panels at the upper right show the details in terms of units of excess demand at the end of the auction [quantity demanded at the margin – supply = 10,509-10,332] and the entire demand curve as revealed at end of the auction. We formalize these measures of excess demand in the next section.

This multi-round bidding process thus induces a price adjustment process analogous to tâtonnement with uncertainty in how newly submitted bid functions collectively lead to further price adjustments. As prices converge over time, bidders can eventually develop confidence that they will become provisional winners for quantities bid at prices that exceed previous round provisional prices.

5.2 Measuring and satisfying excess demand revealed at the margin

Given the implicit assumption that bidders have stable preferences, we now empirically evaluate the extent to which demand is revealed at the margin as the system progresses

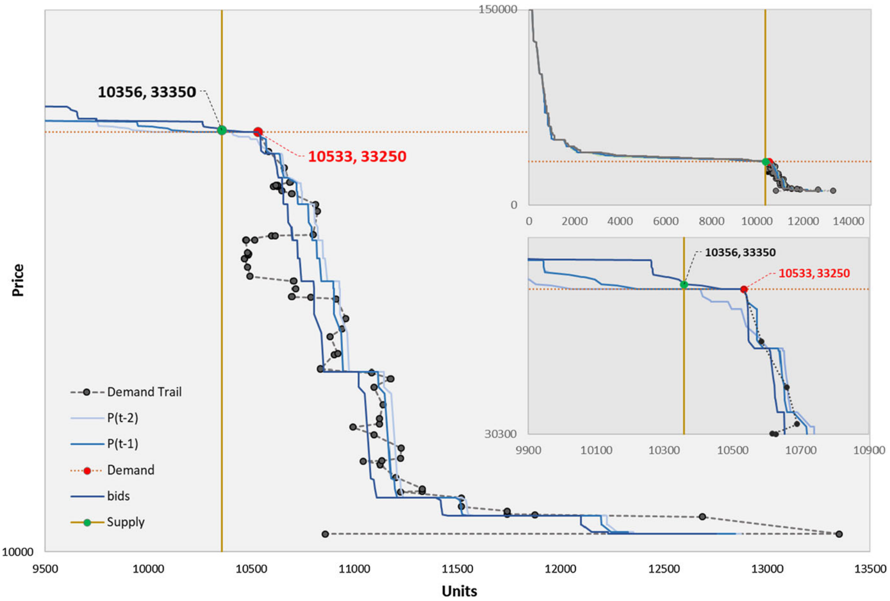

Fig. 6 Marginal demand revelation for unconstrained hotel licenses

through rounds. To begin, define the round t Revealed Demand at any price p as the difference between the round t demand and the round t-1 demand:

$$\Delta X_i^t(p) = X_i^t(p) - X_i^{t-1}(p)$$

Note that if a bidder submits an identical bid schedule in two subsequent rounds, $\Delta X_i^t(p)$ will be zero for all values of p.

Clearly, both the Demand and Revealed Demand measures can be aggregated to the market (area-venue) level by summing over bidders within each market. Define the aggregated (market-level) demand schedules as:

$$X_{aH}^t(p) = \sum_{i \in I} X_i^t(p)\, h_i 1\{a_i = a\}, \text{ and, } X_{aC}^t(p) = \sum_{i \in I} X_i^t(p)(1 - h_i) 1\{a_i = a\}$$
$$\Delta X_{aH}^t(p) = \sum_{i \in I} \Delta X_i^t(p)\, h_i 1\{a_i = a\}, \text{ and, } \Delta X_{aC}^t(p) = \sum_{i \in I} \Delta X_i^t(p)(1 - h_i) 1\{a_i = a\}$$

These measures characterize the aggregate revealed demand in a market as rounds of the mechanism progress, with the higher-level aggregates characterizing total Hotel, Club, and system-wide revealed demand for licenses at any given price.

Given binding incentives at the market clearing price based on the last round's submitted bid schedules, this "opening" price provides the most relevant measure of revealed demand. As the price bidders would have to exceed to obtain additional units relative to their provisional allocation, this price sets bidders' expectations for what bid values are likely to be awarded additional licenses. We adopt this measure, $\Delta X_i^t\left(p_i^{t-1}\right)$, as the empirical measure of Revealed Excess Demand at the Margin.

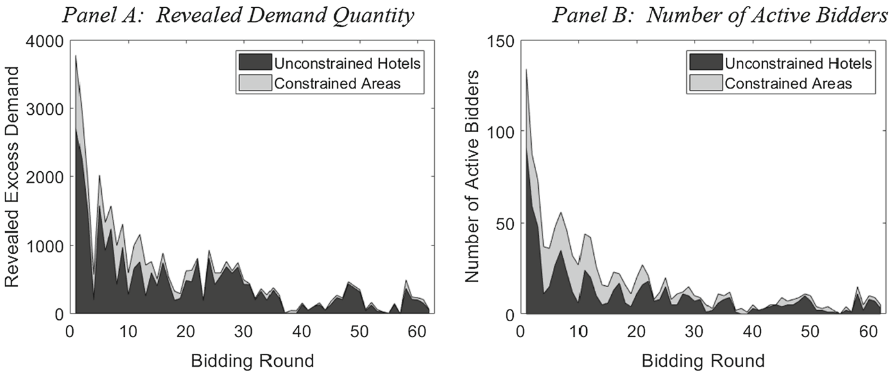

Fig. 7 Revealed excess demand at the margin

Definition 1 The **Revealed Demand at the Margin** for licenses represents the increase in quantity demanded for a license during a bidding round due to revisions in participants bid schedules, valued at the closing price of the previous round. For hotels and clubs in Area a, respectively, this is calculated as:

$$\Delta X_{aH}^{t}\left(p^{t-1}\right) = \sum_{i\in I} \Delta X_{i}^{t}\left(p^{t-1}\right) h_i 1\left\{a_i = a\right\}, \text{ and,}$$
$$\Delta X_{aC}^{t}\left(p^{t-1}\right) = \sum_{i\in I} \Delta X_{i}^{t}\left(p^{t-1}\right) (1-h_i)\, 1\left\{a_i = a\right\}$$

Figure 7 presents the time series of the total Revealed Excess Demand at the Margin from round 2 forward, disaggregated by the type of venue revealing demand. Panel A presents the number of units of revealed excess demand while Panel B reports the number of bidders whose changes in bid schedules revealed demand. As is apparent from the graph, all revealed excess demand is generated by revisions for unconstrained Hotels and bidders in constrained areas, with all revealed demand from Clubs associated with constrained areas. Turning toward the dynamics of aggregate Revealed Excess Demand at the Margin, note the downward trend and convergence toward zero in the later rounds of the market after a few minor bumps in later rounds.

The revealed excess demand in the last round of bid revisions is attributed to the activity of a small number of bidders in only three markets for licenses in constrained areas. The isolated nature of these revisions suggest the system had largely converged by this point. Since these markets' allocations met the constrained maximum for these areas, any additional demand revelation would not change either the area-level allocation or the allocations in unconstrained markets. Rather, any changes to the allocation would only affect the allocation of licenses among those establishments operating within these three markets along with their associated prices.

Result 4: Exhaustion of Revealed Demand at the Margin. Through the rounds of bidding, the auction mechanism exhausted the Revealed Demand at the Margin, demonstrating convergence to an allocation with negligible excess demand.

Fig. 8 Bidding stations in Melbourne convention center

6 Implementation, revenue, and system convergence

The auction took place in two phases.[16] First was an initial round of bid submissions without price or bid revelations that was open for two weeks. During this period an individual bidder could examine or change their own bids as practice with the bidding and information interface. Bidders were required to submit a bid in the initial round to be eligible to participate in the later one-day auction that finalized the entitlement allocation. The initial two-week time period for bid submission during the first round was similar in all respects to the bidding features of later rounds. It was open for many days to ensure that interested bidders had an opportunity to become familiar with the auction interface, consult with advisors when constructing their initial round bid, and communicate with "coaches" who provided technical assistance with bid preparation and submission. Prices and allocations were announced for the first time at the beginning of a one-day, on-site auction that lasted about 10 hours and an additional 62 rounds.

This second phase occurred on-site at a secured convention center in Melbourne. Upon check-in bidders were assigned to a bidding station containing a visually isolated computer workstation and seating for a bidding "team" of up to two individuals as illustrated in Fig. 8. A total of 363 bidding teams participated in the on-site auction.

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[16] The government's original plan was to allocate all 27,500 entitlements in the auction, to take place in early 2010. The government later decided, after the auction rules had been largely designed, to offer existing clubs a capped number of entitlements at a set price. Eligible clubs could buy an entitlement for each gaming machine currently operating at their venue, up to a cap of 40 entitlements per venue. The offer price was based on a percentage of the individual venue's historic gaming revenue, and thus differed across clubs. Most clubs (236 out of 247 eligible) bought at least one offered entitlement, and in aggregate they purchased 8,712 of the 13,750 entitlements available to clubs (63 percent). Following this pre-auction sale, which took place in October and November of 2009, 5,038 club entitlements remained for sale in the auction along with the original 13,750 entitlements available for hotels.

Bidder cell phones were collected and they had no access to public phones. Bidders could not walk through the bidding area unmonitored. They could talk to the bidder on their own team but not bidders on other teams. Coaches were available for assistance or to interface with auctioneers should problems arise.

The auction performed smoothly and ended as planned with no technical difficulties. Price discovery was slower than in the testbed experiments. Unlike in an experiment with induced values, the underlying entitlement values are not observable. Nevertheless, based on an analysis of the bidding behavior we conclude that prices and allocation quantities appear to reflect the conditions of an efficient allocation. The auction permitted entry by new venues as anticipated since bidders other than existing venues were successful in acquiring entitlements.

In its final allocation, the system allocated 18,788 licenses to 428 establishments, generating a total revenue of AU$615M, or about US$555M based on prevailing exchange rates. Table 3 presents summary statistics on the number of bidders in each market, the number of areas assigned their maximal allocations (“Constrained”), and the components of the system’s total revenue. Hotel establishment licenses were capped at their maximal total allocation of 13,750 units. The remaining licenses were allocated to clubs, which collectively generated 12% of the system’s revenue. Sixteen areas’ allocations (out of 88 total) reached the constrained maximum allowance, representing nearly 41% of system-wide revenue.

Figure 9 presents the evolution of prices within the auction mechanism at the end of each round, reporting the base prices for hotel and club licenses (Panel A) as well as the area price premia for licenses allocated to areas with constrained allocations (Panel B). While base prices weakly increase in each round, area price premia can decline due to increases in the base prices. As a result, the price paid by clubs for licenses in constrained areas could decline. For instance, between Period 59 and 60, the Hotel Base Price increases by $914 from $31,261 to $32,175, with Area 185 Price Premium decreasing by $914 from $38,739 to $37,825. The allocation of licenses to hotels in Area 185 and the price paid by those hotels ($70,000) didn’t change during these periods. The price paid for club licenses in Area 185, however, declined from $44,239 to $43,325, while the allocation remained constant since there were no inframarginal bids.

Figure 10 presents the time series of total revenue realized by the system’s provisional allocations, broken down by market segments (unconstrained clubs, unconstrained hotels, constrained clubs, and constrained hotels). The solid black line plots the total reported surplus, based on the total stated willingness to pay from submitted bid schedules for entitlements.

7 Estimating multi-market price formation dynamics and testing stability

Using demand revealed at the margin as a measure of unobserved excess demand for licenses characterizes price adjustment as following a tâtonnement-like process. We use the rich bidding data to estimate the parameters of this price adjustment process and empirically verify that it satisfies well-known stability conditions of general equi-

Table 3 Active bidders, total value, allocations, and revenue

	Number of bidders	Number of areas	Total bid quantity	Total bid value	Awarded quantity	Average price	Total revenue
Clubs							
Unconstrained	103	72	3,757	58,688,745	3,693	5,500	20,311,500
Constrained	64	16	1,617	105,545,885	1,345	36,558	49,171,047
Clubs Total	167	88	5,374	164,234,630	5,038	13,792	69,482,547
Hotels							
Unconstrained	184	72	12,751	498,270,158	10,356	33,350	345,372,600
Constrained	77	16	4,479	280,848,522	3,394	59,019	200,311,424
Hotels Total	261	88	17,230	779,118,680	13,750	39,686	545,684,024
Overall Total	428	176	22,604	943,353,310	18,788	32,743	615,166,571

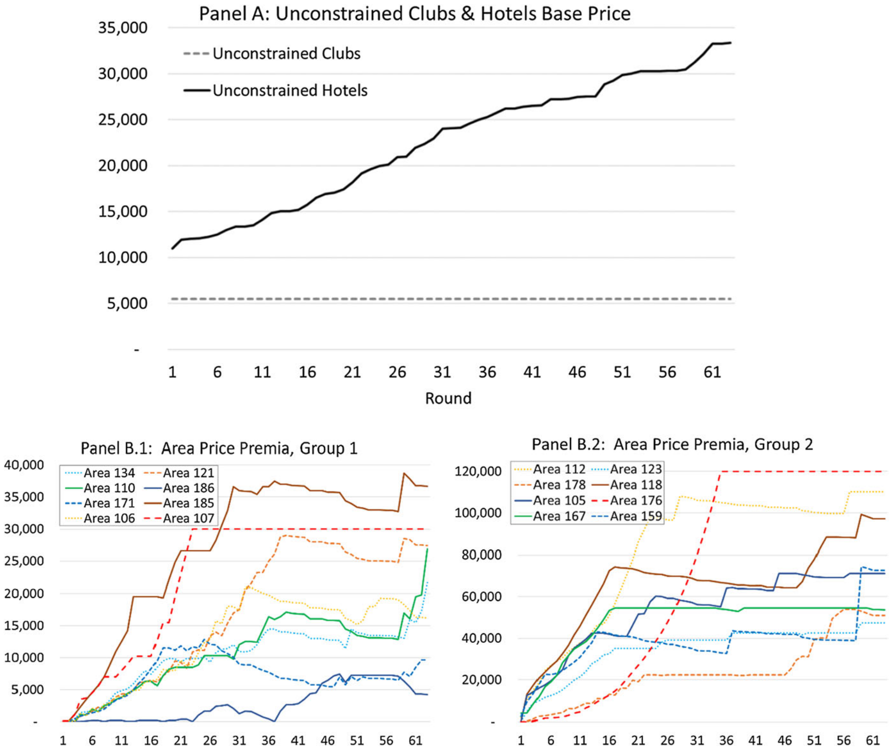
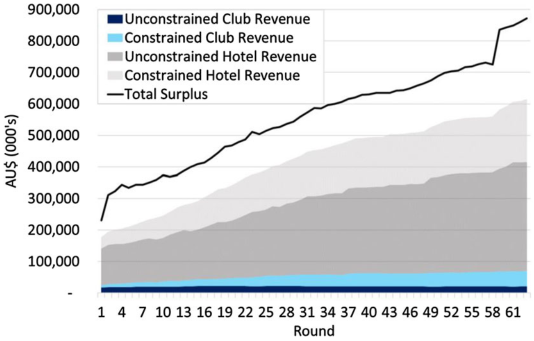


Fig. 9 Time series of prices in unconstrained and constrained markets

Fig. 10 Time series of total revenue from interim allocations each round



librium theory. By satisfying and subsequently reducing revealed excess demand at the margin, the system guides the allocation to a general equilibrium across market segments. First, we demonstrate that the licenses in different markets represent gross substitutes. Second, we show that the evolution of allocations as bidders revise their bid schedules corresponds to adjustments in prices that reduces excess demand. Combined, these results suggest the price adjustment mechanism follows the principles of tâtonnement. Since tâtonnement processes converge to the unique, stable, general equilibrium for gross substitutes, we conclude that the market system's final allocation matches that of a general equilibrium.

7.1 Gross substitutes, stability, and tâtonnement convergence in multiple-markets

From a theoretical perspective, the gross substitute property of preferences, i.e., that excess demand weakly increases in response to an increase in the price for another good, is a sufficient condition for stability of a general equilibrium system.[17] To establish the gross substitute property for licenses in different markets, consider the response of derived supply and demand for licenses in market i as the price for licenses in market j increases. On the supply side, the price increase in market j could increase the number of licenses allocated to market j, weakly reducing the number of licenses available in market i.[18]

On the demand side, bidders are not allowed to reduce the prices in their submitted bid schedules but are allowed to increase their prices. As a result, the aggregated demand schedule is weakly increasing across rounds by construction. Using the notation of derived supply and demand in round t from the previous section, these arguments establish $\frac{\partial S_i^t(p_i^t)}{\partial p_j^t} \leq 0$ and $\frac{\partial D_i^t(p_i^t)}{\partial p_j^t} \geq 0$. Combining the inequalities, excess demand in market i increases weakly in response to changes in other markets' prices, establishing gross substitute property: $\frac{\partial\left[D_i^t(p_i^t)-S_i^t(p_i^t)\right]}{\partial p_j^t} \geq 0$.

Having established the gross substitutes property, we now relate price adjustments to a tâtonnement-like process driven by excess demand. As in the model with a hypothetical Walrasian auctioneer, all provisional allocations and prices are announced based on reported demand with no actual trading taking place, a key feature of tâtonnement. Through revisions to the submitted bid schedules, prices mechanically increase in response to excess demand revealed at the margin pushing up against an inelastic supply curve. This price increase is directly attributable to excess demand in the market for that license. Section 7.2 demonstrates the empirical properties of these dynamics using a structural model of tâtonnement to characterize the adjustment process for prices in response to excess demand.

Result 5: Gross Substitutes, Tâtonnement, and General Equilibrium Stability. Licenses in different market segments represent gross substitutes. Since the auction's

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[17] This classical economic result goes back to Arrow and Hurwicz (1958) and Hahn (1958) and is commonly presented in core textbooks such as Arrow and Hahn (1971); Mas-Colell et al. (1995), and McKenzie (2002). Analyzing markets with indivisibility is considered by Kelso and Crawford (1982), who identify the gross substitutes property as a sufficient condition for the existence of Walrasian equilibria with indivisible goods. Gul and Stacchetti (1999) study the efficiency of Walrasian equilibrium in economies with indivisibilities satisfying gross substitutes.

[18] Technically speaking, the supply of licenses in the mechanism as determined by the vector of prices is defined as a correspondence. The uniquely realized supply from this correspondence minimizes the total value of excess demand in all markets. As such, while realized supply is endogenously determined within the mechanism, excess demand is well-defined and its first order dynamics satisfy the gross substitutes property.

price adjustment mechanism corresponds to a tâtonnement process, theory suggests its convergence leads to a stable general equilibrium allocation.

7.2 Empirical stability and dynamics of multiple-market equilibrium convergence

Stability is an empirical property of price formation as well as a theoretical property of tâtonnement. The multi-round bidding format gives rise to a price adjustment process analogous to tâtonnement with the demand revealed at the margin providing a measure of excess demand. The dynamic relationship between price changes and the revealed demand at the margin characterizes the equilibration process in these markets. Here, we demonstrate the observed process of price adjustment satisfies conditions for stability and general equilibrium convergence to prices supporting an efficient allocation across markets.

Classical analysis of general equilibrium systems[19] characterizes the forces driving price equilibration by differentiating excess demand functions $z_t(p)$ with respect to prices:

$$\frac{\delta}{\delta t} z_t\left(p\right) = \nabla_p z_t\left(p\right) \frac{\delta}{\delta t} p_t + \varepsilon_t = B^{-1} \Delta p_t + \varepsilon_t \tag{2}$$

Because $B^{-1} = \nabla_p z_t\left(p\right)$ is the gradient of current excess demand with respect to prices, the matrix B is sometimes referred to as the Inverse Jacobian of the excess demand function. Solving for Δp_t in Eq. (2) and letting $u_t = B\varepsilon_t$ represents the price adjustment process as a linear function of excess demand that resembles a regression equation.

$$\Delta p_t = B z_t\left(p\right) + u_t \tag{3}$$

Definition 2 The **Empirical Inverse Jacobian** represents the expected change in a price adjustment process predicted by excess demands for goods under a set of disequilibrium prices. Given a time series of price changes and measurements of excess demand in each market, the Empirical Inverse Jacobian can be estimated directly by the Seemingly Unrelated Regression model in Eq. (3).

McFadden (1968) provides sufficient conditions for the matrix B to characterize a stable system. These conditions impose two restrictions on the coefficients that predict price movements in response to excess demand. First, the diagonal entries in B must be positive, so that $\beta_{mm} > 0$ for all m. Second, the determinant of the (m, m) principal

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[19] For brief outlines, see Negishi (1989) for a discussion of tâtonnement and its theoretical background. Reviews of general equilibrium theory can also be found in McKenzie (2002) and Fisher (1989). More extensive discussions of the gross substitute case are found in Arrow and Hahn (1971), Negishi (1972, 1989), Mukherji (1995, 2002), and Mukherji and Guha (2011). Experimental work on stability of general equilibrium can be found in Gillen et al. (2021), which demonstrates that double auction markets that do not satisfy general equilibrium stability conditions follow excess demands and as predicted by the dynamic model, diverge from an interior competitive equilibrium.

minor for the matrix B must be weakly negative for all m. We can thus evaluate whether the system's response to excess demand is stable by testing these two conditions.

Here, we pose two questions. First, as an empirical characterization of equilibration dynamics, do observed prices adjust according to measures of excess demand? We approach this question by directly estimating the price adjustment process in Eq. (3). In so doing, we characterize the "Empirical Inverse Jacobian" and test the significance of its estimated coefficients. Second, does this Empirical Inverse Jacobian satisfy the conditions for stability that would lead to price convergence to equilibrium according to classical analysis of multi-market economic systems? These conditions are well understood from a purely theoretical perspective from which we derive novel econometric tests. The rich data available in the Victoria Gaming Auction provides novel empirical insight into these basic and important theoretical properties.

7.3 Estimating price response to excess demand

We begin by considering the first classical condition for stability of the system presented in Eq. (3). This condition requires the price for a given type of license respond positively to excess demand for that license. Following the analysis from Section 4, the constraints on the system effectively generated eighteen (18) different market segments or types of licenses: Unconstrained Clubs, Unconstrained Hotels, and sixteen (16) Constrained Areas with Area Price Premia. Let N denote the set of derived markets,[20] and let $z_{nt}(p_{t-1})$ denote the excess demand in market n and round t. Consider the price adjustment process for market m, $\Delta p_{mt} = p_{mt} - p_{mt-1}$:

$$\Delta p_{mt} = \beta_{m1} z_{1t}(p_{t-1}) + \cdots + \beta_{mm} z_{mt}(p_{t-1}) + \cdots + \beta_{mN} z_{Nt}(p_{t-1}) + u_{mt} \tag{4}$$

The regression equation (4) relates the expected price changes for market m to the level of disequilibrium in each of the individual markets including the excess demand in m itself, providing a predictive model for price changes driven by excess demand.

Classical Conjecture 1: Prices Adjust Positively in Response to Excess Demand. Consistent with a tâtonnement price adjustment process, auction prices are predicted to adjust positively (negatively) in response to excess demand (supply) for licenses.

Estimating the model requires first specifying a measure of excess demand for each market in each round. "Closing" prices, or the prices associated with any given allocation, are determined to clear markets. Therefore, zero excess demand exists based on stated bid functions at closing prices and provisional allocations. The Revealed Excess Demand at the Margin introduced in section 5.2, however, provides a natural proxy of excess demand at the opening price for a round. The measure takes advantage of reported demand at two different price points where bidders' incentives are binding, at least in determining provisional allocations. As Revealed Demand is reported in terms of licenses for each specific market, these measures need to be translated into a

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[20] Since no variation in price or excess demand exists for the Unconstrained Clubs, we exclude that market from the analysis.

Table 4 Price adjustment in response to excess demand

	UC Hotel	Area 105	Area 106	Area 107	Area 110	Area 112
$\hat{\beta}_{mm}$	1.87E−05	1.57E−02	−1.81E−04	1.19E−04	−2.72E−04	2.09E−04
Std Error	4.07E−06	5.21E−03	1.30E−04	2.39E−04	1.11E−03	6.78E−04
t-Stat	4.60**	3.01**	−1.39	0.50	−0.24	0.31
	Area 118	Area 121	Area 123	Area 134	Area 159	Area 167
$\hat{\beta}_{mm}$	6.83E−04	2.55E−04	−3.92E−04	2.69E−05	−1.53E−02	2.20E−03
Std Error	6.16E−04	2.55E−04	2.45E−04	2.16E−05	2.80E−02	9.36E−04
t-Stat	1.11	1.00	−1.60	1.25	−0.55	2.35**
	Area 171	Area 176	Area 178	Area 185	Area 186	
$\hat{\beta}_{mm}$	9.23E−05	−2.65E−04	1.45E−04	2.76E−04	2.05E−05	
Std Error	1.71E−04	7.66E−04	4.79E−04	2.14E−04	1.72E−04	
t-Stat	0.54	−0.35	0.30	1.29	0.12	

common numeraire for evaluation across markets. Consequently, we measure excess demand by valuing revealed demand according to the opening price of a round:

$$z_{mt}\left(p_{t-1}\right) = p_{m,t-1}\sum_{a,v\in m}\Delta D_{av}^{t}\left(p_{t-1}\right)$$

Table 4 reports the estimated values for the coefficients β_{mm} along with their standard errors, with estimates for all parameters appearing in Appendix D.[21] The estimated coefficients suggest that prices typically respond positively to excess demand, given 12 of the 17 estimated β_{mm} coefficients are positive, with three statistically significant, and none of the negative estimates are significantly negative. These results suggest an affirmative answer to our first question of whether prices respond to excess demand. Further, the direction of the price response is consistent with theory suggesting positive excess demand leads to rising prices and prices drop in response to negative excess demand.

A practical challenge arises in analyzing the regression specification in (4) due to the need to fit 17 coefficients with only 62 rounds of data for each of the 17 market segments (recalling the unconstrained club market is excluded due to lack of variation in price and excess demand). Given the number of free parameters in regression equation (4), one might be concerned that the model is over parameterized as evidenced by the relatively large standard errors and limited significance of estimates in Table 4. To reduce the dimensionality of the problem, we evaluate price changes with respect to the excess demand for licenses in a single market and the total excess demand for licenses in all other markets. That is, define the composite measure of excess demand

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[21] As discussed in the next section, this regression equation represents a system of Seemingly Unrelated Regressions. We estimate these regressions on an equation-by-equation basis in this section to focus on each individual market's price response to excess demand within that market. In the next section, when we consider cross-market restrictions necessary for stability, we exploit the SUR structure to establish joint asymptotic normality of all estimates and justify a parametric bootstrap as a device for calculating standard errors for inference.

Table 5 Excess demand and price dynamics

Coefficient	Estimate	Std Error	t-Stat	RE Std Dev
Panel A: Random effects pooled results across markets				
$\hat{\beta}_1$	0.205	0.066	3.12	0.24
$\hat{\beta}_2$	0.178	0.046	3.88	0.14

Market	$\hat{\beta}_{j1}$	t-Stat	$\hat{\beta}_{j2}$	t-Stat	Market	$\hat{\beta}_{j1}$	t-Stat	$\hat{\beta}_{j2}$	t-Stat
Panel B: Fixed effects individual market results									
UC Hotel	0.335	2.32	0.025	0.17	Area 134	0.595	5.30	−0.081	−0.72
Area 105	0.516	4.17	0.131	1.06	Area 159	−0.073	−0.50	0.322	2.18
Area 106	−0.054	−0.42	0.200	1.55	Area 167	−0.023	−0.19	0.507	4.17
Area 107	0.261	1.90	0.160	1.17	Area 171	−0.021	−0.13	0.297	1.89
Area 110	0.289	2.31	−0.004	−0.03	Area 176	−0.301	−1.98	0.261	1.71
Area 112	0.587	5.23	0.096	0.86	Area 178	−0.123	−0.87	0.014	0.10
Area 118	0.512	5.82	0.440	5.00	Area 185	0.288	2.55	0.372	3.29
Area 121	0.000	0.00	0.151	0.99	Area 186	0.077	0.61	−0.171	−1.35
Area 123	0.532	5.72	0.371	3.99					

for other markets as $z_{(-j)t}\left(p_{t-1}\right) = \sum_{n \neq j} z_{nt}\left(p_{t-1}\right)$ and consider the simplified regression:

$$\Delta p_{mt} = \beta_{m1} z_{mt}\left(p_{t-1}\right) + \beta_{m2} z_{(-m)t}\left(p_{t-1}\right) + \varepsilon_{mt} \tag{5}$$

This specification concentrates the influence of excess demand across other markets, reducing the dimensionality of the regression to enable estimates that are more precise.

Table 5 represents the results for estimating regression specification (5) using a maximum likelihood mixed effects model for heterogeneous coefficients across markets (Panel A) and OLS fixed effects model (Panel B). For ease of interpreting the coefficients, we standardize all independent and dependent variables to mean zero and unit variance by market. Overall, the results demonstrate that positive excess demand is associated with an expected increase in prices in both the mixed-effects and fixed-effects specifications. Prices respond positively to market-specific excess demand in 11 of the 17 markets featuring positive estimates for β_{m1} with t-Statistics exceeding the traditional threshold for 5% significance in eight of these markets. The mixed effects model consolidates these results, further verifying the expected positive sign of β_1 consistent with theoretical restrictions of stability. This robustness result further affirms the results demonstrated in Table 4: the empirical evidence shows that prices change in response to excess demand as posited by theoretical analysis.

Interestingly, Table 5 also demonstrates significant sensitivity for prices to respond to excess demand in other markets. Four of the individual-market regressions demonstrate a significant price response to aggregate excess demand in other markets. Further, the estimated value of β_2 in the random effects model is positive and highly significant. These estimates provide evidence of general equilibrium dynamics in the system that cause prices to shift in response to excess demand in other markets and raise the question of whether these effects might lead to instability in the price adjustment process addressed in the next section.

Result 6: Prices Adjust Positively in Response to Excess Demand. Consistent with the prediction of Classical Conjecture 1, the auction adjusts prices positively in response to excess demand as measured by the Revealed Demand at the Margin.

- In the Empirical Inverse Jacobian, twelve of seventeen markets predict a positive price adjustment to demand revealed at the margin, three of which are statistically significant.
- Pooling excess demand for other markets to create a more parsimonious model yields similar results, with a random-effects specification yielding a positive and statistically significant relationship.
- Estimating the parsimonious model using fixed-effects, eleven of the seventeen markets predict a positive price adjustment to demand revealed at the margin with eight of these demonstrating statistical significance.

7.4 Testing stability of price adjustment process

Given our measures of excess demand and observed interim prices, we can treat tâtonnement as an empirical process rather than a purely theoretical construct. That is, treating the dynamic price adjustment process specified in Eq. (3) as regression equation (4) allows us to estimate the Empirical Inverse Jacobian for the price adjustment process. By estimating the rates of adjustment for prices in response to observed excess demand, we can estimate and test the hypothesis that observed price adjustments are consistent with classical restrictions of stability under which prices converge to equilibrium.

The regression specifications (4) and (5) also provide a device for evaluating the stability of the observed price dynamics. A variety of classical models for disequilibrium price dynamics characterize the system of price dynamics in response to prevailing excess demand in multiple markets. These models consider the full system of price changes:

$$\begin{bmatrix} \Delta p_{1t} \\ \vdots \\ \Delta p_{Nt} \end{bmatrix} = \Delta p_t = B z_t\left(p_{t-1}\right) + \varepsilon_t = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1N} \\ \vdots & \ddots & \vdots \\ \beta_{N1} & \cdots & \beta_{NN} \end{bmatrix} \begin{bmatrix} z_{1t} \\ \vdots \\ z_{Nt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \vdots \\ \varepsilon_{Nt} \end{bmatrix} \quad (6)$$

Recalling McFadden (1968) two conditions for the matrix B to characterize a stable system, we test if (1) the diagonal entries in B are positive, so that $\beta_{mm} > 0$ and (2) the determinants of the diagonal principal minors for the matrix B are all weakly negative.

We have already evaluated condition (1) in the previous subsection, with the results in Tables 4 and 5 providing supportive evidence that prices respond positively to excess demand within a market. Indeed, results for the mixed effects model suggests fewer than 20% of samples will yield negative estimates for β_{m1}, which corresponds to the β_{mm} coefficient in model (4). Considering the regression estimates presented in Table 5, we note positive coefficient estimates suggest that the prices increase with excess demand in twelve of the seventeen markets. While five markets have negative coefficient estimates, suggesting that prices actually fall in the presence of significant excess demand, none of these are statistically significant.

Classical Conjecture 2: Price Adjustment Stability is Robust to Cross-Market Influences. The Empirical Inverse Jacobian satisfies classical tâtonnement conditions for stability:

1) Its diagonal entries are positive (as stated in Hypothesis 1), and,
2) The determinants of the principal minors along the diagonal of the Empirical Inverse Jacobian are weakly negative.

The second condition for determining stability is somewhat more challenging to test and, to our knowledge, has not yet been performed in a multiple market economic system. From an econometric perspective, the test is facilitated by the Seemingly Unrelated Regression (SUR) representation implied by (6). Given coefficient estimates in the matrix B, computing the determinant of the principal minors for each of its diagonal elements is straightforward. Since the determinant is a continuous function, a delta-method approximation will establish asymptotic normality and a parametric bootstrap is available to construct confidence intervals. Appendix B reports he details of this inference technique, including the parametric bootstrap.

Table 6 presents the estimates of the relevant determinants for each market, along with their 95% confidence intervals. Panel A characterizes confidence intervals for the maximum of the determinants across all principal minors of the B matrix from Regression (6). Though the bootstrapped mean of this statistic is positive, it is quite close to zero and an order of magnitude smaller than the bootstrapped standard error of the estimate so that the 95% confidence interval clearly includes zero. An asymptotic approximation assigns a p-value of only 0.34 for the null hypothesis that the maximum determinant is weakly negative, which does not support rejecting the second condition of stability at any reasonable level.

Panel B of Table 6 presents the estimated principal minor determinants of the Empirical Inverse Jacobian for each market along with their 95% confidence intervals. Though many markets' point estimates for these determinants are positive, the 95% confidence interval always includes zero in every case and their magnitudes are extremely small. One challenge associated with the test relates to the relatively large number of parameters involved relative to the number of rounds for which we have bidding data, and the large standard errors suggest the test features limited power in this sample.

Overall, our analysis of the Empirical Inverse Jacobian of excess demand demonstrates that the price dynamics observed in the Victoria Gaming Auction are consistent with a stable equilibration process. First, prices in a market shift positively in response to excess demand, increasing when excess demand in the market is positive and decreasing when excess demand is negative. Second, excess demand in other markets does not generate unstable general equilibrium effects in the price adjustment process. Showing the Empirical Inverse Jacobian statisfies these stability conditions provides further evidence that the price adjustment process leads to a stable equilib-

Table 6 Estimates and bootstrap confidence intervals for determinant test of stability

Panel A: Bootstrap confidence interval for maximum principal minor determinant across markets

Lower 95%	Estimate	Upper 95%
−1.14E−42	3.05E−43	1.75E−42

Panel B: Bootstrap confidence interval for principal minor determinants by market

Market	Lower 95%	Estimate	Upper 95%	Market	Lower 95%	Estimate	Upper 95%
Uncon Hotel	−2.17E−42	1.26E−44	2.19E−42	*Area 134*	−2.32E−44	−7.17E−47	2.31E−44
Area 105	−1.77E−44	1.55E−48	1.77E−44	*Area 159*	−1.43E−45	9.70E−48	1.45E−45
Area 106	−4.59E−44	−1.14E−46	4.56E−44	*Area 167*	−1.07E−45	−1.33E−48	1.07E−45
Area 107	−3.73E−45	4.38E−48	3.73E−45	*Area 171*	−6.80E−44	−4.09E−47	6.80E−44
Area 110	−2.31E−44	−1.37E−47	2.31E−44	*Area 176*	−4.88E−45	−1.92E−48	4.88E−45
Area 112	−1.25E−44	2.31E−47	1.26E−44	*Area 178*	−1.60E−44	−4.43E−47	1.59E−44
Area 118	−1.70E−44	2.60E−47	1.71E−44	*Area 185*	−3.02E−44	4.26E−47	3.03E−44
Area 121	−1.46E−44	8.51E−48	1.46E−44	*Area 186*	−1.10E−43	3.29E−46	1.10E−43
Area 123	−6.05E−44	−6.56E−47	6.03E−44				

rium in prices. Beyond informing the scientific interest in empirically investigating conditions for tâtonnement convergence, this stable equilibrium further supports the efficiency of the final allocation achieved by the auction mechanism.

Result 7: The Price Adjustment Process Satisfies Tâtonnement Conditions for Convergence to General Equilibrium.

We estimate and test the determinants for the diagonal principal minors of the Empirical Inverse Jacobian using a delta method approximation, finding that:

- The 95% confidence intervals for all seventeen determinants include zero.
- Eight of the seventeen principal minors have negative estimated determinants.
- The 95% confidence interval for the maximal determinant of the seventeen principal minors includes zero. The one-sided hypothesis test that the maximal determinant is weakly negative receives a p-value of only 0.34 and is not rejected.

8 Conclusion and after market evaluation

This paper reports the design and implementation of a Victoria Gaming Auction mechanism to solve a complex allocation problem that involved 176 interdependent markets and prices, 18,788 entitlements and 363 bidders. Social policies led to constraints on the distribution of gambling activities in a highly regulated industry. Addressing these government concerns presented a challenging policy design problem. The analysis approached this problem by starting with theoretical properties of an efficient allocation, identified as the solution to a constrained surplus maximization problem in which participants have well formed, but unobserved, preferences. Extensive experimental testbedding determined the performance of the auction mechanism, including practical elements of its function, rules, and technical issues. The successful transition from the lab to the field is supported by theoretical principles, and verified by the empirical properties from the time-series of observed price dynamics and underlying excess demand. The mechanism itself is based on competitive economic theory of markets with practical features suggested by some of the prominent features of the classical tâtonnement theory of price adjustment and refined through an experimental testbedding process.

In the end, we demonstrate that the mechanism achieved its basic design and assessment goals. First, it did what is was designed to do. The resulting allocation satisfied basic properties of efficiency subject to the fact that the complex legal, political and social goals were met. Secondly, the data demonstrate that the design success can be attributed to the underlying principles from which the design was constructed. The results were not a consequence of luck or some arbitrary random events.

A competitive theory of general equilibrium underlies the principles determining allocations within the mechanism. If individuals prefer to buy more entitlements at a stated price, they can attempt to obtain additional licenses by simply increasing their bids. The decisions result in measurements naturally interpreted as market demand functions. The auction responded to such revealed demand functions and resulting excess demands by adjusting allocations and prices to reflect an efficient allocation and equilibrium supporting prices given value revelations of participants. In the last round of the auction's operation, only five bidders made small adjustments to their bid schedules, suggesting that bidders were satisfied with their allocations at the prevailing prices. In effect, participants' unwillingness to revise their bid schedules suggests ex post efficiency of the final allocation.

The bidding process also provided new insights into the dynamics by which demand and excess demand are revealed through the auction mechanism. Since bidders' incentives bind only at the interim prices and allocations announced between rounds, bidders truthfully revealed demand at these prices and allocations. This principle of "demand revelation at the margin" allows us to measure excess demand and demonstrate that excess demand diminishes as bidding rounds progress and prices increase. This property is closely related to a general principle–treated as a theoretical axiom since Walras–that if excess demand of a commodity is negative then other things being equal, its price will fall.[22]

Finally, we observe how relative prices evolve across bidding rounds in response to these revealed excess demands. This provides a unique opportunity to evaluate classical properties of multi-market, equilibrating dynamics. We define the Empirical Inverse Jacobian as the empirical counterpart to the inverse Jacobian of excess demand governing price dynamics under tâtonnement price adjustments. We estimate this Empirical Inverse Jacobian using the Victoria Gaming Auction interim prices, bids, and allocations to compute observed price changes and imputed excess demand. We derive tests to show the Empirical Inverse Jacobian satisfies classical conditions for stability, with results supporting the hypothesis that prices converge toward their equilibrium values as revealed by the model. Though abstract, theoretical considerations based on the Sonnenshein-Mantel-Debreu Theorem (Mas-Colell et al. 1995, Chapter 17) can be interpreted as calling into question theoretical micro foundations of general equilibrium dynamics, our focus on observed system behavior renders such considerations irrelevant for the application we study. Because empirical evidence suggests the system satisfies conditions necessary for convergence to a stable general equilibrium, we can conclude that such convergence does occur. These results demonstrate the importance and power of classical general equilibrium theory in addressing real-world market design problems.

In sum, the Victoria Gaming Auction delivered a stable and efficient allocation for licenses across a large number and variety of markets and establishments. This allocation satisfied policy constraints while generating revenues of AU\$614 million for the Victoria Government in a ten-hour period plus an additional AU\$366 million from the pre-auction offer to existing bidders – a total of AU\$980 million. Its success demonstrates the effectiveness of combining economic theory with experimental testbedding in applied mechanism design. It also shows the usefulness of using laboratory experiments for revealing the content and reliability of basic economic principles in the context of a multidisciplinary and politically and legally sensitive policy. Finally, evaluating the mechanism's performance and analyzing the time series of prices and

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[22] See the discussion in Mukerji (2002, p. 74), or in McKenzie (2002, p. 54). Walras (1954, p. 170) notes the property as fundamental: "If the demand for any one commodity is greater than the offer, the price of that commodity in terms of the numeraire will rise; if the offer is greater than the demand, the price will fall".

demand provides new insights into the market and excess demand forces, at the heart of general equilibrium theory, driving price equilibration in multiple market settings.

Could the auction be used in other places? The auction rests on the most basic and general of economic theories. In contrast to popular auction theory, special features of game theory play no role and the technical aspects used in the construction of the auction are widely used in economics and management. The implementation and testing are well known to experimental economics. The auction could have a wide range of applications due to the generality of the basic principles and the common use of (linear) constraints to state and represent social policies.

Declarations

Conflict of interest The authors have no relevant financial or non-financial interests to disclose. Financial support for this research is indicated in the acknowledgement footnote.

Appendix A: Experimental Testbeds

A.1. Introduction

We assessed the new form of auction in various possible environments and under alternative auction rules through a new application of testbed laboratory experimental methods. The exploratory methodology provided a source of experience with market organizations prior to their implementation in naturally occurring, uncontrolled, possibly extreme or unlikely environments. The experiments provide insights that helped formulate judgements about the reliability and robustness of the theory and basic economics principles used to guide the market's implementation. The approach is especially attractive when tools based on random samples, independence, and tests of clear theoretical predictions are not available due to constrained research budgets, vague and changing policy goals, and complex or incomplete theory. In complex field applications, the exploratory research provided useful information about qualitative patterns as opposed to quantitative, statistical replicability.

This appendix presents the structure of the testbed experiments we developed to guide judgements about auction design details, auction implementation and auction control. The words "experience" and "judgement" are appropriate since parameter choices, experimental designs, or numbers of observations were not chosen to test hypotheses formally, but rather to guide the subjective assessments of the auction designers about how the auction might perform in the field. It was impossible to remove all uncertainties about how the auction might perform. The tests described here focus on issues related to auction scale, efficiency, sector prices and time controls. Only a limited number of observations were possible relative to a very large body of uncertainties and parameters.

A.2. Testbed Scope

The testbeds were used as support for a wide range of decisions not discussed in detail here. The omitted issues include software development, network and hardware architecture reliability (including procedures related to computational speeds), hacking vulnerability, sabotage, catastrophic failures (fire or power outage), and monitoring tools. Additional decisions included screen displays, colors, language, information locations and popup help screens. Considerable effort was devoted to simplify for participants the form of bids, bid sheets, bidding screens and bidder feedback. The testbeds were used to determine procedural matters such as methods of problem detection/recovery as well as timing between and during periods. Decisions regarding important features of the auction architecture were based on testbed experiences, including the structure of bids (functions or scalars), submitted as demand or inverse demand, the use of rounds as opposed to continuous time, and the timing of and time between rounds. Testbeds also focused on different increment requirements, the form and timing of stopping rules (new bids, number of people attempting to acquire increased quantity, amount of the attempted quantity increase, the number of markets changing price), procedures to prevent strategic delays, and methods of dealing with bidder “mistakes.” The testbeds used “coaches” to facilitate instructions, report problems to experimenters, and quickly address local problems like typos that might need interventions or corrections. The testbeds were also used to train the coaches that would be used in the actual auction.

A.3. Performance tests: scale, efficiency, convergence

A.3.1 Scale

Parameters for testbed experiments were typically scaled up using a base module constructed from the prominent properties of the upcoming field application. This scalability was considered fundamental. The scale of the first experiments was very small. Whether or not the basic principles used in the design would withstand scale increases was unknown. In the base module, eight agents were assigned to eight “types” representing the two forms of establishments (hotel or club) and four areas (two urban and two rural). Constraints were imposed on the total number of experimental units (“licenses”) that could be acquired by each type. Following the features of the actual auction, each area was also constrained. Induced individual demands for licenses were linear and market demands were based on the additive aggregation of the competitive model. Each bidder operated as a single establishment and in a given area with no preferences for entitlements in other establishments or in other areas. Entitlement preferences for an individual agent were of the form $Value = A - bx$ where A is the value of the first license and x is the number of licenses the individual acquired.

Experiments were conducted at different scales implemented using two different dimensions. The first dimension increased the number of agents of each “type”. At small scale, the number of agents of each type was always one agent per type in each market. This was increased to two and to three for some expansion of scale. The

Table 7 Base experimental module parameters: ten markets and ten agents - five Y markets (hotels) and five Z markets (clubs)

		A Metro	A Metro	B Rural	B Rural	B Rural	Market Supply
Y: Hotel Markets	Market ID	A1Y	A2Y	B3Y	B4Y	B5Y	$\bar{Y} = 300$
	A	240,000	230,000	8,000	7,000	7,000	
	b	1,000	1,000	50	50	50	
Z: Clubs Markets	Market ID	A1Z	A2Z	B3Z	B4Z	B5Z	$\bar{Z} = 300$
	A	192,000	184,000	6,000	5,600	5,600	
	b	800	175	150	100	100	
Area Supply		$\bar{A}_1 = 200$	$\bar{A}_2 = 175$	$\bar{B}_3 = 150$	$\bar{B}_4 = 100$	$\bar{B}_5 = 100$	

second dimension increased the number of areas. The base module had four areas and scale increases were implemented by adding base modules of individual areas or all areas. So as scale expanded the number of areas increased in the series 4, 8, 12, 16 …, if all areas were replicated. Each area had two markets (clubs and hotels) so as scale increased the number of markets in an experiment expanded in the series 8, 16, 24, 32 …. Alternatively, other numbers of, say, 10 markets could be studied and, due to the linearity in the parameters and the convenience of adding or subtracting from the number of licenses offered, adjustments to equilibrium predictions were easily computed. Similarly, the consequences of constraint rearrangements were easily recalculated. Table 7 shows example individual preference parameters A and b for each type/market and these were the same for all agents in a given market. Parameters were the same across all experiments unless the upper level of demand and prices were adjusted. Such adjustments would simply change the price predictions by a constant.

A total of 40 experiments were conducted ranging from two areas with 4 agents to 60 markets with 140 agents. Simulations with artificial agent bidders were conducted for 200 markets with 800 agents. The experiments explored a wide range of issues, including software reliability, subject displays and feedback, forms of auctions, training, auction controls, disruptions (bidding errors, computational speed, power outage) auction management, monitoring, convergence speed and basic economic theory. The discussions here will focus on the results of a single experiment that illustrates a broad pattern of questions and results. Aside from subject mistakes and software issues, scale made little difference to assessments.

Experiment 100223 engaged ten markets with three agents in each for a total of 30 (human) agents. This was accomplished by adding two agents to each market, each with the linear demands described in Table 7. Since the experiment has three rather than one agent per market the supplies must be multiplied by 3 to 900 Hotel licenses and 900 Club licenses. Individual demand functions in a given market are the same for each agent in the market.

The price discovery and allocation results of the auction were tested from several different perspectives using the model as the assessment tool. Those include prices in individual markets and collections of markets, income of participants (and income distribution), accuracy of the Lagrangian tools as predictors, convergence process and speed, termination procedures, total revenue, and efficiency. Details of individual behaviors were examined, including revelation at the margin and non-revelation of infra marginal units. The market closing and ending were also studied since the time constraints on the auction required that the action end within an appropriate time and thus required some control that would not interfere with the price discovery process.

A.3.2 Revenue, Efficiency and Prices

Figure 11 presents the time series of revenue and total value as revealed by the sum of bids across 17 auction rounds taken from Experiment 100223. The figure illustrates three important properties. First, the system converges as opposed to behaving randomly or oscillating. Secondly, it operates efficiently as revealed by the observed social welfare reaching a level nearly as high as the optimal allocation. Third, the performance is close to theory as is illustrated by the convergence to predicted total revenue and observed social welfare relative to the optimum. These relationships can only hold if all equations are close to the theoretical solution.

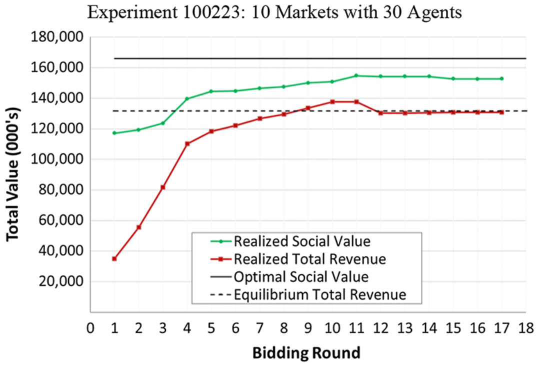
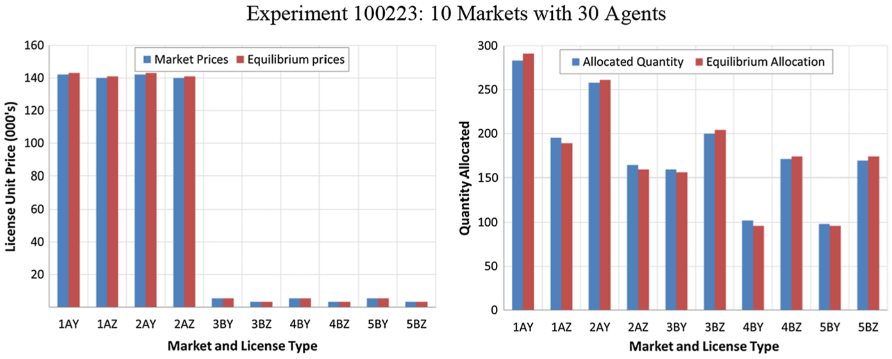


Fig. 11 Time series of revenue and total value in testbed market



Panel A: Final Market Prices

Panel B: Final Market Allocations

Fig. 12 Final prices and allocations in testbed market

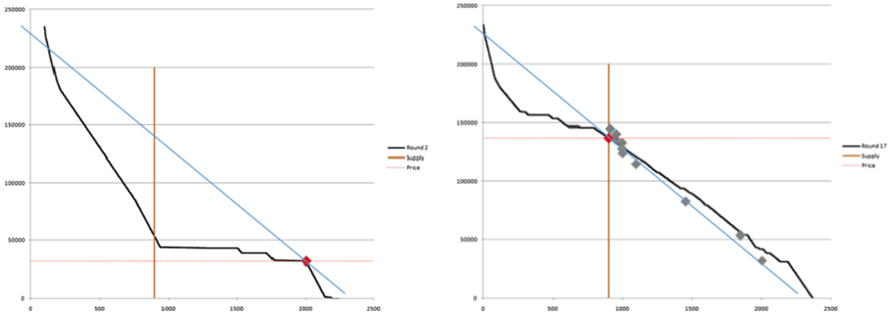
Figure 12 displays the final prices in each of the 10 markets. In the top panel presents the prices in each separate market in relation to the theoretical predictions. The dramatic difference in demand relative to supply between the metropolitan and rural areas is realized by the final prices. The bottom panel illustrates final quantities in each area compared to the predictions of the model as derived from the induced parameters. As can be seen the equilibrium predictions are very close to the final outcomes.

A.3.3 Bidding Behavior

The following figures illustrate prominent features of individual bidding behavior. The left Panel A of Fig. 13 illustrates the pattern of bidding for hotels in the first period. The induced preferences are linear so bids that reveal true preferences would follow the linear pattern. Panel A indicates the aggregated bid schedule is similar to a hyperbola meaning that the individuals discount infra marginal bids. As periods progress the bids increase with each round, however, approaching the underlying market demand functions. The bids for hotels in the last period are shown in Panel B to the right. The revealed bids beyond the fixed supply are very close to the induced linear demand. The points on the curve record the maximum quantity demanded at the stated price. This is the principle of demand revelation at the margin discussed in the next paragraph and figure.

Bidding behavior in the testbed experiments exhibits the property we discuss prominently in the main text: Revelation of Excess Demand at the Margin. Figure 14 demonstrates for three experiments that revelation of quantity demanded occurs near the margin of where the stated price equals the demand prices. That is, let the induced quantity demanded function for an individual be $D(P)$ and let the individual demand function revealed by the individual's bid be $B(P)$. If P^* is the market price then $B(P^*)$-$D(P^*)$ is the quantity revealed at the margin. If $B(P^*)$-$D(P^*) = 0$ the demand is fully revealed at the margin but if $B(P^*)$-$D(P^*) <0$ the revelation falls short at the margin. The middle figure tracks relative proportion of revelation shortfalls of this

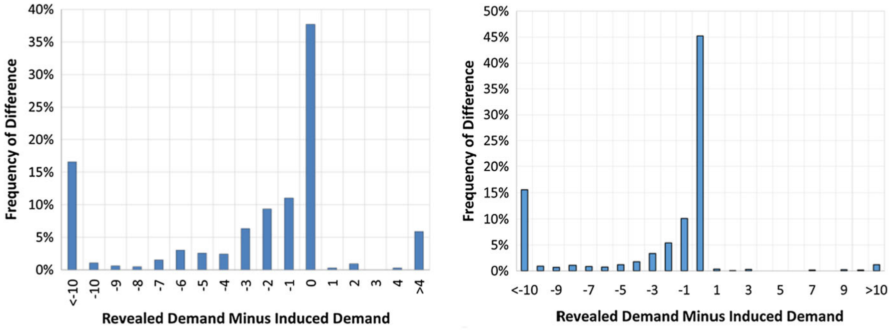
Experiment 100223: 10 Markets with 30 Agents



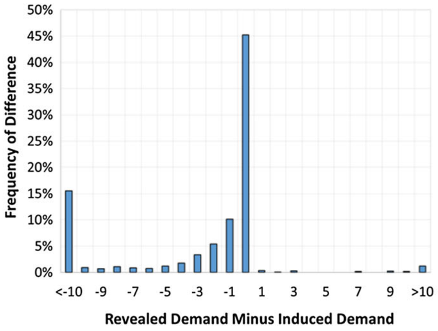
Panel A: First Period Bidding

Panel B: Last Period Bidding

Fig. 13 First period and last period hotel bids



Panel A: 100331 *Panel B: 091115*



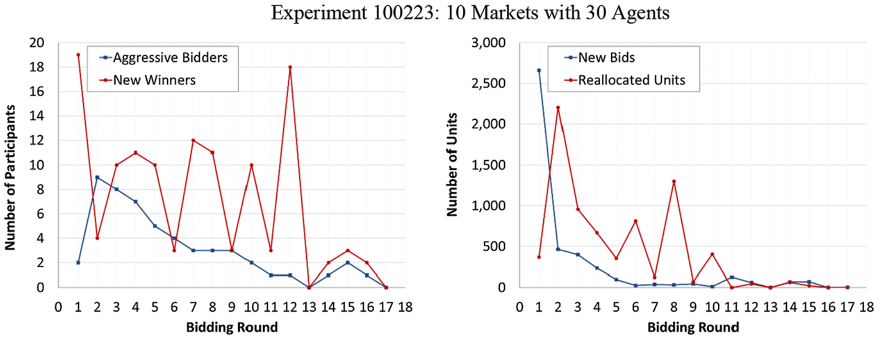
Panel C: 100210

Fig. 14 Revealed demand minus payoff demand in last period

revelation process for all subjects in all periods of three experiments as the prices change with advancing periods. The total demand function is seldom revealed but the value at the margin (the revealed quantity demanded at the stated price) is frequently revealed within one unit. For the three experiments the shortfall is either zero or one unit for 35%-40% of the individual bids. Revelation failures that occur are typically shortfalls, below the induced demand.

A.3.4 Convergence

Bidding activity at the ending of auctions suggest convergence to the equilibrium of the model. The two panels of Fig. 15 display bidding activity across the seventeen rounds of Experiment 100223 and reveal bidding near the end of the auction in relation to the auction closing rules. Bidding for infra-marginal units is less constrained by theory since bid prices do not affect price paid due to the auction’s price determination rule. The ending itself depends on both the number of bidders with new bids and the number of markets with changed prices. Panel A on the left measures the number of aggressive bidders attempting to increase their allocated quantity and the number of bidders who are awarded increased allocations. These are defined as aggressive bids and are measured in attempted number of units increase at the stated market price. The


Experiment 100223: 10 Markets with 30 Agents

Panel A: Active Bidders (New/Aggressive)

Panel B: New Bid Units and Reallocations

Fig. 15 Time series of new bids and reallocation activity

panel on the right measures the associated quantities of units associated with new bids and reallocated quantities due to this bidding activity as the auction progressed. As can be seen both are converging toward zero, which indicates the auction was approaching a natural ending.

Appendix B: Statistical Properties of Empirical Inverse Jacobian

This appendix addresses the sampling properties the Empirical Inverse Jacobian specified in the regression equation (6):

$$\begin{bmatrix} \Delta p_{1t} \\ \vdots \\ \Delta p_{Nt} \end{bmatrix} = \Delta p_t = Bz_t\left(p_{t-1}\right) + \varepsilon_t$$
$$= \begin{bmatrix} \beta_{11} & \cdots & \beta_{1N} \\ \vdots & \ddots & \vdots \\ \beta_{N1} & \cdots & \beta_{NN} \end{bmatrix} \begin{bmatrix} z_{1t} \\ \vdots \\ z_{Nt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \vdots \\ \varepsilon_{Nt} \end{bmatrix} \qquad \text{(B.1)}$$

The sampling properties of this regression equation are straightforwardly derived within a SUR framework. Under the usual regularity conditions, the estimated coefficients in the B matrix are consistent and asymptotically normally distributed. Given this asymptotic normality for the coefficients in the B matrix, testing the two conditions necessary for stability of the Empirical Inverse Jacobian is a relatively straightforward exercise.

B.1 Asymptotic Distribution and Testing Hypotheses for Individual Coefficients

The first condition for stability of the Empirical Inverse Jacobian holds that the diagonal entries in the B matrix are non-negative, setting up the null hypothesis that these coefficients are negative against the alternative that they are weakly positive. The usual test statistics for evaluating this null hypothesis can be estimated using the asymptotic approximation and asymptotic standard errors without complication as presented in section 7.3.

B.2 Asymptotic Distribution and Testing Hypotheses for Determinants of Principal Minors

The second condition for stability of the Empirical Inverse Jacobian holds that the determinants of each principal minor are weakly negative. Denoting the principal minor for the m^{th} market by

$$B_{(-m)} = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1,m-1} & \beta_{1,m+1} & \cdots & \beta_{1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \beta_{m-1,1} & \cdots & \beta_{m-1,m-1} & \beta_{m-1,m+1} & \cdots & \beta_{m-1,N} \\ \beta_{m+1,1} & \cdots & \beta_{m+1,m-1} & \beta_{m+1,m+1} & \cdots & \beta_{m+1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \beta_{N,1} & \cdots & \beta_{N,m-1} & \beta_{N,m+1} & \cdots & \beta_{N,N} \end{bmatrix}.$$

Letting $\tau_m \equiv \det\left(B_{(-m)}\right)$ denote the determinant for this principal minor matrix, we seek to test:

$$H_0 : \tau_m \leq 0 \, \text{versus} \, H_1 : \tau_m > 0.$$

Note that the estimated coefficients are all jointly asymptotically normally distributed and the determinant is a continuous function. As such, the estimated determinant of the principal minor, which replaces the entries of $B_{(-m)}$ with their OLS estimates (denoted $\hat{B}_{(-m)}$), is also consistent and asymptotically normal:

$$\hat{\tau}_m \equiv \det\left(\hat{B}_{(-m)}\right) \to N\left(\tau_m, \sigma^2_{\tau_m}\right)$$

The asymptotic variance $\sigma^2_{\tau_m}$ can, in principle, be found using a delta-method approximation. In practice, though, such an analytical exercise would prove exceedingly complicated. In practice, this asymptotic variance can be approximated using numerical methods applying a bootstrap technique.

We adopt a parametric bootstrap to estimate the variance $\sigma^2_{\tau_m}$. The bootstrap algorithm we adopt, and our methodology for constructing confidence intervals, proceeds as follows:

Algorithm 1: Bootstrap Confidence Intervals for Principal Minor Determinants

Step 1: Estimate the regression model B.1

Step 2: Recover the estimated coefficients $\hat{B}$ and residuals $\left\{\hat{\varepsilon}_t\right\}_{t=1}^{T}$, where, $\hat{\varepsilon}_t = \left[\hat{\varepsilon}_{1t}, \ldots, \hat{\varepsilon}_{Nt}\right]'$

Step 3: Estimate the covariance matrix for residuals from $\left\{\hat{\varepsilon}_t\right\}_{t=1}^{T}$ applying the usual degree of freedom correction, denoted $\hat{\Sigma}_{\varepsilon}$.

Step 4: Generate K Bootstrap Samples:

For $k = 1, \ldots, K$

Step k.1: Draw simulated residuals $\left\{\tilde{\varepsilon}_t^{(k)}\right\}_{t=1}^{T}$, where, $\tilde{\varepsilon}_t^{(k)} \sim N\left(0, \hat{\Sigma}_{\varepsilon}\right)$

Step k.2: Calculate $\left\{\Delta p_t^{(k)}\right\}_{t=1}^{T}$, where, $\Delta p_t^{(k)} = \hat{B} z_t\left(p_{t-1}\right) + \tilde{\varepsilon}_t^{(k)}$

Step k.3: Estimate regression model: $\Delta p_t^{(k)} = \tilde{B}^{(k)} z_t\left(p_{t-1}\right) + u_t^{(k)}$

Step k.4: Compute the principal minor determinants, $\tau_m^{(k)} = \det\left(\tilde{B}_{(-m)}^{(k)}\right)$

Next k

Step 5: Compute the bootstrap standard error, $\hat{\sigma}_{\tau_m}^2$, for the m^{th} principal minor determinant from the sample $\left\{\tau_m^{(k)}\right\}_{k=1}^{K}$.

Step 6: Compute the 95% confidence interval using the usual critical values:

$$\hat{C}_{0.95}\left(\hat{\tau}_m\right) = \left[\hat{\tau}_m - 1.96\hat{\sigma}_{\tau_m}, \hat{\tau}_m + 1.96\hat{\sigma}_{\tau_m}\right]$$

Applying this algorithm for each of the markets under consideration provides a straightforward mechanism for calculating the confidence intervals reported in Table 6, Panel B.

B.3 Testing Hypotheses for Maximum Determinants of All Principal Minors

The restriction that the determinants of all principal minors are weakly negative can be analyzed by considering the maximum of the determinants for each of the principal minors considered in the previous subsection. Specifically, define:

$$\tau = \max_{m=1,\ldots,N}\left\{\tau_m\right\}$$

As the max operator is another continuous function, estimates for τ satisfy all of the asymptotic sampling properties from the previous section, so that:

$$\hat{\tau} \equiv \max_{m=1,\ldots,N}\left\{\hat{\tau}_m\right\} \rightarrow N\left(\tau, \sigma_\tau^2\right)$$

As above, the asymptotic variance can in principal be solved for analytically, but in practice can be estimated using the bootstrap samples generated from Algorithm 1.

We begin by applying Algorithm 1, Steps 1 through 3. We then perform Algorithm 1, Step 4 for $m = 1, \ldots, N$. We generate the bootstrap sample $\left\{\tau^{(k)}\right\}_{k=1}^{K}$, with $\tau^{(k)} = \max_{m=1,\ldots,N}\left\{\tau_m^{(k)}\right\}$, and use this sample to calculate the bootstrap mean $\hat{\tau}$ and bootstrap variance $\hat{\sigma}_\tau^2$. Given these estimates, which are consistent for the population parameters, we can then construct the 95% confidence interval as:

$$\hat{C}_{0.95}\left(\hat{\tau}\right)=\left[\hat{\tau}-1.96\hat{\sigma}_{\tau},\,\hat{\tau}+1.96\hat{\sigma}_{\tau}\right]$$

This confidence interval is reported in Table 6, Panel A.

Appendix C: Detailed demand and supply figures for area price premia

See Fig. 16.

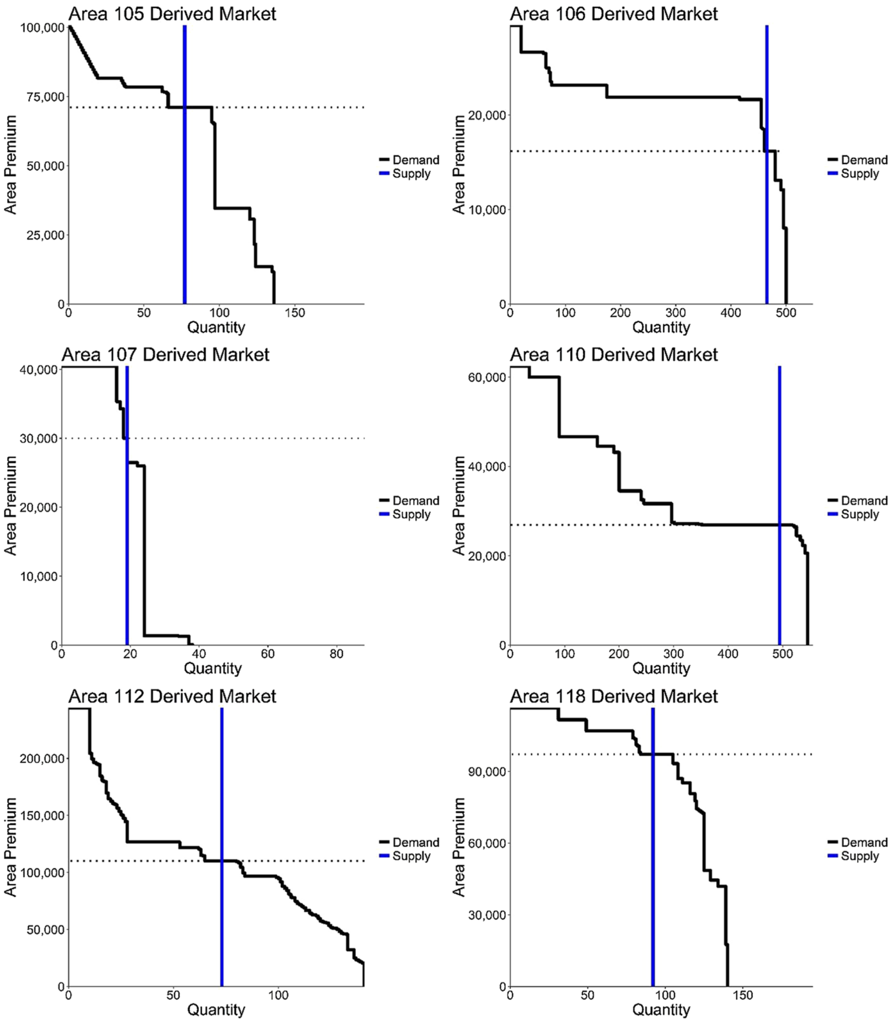


Fig. 16 Detailed demand and supply figures for area price premia

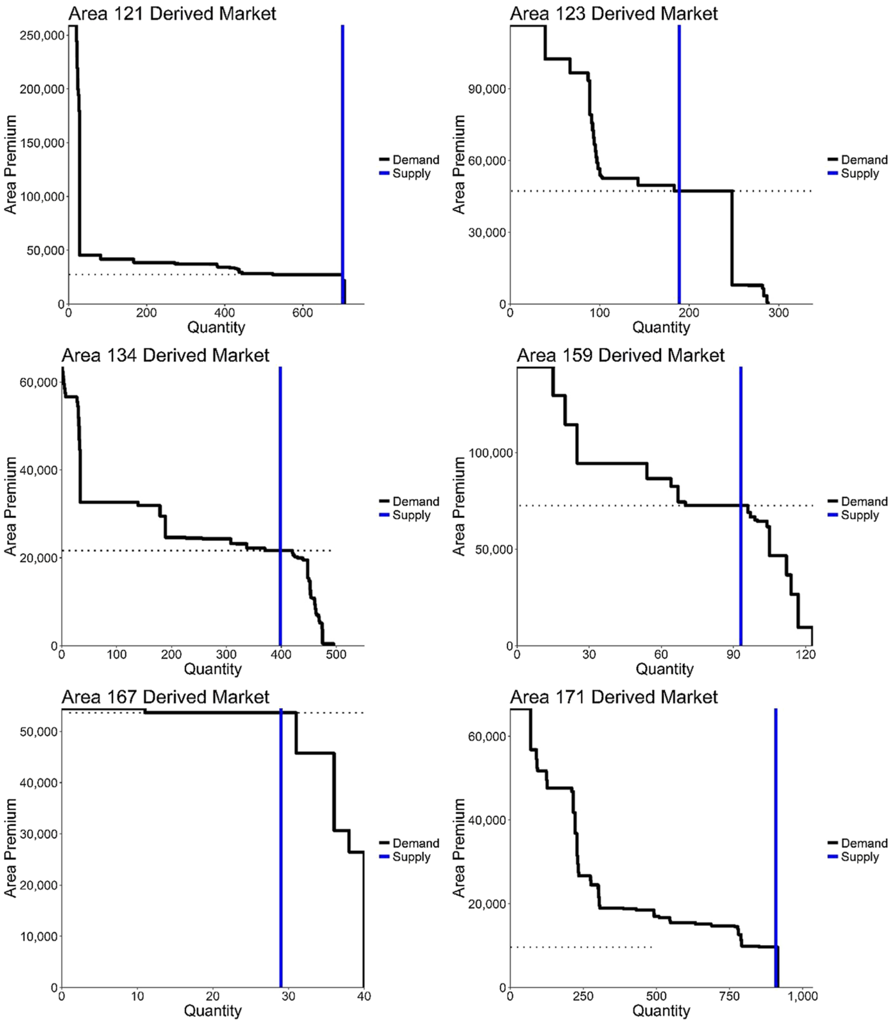


Fig. 16 continued

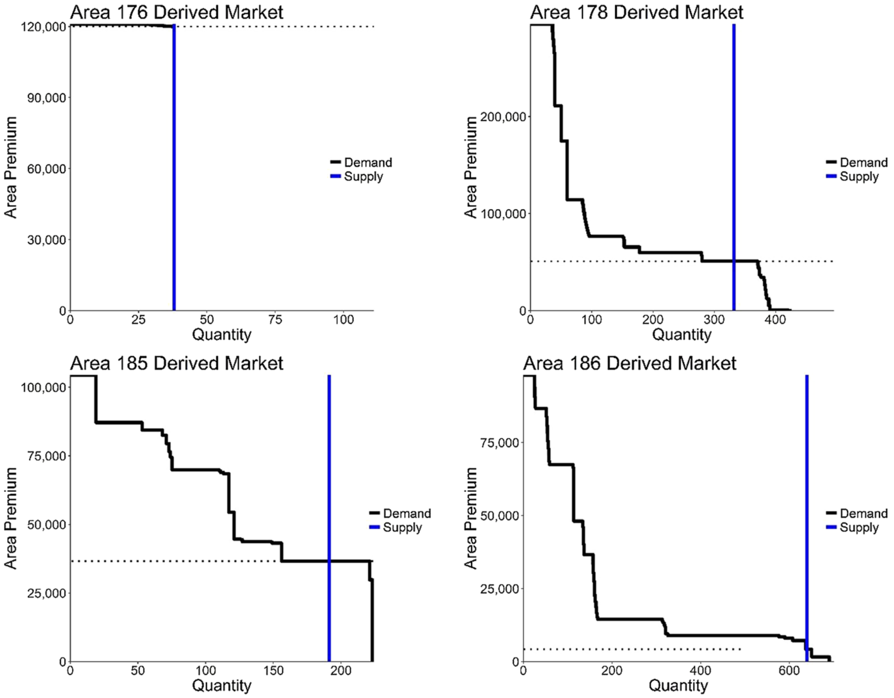


Fig. 16 continued

Appendix D: Full regression results for regression model 6

Table 8 .

		Revealed Excess Demand for Licenses in Market																
Market Price Response		*UC Hotel*	*Area 105*	*Area 106*	*Area 107*	*Area 110*	*Area 112*	*Area 118*	*Area 121*	*Area 123*	*Area 134*	*Area 159*	*Area 167*	*Area 171*	*Area 176*	*Area 178*	*Area 185*	*Area 186*
UC Hotel	Coeff	1.87E-05	-6.25E-05	9.71E-05	2.94E-04	-3.68E-05	-3.14E-05	4.80E-05	-2.31E-04	4.61E-05	9.30E-05	-8.92E-04	2.49E-03	-8.21E-05	-2.51E-05	-5.28E-05	2.98E-05	-3.65E-05
	Std Err	(4.07E-06)	(4.32E-05)	(3.25E-05)	(3.91E-04)	(4.43E-05)	(3.78E-05)	(2.15E-05)	(1.58E-04)	(2.84E-05)	(4.28E-05)	(4.16E-04)	(1.08E-03)	(4.74E-05)	(1.37E-04)	(5.31E-05)	(3.72E-05)	(3.85E-05)
Area 105	Coeff	1.18E-03	1.57E-02	5.27E-05	-5.23E-04	4.97E-04	-1.65E-04	8.28E-04	1.04E-03	1.30E-04	-2.63E-05	2.62E-04	-2.69E-03	-1.62E-04	-6.95E-04	1.29E-03	-1.28E-04	1.57E-04
	Std Err	(5.75E-04)	(5.21E-03)	(2.86E-04)	(4.32E-04)	(2.10E-03)	(5.70E-04)	(7.07E-04)	(5.03E-04)	(6.31E-04)	(5.41E-05)	(5.53E-03)	(1.44E-02)	(5.13E-04)	(1.82E-03)	(4.96E-04)	(3.78E-04)	(5.90E-04)
Area 106	Coeff	6.76E-06	2.12E-04	-1.81E-04	-6.31E-03	8.32E-04	-1.16E-04	1.29E-05	-1.20E-04	-5.77E-04	6.62E-05	6.34E-04	6.35E-03	1.85E-04	1.74E-03	1.78E-04	-1.24E-04	-9.00E-05
	Std Err	(1.97E-04)	(2.62E-04)	(1.30E-04)	(2.37E-03)	(9.55E-04)	(2.59E-04)	(3.22E-04)	(2.29E-04)	(2.87E-04)	(2.46E-05)	(2.52E-03)	(6.54E-03)	(2.34E-04)	(8.30E-04)	(2.26E-04)	(1.72E-04)	(2.69E-04)
Area 107	Coeff	1.86E-03	-1.02E-04	-9.60E-05	1.19E-04	-8.42E-04	-9.53E-06	6.71E-05	5.13E-04	9.46E-05	5.21E-05	-6.20E-03	5.11E-03	-2.43E-04	1.34E-03	5.64E-05	1.08E-04	-2.43E-04
	Std Err	(2.88E-03)	(3.18E-04)	(1.58E-04)	(2.39E-04)	(1.16E-03)	(3.15E-04)	(3.91E-04)	(2.78E-04)	(3.49E-04)	(2.99E-05)	(3.06E-03)	(7.95E-03)	(2.84E-04)	(1.01E-03)	(2.74E-04)	(2.09E-04)	(3.27E-04)
Area 110	Coeff	-8.24E-05	-2.01E-04	2.28E-04	-3.90E-03	-2.72E-04	1.70E-03	-2.10E-04	-4.59E-04	-1.53E-04	2.67E-05	6.39E-05	8.14E-03	-1.98E-04	2.52E-04	5.01E-05	6.09E-04	1.58E-04
	Std Err	(3.12E-04)	(3.04E-04)	(2.29E-04)	(2.76E-03)	(1.11E-03)	(3.01E-04)	(1.51E-04)	(2.66E-04)	(2.00E-04)	(2.86E-05)	(2.93E-03)	(7.61E-03)	(3.34E-04)	(9.65E-04)	(3.74E-04)	(2.62E-04)	(2.71E-04)
Area 112	Coeff	3.33E-03	-6.28E-05	1.06E-03	8.09E-03	-3.55E-04	2.09E-04	-2.33E-04	2.10E-03	-3.65E-04	-2.32E-05	-4.80E-03	8.64E-03	-4.72E-04	1.73E-03	8.51E-04	8.16E-04	5.54E-05
	Std Err	(5.99E-04)	(6.85E-04)	(5.15E-04)	(6.20E-03)	(7.03E-04)	(6.78E-04)	(3.40E-04)	(2.50E-03)	(4.50E-04)	(6.44E-05)	(6.59E-03)	(1.71E-02)	(7.52E-04)	(2.17E-03)	(8.42E-04)	(5.90E-04)	(6.11E-04)
Area 118	Coeff	1.46E-03	-5.52E-04	-7.13E-05	1.33E-02	2.60E-04	1.30E-04	6.83E-04	-2.18E-04	3.12E-05	1.39E-05	5.41E-03	4.82E-03	-2.20E-05	-3.66E-04	-4.85E-04	5.49E-04	1.38E-03
	Std Err	(2.49E-04)	(5.01E-04)	(3.76E-04)	(4.54E-03)	(1.83E-03)	(4.96E-04)	(6.16E-04)	(4.38E-04)	(5.50E-04)	(4.71E-05)	(4.82E-03)	(1.25E-02)	(4.47E-04)	(1.59E-03)	(4.32E-04)	(3.29E-04)	(5.14E-04)
Area 121	Coeff	5.78E-04	-2.54E-04	1.98E-04	-1.73E-03	4.59E-04	4.19E-05	-2.02E-04	2.55E-04	3.83E-05	5.34E-05	-1.37E-03	-3.42E-03	-4.40E-04	2.69E-04	3.48E-04	2.90E-04	4.28E-04
	Std Err	(1.06E-03)	(2.92E-04)	(2.19E-04)	(2.64E-03)	(2.99E-04)	(2.89E-04)	(1.45E-04)	(2.55E-04)	(1.91E-04)	(2.74E-05)	(2.81E-03)	(7.29E-03)	(3.20E-04)	(9.25E-04)	(3.58E-04)	(2.51E-04)	(2.60E-04)
Area 123	Coeff	5.33E-04	1.87E-04	7.75E-05	2.74E-03	2.51E-04	-1.64E-04	9.01E-06	-3.19E-05	-3.92E-04	3.86E-05	4.11E-03	-8.11E-03	5.56E-04	7.24E-04	1.84E-04	-2.38E-04	6.55E-04
	Std Err	(1.46E-04)	(2.23E-04)	(1.68E-04)	(2.02E-03)	(8.14E-04)	(2.21E-04)	(1.11E-04)	(1.95E-04)	(2.45E-04)	(2.10E-05)	(2.14E-03)	(5.57E-03)	(1.99E-04)	(7.07E-04)	(2.74E-04)	(1.92E-04)	(2.29E-04)
Area 134	Coeff	1.42E-03	5.22E-05	5.18E-06	-2.67E-03	-3.25E-04	-2.29E-04	1.16E-04	7.31E-04	-1.87E-04	2.69E-05	-1.09E-03	-9.52E-04	-1.89E-04	3.29E-04	-3.50E-04	2.72E-04	1.01E-04
	Std Err	(2.28E-04)	(2.30E-04)	(1.73E-04)	(2.08E-03)	(2.36E-04)	(2.01E-04)	(1.14E-04)	(8.38E-04)	(1.51E-04)	(2.16E-05)	(2.21E-03)	(5.74E-03)	(2.52E-04)	(7.29E-04)	(2.82E-04)	(1.98E-04)	(2.05E-04)
Area 159	Coeff	-4.59E-03	6.19E-03	-4.04E-04	-9.98E-04	2.50E-06	5.01E-04	1.35E-03	-2.18E-04	-2.70E-04	4.39E-05	-1.53E-02	-1.31E-03	1.58E-03	-5.53E-04	-8.98E-04	1.36E-03	4.85E-03
	Std Err	(1.08E-02)	(1.01E-02)	(5.57E-04)	(8.42E-04)	(4.09E-03)	(1.11E-03)	(1.38E-03)	(9.80E-04)	(1.23E-03)	(1.05E-04)	(2.80E-02)	(3.55E-03)	(9.99E-04)	(1.12E-03)	(9.66E-04)	(7.35E-04)	(1.15E-03)
Area 167	Coeff	-6.09E-03	-7.38E-03	3.21E-04	2.98E-04	1.44E-03	2.76E-04	6.80E-04	2.96E-04	6.27E-05	-1.80E-05	6.38E-03	2.20E-03	4.22E-04	2.79E-04	-1.15E-04	2.19E-05	-3.87E-04
	Std Err	(7.38E-03)	(2.67E-03)	(1.47E-04)	(2.22E-04)	(1.08E-03)	(2.92E-04)	(3.63E-04)	(2.58E-04)	(3.24E-04)	(2.78E-05)	(2.84E-03)	(9.36E-04)	(2.63E-04)	(2.95E-04)	(2.54E-04)	(1.94E-04)	(3.03E-04)
Area 171	Coeff	-1.41E-04	2.43E-05	-1.74E-04	-1.29E-03	-7.39E-04	-7.46E-05	4.98E-05	2.49E-05	8.36E-05	2.68E-05	-4.80E-04	3.73E-03	9.23E-05	1.05E-03	4.70E-06	-8.84E-06	3.79E-04
	Std Err	(2.11E-04)	(1.92E-04)	(1.44E-04)	(1.74E-03)	(7.00E-04)	(1.90E-04)	(9.53E-05)	(1.68E-04)	(1.26E-04)	(1.81E-05)	(1.85E-03)	(4.80E-03)	(1.71E-04)	(6.09E-04)	(2.36E-04)	(1.65E-04)	(1.97E-04)
Area 176	Coeff	-5.36E-04	-1.68E-02	-2.75E-04	1.97E-03	7.29E-03	9.13E-05	-3.93E-04	5.06E-04	-2.01E-03	1.76E-04	-7.03E-03	1.56E-02	4.91E-04	-2.65E-04	1.50E-03	6.58E-05	-2.02E-04
	Std Err	(2.43E-03)	(6.94E-03)	(3.81E-04)	(5.76E-04)	(2.80E-03)	(7.59E-04)	(9.42E-04)	(6.71E-04)	(8.41E-04)	(7.21E-05)	(7.37E-03)	(1.92E-02)	(6.84E-04)	(7.66E-04)	(6.61E-04)	(5.03E-04)	(7.87E-04)
Area 178	Coeff	-3.97E-04	-1.55E-06	5.36E-05	4.40E-03	-1.19E-03	-7.12E-05	1.22E-03	-4.27E-04	5.06E-04	5.16E-05	-6.09E-03	-7.38E-03	-1.34E-04	6.32E-04	1.45E-04	-2.36E-04	-8.02E-05
	Std Err	(6.82E-04)	(5.55E-04)	(4.17E-04)	(5.03E-03)	(2.03E-03)	(5.50E-04)	(2.76E-04)	(4.86E-04)	(6.09E-04)	(5.22E-05)	(5.34E-03)	(1.39E-02)	(4.95E-04)	(1.76E-03)	(4.79E-04)	(3.64E-04)	(5.70E-04)
Area 185	Coeff	7.89E-04	4.97E-05	3.92E-04	-3.46E-03	1.46E-03	-1.10E-04	-8.63E-05	4.16E-04	-3.24E-04	-3.16E-06	1.28E-04	-9.81E-05	7.24E-05	-2.65E-04	6.40E-04	2.76E-04	9.74E-04
	Std Err	(2.80E-04)	(3.25E-04)	(2.44E-04)	(2.95E-03)	(1.19E-03)	(3.22E-04)	(1.62E-04)	(2.85E-04)	(3.57E-04)	(3.06E-05)	(3.13E-03)	(8.13E-03)	(2.90E-04)	(1.03E-03)	(4.00E-04)	(2.14E-04)	(3.34E-04)
Area 186	Coeff	1.31E-04	1.53E-04	2.17E-05	-1.51E-04	-1.69E-04	-1.27E-04	2.84E-05	-3.64E-05	-1.98E-05	2.03E-05	-1.49E-04	5.10E-04	-1.05E-04	9.61E-05	-9.24E-05	-1.07E-05	2.05E-05
	Std Err	(1.49E-04)	(1.67E-04)	(1.26E-04)	(1.52E-03)	(6.11E-04)	(1.66E-04)	(8.32E-05)	(1.47E-04)	(1.10E-04)	(1.58E-05)	(1.61E-03)	(4.19E-03)	(1.84E-04)	(5.31E-04)	(2.06E-04)	(1.44E-04)	(1.72E-04)

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