# Online Appendix: Contingent Reasoning and Dynamic Public Goods Provision 

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This document contains the online Appendix for Calford and Ca-
son (Forthcoming).
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## A. Theory

For ease of exposition we present the game with a continuous signal space on the interval $[0,1]$, and also assume that $D_{0} \in[0,1]{ }^{1}$ Our experimental implementation, as discussed in the main text, uses a discrete signal space on the interval $[0,100]$ to avoid the need to use decimal notation.

Formally, the game is a Bayesian game given that each player has a private signal. We demonstrate, however, that all agents use cutoff strategies and that the equilibrium can be parsimoniously represented by the corresponding cutoffs without the need to carry extra notation for beliefs. We assume that the exante probability of selecting the public good is strictly positive and identical

[^0]for all agents; this assumption rules out asymmetric equilibrium and the trivial equilibrium where no one ever selects the public good.
Denote an agent's beliefs about the likelihood that another agent will select the public good, as a function of the other agent's signal, by $\beta:[0,1] \rightarrow[0,1] .{ }^{2}$ Given $\beta\left(s_{j}\right)$ we can define $b=\int_{s_{j}=0}^{1} \beta\left(s_{j}\right)>0$ to be the expected probability that another agent will select the public good, $\mathbb{E}\left[s_{j} \mid P G\right]=\int_{s_{j}=0}^{1} \beta\left(s_{j}\right) s_{j}$ to be the expected value of the other agent's signal conditional on the agent selecting the public good, and $\mathbb{E}\left[s_{j} \mid R G\right]=\int_{s_{j}=0}^{1}\left[1-\beta\left(s_{j}\right)\right] s_{j}$ to be the expected value of the other agent's signal conditional on the agent selecting the private good. Denote $\mathbb{E}[P G]$ and $\mathbb{E}[R G]$ to be the expected payoff for selecting the public good or private good, respectively ${ }^{3}$

LEMMA 1: All agents will play cutoff strategies in the static treatment. That is, there exists a $y_{i}$ such that agent $i$ will choose the public good if $s_{i} \geq y_{i}$ and choose the private good if $s_{i}<y_{i}$.

## PROOF:

$$
\mathbb{E}[R G]=D_{0}+1, \text { and }
$$

$\mathbb{E}[P G]=(1-b)^{2} \mathbb{E}[R G]+2(1-b) b\left[s_{i}+\mathbb{E}\left[s_{j} \mid P G\right]+\mathbb{E}\left[s_{j} \mid R G\right]\right]+b^{2}\left[s_{i}+2 \mathbb{E}\left[s_{j} \mid P G\right]\right]$.
$\mathbb{E}[R G]$ is independent of $s_{i}$ while, because of the assumption that $b>0, \mathbb{E}[P G]$ is strictly increasing in $s_{i}$. The result follows.

We therefore proceed by restricting attention to cutoff strategies and simplify the notation for beliefs. We employ the following notation: $y$ denotes the cutoff above which an agent selects the public good whenever $s \geq y ; p$ denotes the belief regarding the probability that others select the public good; and equilibrium

[^1]quantities are appended with an asterisk ( $y^{*}$ and $p^{*}$ ). We focus on symmetric equilibria such that, in equilibrium, $y^{*}=1-p^{*}$.
Lemma 1 is easily extended to each history of the dynamic treatment, at the expense of some extra notation. As a consequence, we document the dynamic treatment as a sequence of cutoff strategies, one for each history. We impose one additional assumption in the dynamic treatment: that the equilibrium is a "no-delay" equilibrium. That is, if at any stage of the game all agents select the private good, then no agent will switch to selecting the public good in any future stage. ${ }_{4}^{4}$ Imposing this assumption pins down beliefs on off-equilibrium paths.

## A1. Static treatment

We begin with the static treatment. In the static treatment the relevant cursedness parameter is $\chi_{H}$ : all inference is conducted with respect to the hypothetical decisions of others. $\psi$ has no role to play in the static treatment, as there is no future to consider.

PROPOSITION 1: In the static treatment the cutoff, $y_{S}^{*}$, satisfies

$$
y_{S}^{*}=\frac{D_{0}-1+\sqrt{1+6 D_{0}-4 \chi_{H} D_{0}+D_{0}^{2}}}{2\left(2-\chi_{H}\right)} .
$$

## PROOF:

Suppose that an agent expects each other player to select the public good with probability $p . \mathbb{E}[R G]=D_{0}+1$, and

$$
\begin{aligned}
\mathbb{E}\left[P G \mid s_{i}\right]= & (1-p)^{2}\left[D_{0}+1\right]+p^{2}\left[s_{i}+2\left(1-\frac{p}{2}+\chi_{H}\left(\frac{p}{2}-\frac{1}{2}\right)\right)\right] \\
& +2 p(1-p)\left[s_{i}+\left(1-\frac{p}{2}+\chi_{H}\left(\frac{p}{2}-\frac{1}{2}\right)\right)+\left(\frac{1-p}{2}+\frac{\chi_{H} p}{2}\right)\right] .
\end{aligned}
$$

[^2]The result follows after setting $\mathbb{E}\left[P G \mid s_{i}=y_{S}^{*}\right]=\mathbb{E}[R G]$, substituting $p=1-y_{S}^{*}$, and solving for $y^{*}$ (choosing the positive arm of the resulting quadratic equation).
Substituting $\chi_{H}=0$ returns the Bayesian Nash equilibrium cutoff, $y_{S}^{N}=$ $\frac{D_{0}-1+\sqrt{1+6 D_{0}+D_{0}{ }^{2}}}{4}$, and substituting $\chi_{H}=1$ returns the fully Cursed equilibrium, $y_{S}^{C}=D_{0}$.

## A2. Dynamic treatment

The dynamic treatment with unawareness, $\psi=1$, involves agents who solve a series of static problems: by definition, agents ignore the future when making any decision. An unaware agent ignores all future information, and also ignores the possibility of transmitting information to others. Therefore, the first stage of the dynamic treatment is functionally identical to the static treatment. That is, $y_{0}^{*}=y_{S}^{*}$ when $\psi=1$.

When the unaware agent arrives at the second stage, they are surprised by the arrival of new information. Importantly, the unaware agent is not able to condition beliefs on the "correct" event in the case that they observe exactly one other player select the public good. ${ }^{5}$ Upon arriving in the second stage, the unaware agent assumes, in equilibrium, that both other players chose the public good with probability $p_{0}^{*}=1-y_{0}^{*}$ in the first stage. Further, the agent evaluates the new information with the cursedness parameter $\chi_{R}$ : the first stage choices of the other players are now realized, rather than hypothetical, events.

LEMMA 2: In the second stage, after observing exactly one other player select the public good in the first stage, the cursed cutoff for an agent with $\psi=1$ in the second stage satisfies:

$$
y_{1}^{*}=\min \left\{\max \left\{D_{0}+\left(1-\chi_{R}\right)\left(p_{0}^{*}-\frac{1}{2}\right), 0\right\}, 1\right\} .
$$

[^3]
## PROOF:

The unaware agent expects to receive the private good if they select the private good in the second stage, ignoring the future possibility to select the public good, such that $\mathbb{E}[R G]=D_{0}+1$. Meanwhile, $\mathbb{E}\left[P G \mid s_{i}\right]=s_{i}+\left[1-\frac{p_{0}^{*}}{2}+\chi_{R}\left(\frac{p_{0}^{*}}{2}-\frac{1}{2}\right)\right]+$ $\left[\frac{1-p_{0}^{*}}{2}+\frac{\chi_{R} p_{0}^{*}}{2}\right]$. Solving $\mathbb{E}[R G]=\mathbb{E}\left[P G \mid s_{i}=y\right]$ yields the required equation. If $y<0$ then the agent always selects the public good, such that $y_{1}^{*}=0$, and if $y>1$ then the agent never selects the public good, such that $y_{1}^{*}=1$.

LEMMA 3: In the second stage, after observing two other players select the public good in the first stage, the cursed cutoff for an agent with $\psi=1$ in the second stage satisfies:

$$
\begin{equation*}
y_{2}^{*}=\max \left\{D_{0}+\left(1-\chi_{R}\right)\left[p_{0}^{*}-1\right], 0\right\} . \tag{A1}
\end{equation*}
$$

## PROOF:

$\mathbb{E}[R G]=D_{0}+1$ and $\mathbb{E}\left[P G \mid s_{i}\right]=s_{i}+2\left[1-\frac{p_{0}^{*}}{2}+\chi_{R}\left(\frac{p_{0}^{*}}{2}-\frac{1}{2}\right)\right]$. Solving for $\mathbb{E}\left[P G \mid s_{i}=y\right]=\mathbb{E}[R G]$ yields the solution. If $y<0$ then the agent always selects the public good, such that $y_{2}^{*}=0$.
For an unaware agent, in contrast to the case with aware agents discussed below, it is possible that the equilibrium cutoff increases from the first stage to the second stage. This is because an unaware agent is surprised by new information in the second stage and, in some cases, this new information may make the public good appear less attractive.

As before, we write $p_{0}$ to denote the expected probability that each other player selected the public good in the first stage, and we now write $p_{1}$ to denote the expected probability that each other player selected the public good in either the first or second stage. That is, $p_{0}=1-y_{0}$ and $p_{1}=\max \left\{1-y_{0}, 1-y_{1}\right\}$.

LEMMA 4: In the third stage, after observing one other player select the public good in the first stage and one other player select the public good in the second
stage, the equilibrium cutoff for the unaware agent in the third stage satisfies:

$$
y_{1,1}^{*}=\min \left\{\max \left\{D_{0}+\left(1-\chi_{R}\right)\left[p_{0}^{*}+\frac{p_{1}^{*}}{2}-1\right], 0\right\}, 1\right\} .
$$

## PROOF:

The equilibrium cutoff must solve $\mathbb{E}\left[P G \mid s_{i}=y\right]=\mathbb{E}[R G]$ where $\mathbb{E}[R G]=1+D_{0}$ and $\mathbb{E}\left[P G \mid s_{i}\right]=s_{i}+\left[1-\frac{p_{0}^{*}}{2}+\chi_{R}\left(\frac{p_{0}^{*}}{2}-\frac{1}{2}\right)\right]+\left[1-\frac{p_{0}^{*}+p_{1}^{*}}{2}+\chi_{R}\left(\frac{p_{0}^{*}+p_{1}^{*}}{2}-\frac{1}{2}\right)\right]$. If $y<0$ then the agent always selects the public good, such that $y_{1}^{*}=0$, and if $y>1$ then the agent never selects the public good, such that $y_{1}^{*}=1$.

Dynamic treatment with awareness. - For an agent with awareness, we proceed via backwards induction. However, the aware and unaware agent agree on how to proceed in the cases where both other players have already selected the public good. Therefore, for an agent with $\psi=0$, we have that $y_{1,1}^{*}=$ $D_{0}+\left(1-\chi_{R}\right)\left[p_{0}^{*}+\frac{p_{1}^{*}}{2}-1\right]$ and $y_{2}^{*}=D_{0}+\left(1-\chi_{R}\right)\left[p_{0}^{*}-1\right]$, whenever these values lie between 0 and 1 , as before.

Note, however, that the aware and unaware agents will, typically, not have the same values of $p_{0}^{*}$ and $p_{1}^{*}$. This implies that the two types of agents also disagree about the cutoff values $y_{1,1}^{*}$ and $y_{2}^{*}$.
We assert that the equilibrium must satisfy $y_{0}^{*} \geq y_{1}^{*} \geq y_{1,1}^{*}$, and establish this monotonocity condition in the following two lemmas.

LEMMA 5: $\quad$ Either $y_{1}^{*}>y_{1,1}^{*}$ or $y_{1}^{*}=y_{1,1}^{*}=0$.

## PROOF:

Suppose that $0<y_{1}^{*} \leq y_{1,1}^{*}$, and consider an agent in the second stage. We seek a contradiction.

In this case, an agent who does not select the public good in the second stage will never do so in the third stage. Therefore, $\mathbb{E}[R G]=D_{0}+1$. Meanwhile, $\mathbb{E}\left[P G \mid s_{i}\right]=s_{i}+1-\frac{p_{0}^{*}}{2}+\chi_{R} \frac{p_{0}^{*}-1}{2}+\frac{1-p_{0}^{*}}{2}+\chi_{R} \frac{p_{0}^{*}}{2}$. In equilibrium, $\mathbb{E}[R G]=$
$\mathbb{E}\left[P G \mid s_{i}=y_{1}^{*}\right]$, which implies that $y_{1}^{*}=D_{0}+\left(1-\chi_{R}\right)\left(p_{0}^{*}-\frac{1}{2}\right)$. Therefore, either $y_{1}^{*}=0=y_{1,1}^{*}$ or $y_{1}^{*}>y_{1,1}^{*}$, a contradiction.

LEMMA 6: $\quad y_{0}^{*} \geq y_{1}^{*}$.

## PROOF:

Consider an agent with signal $s_{i}=y_{0}^{*}<y_{1}^{*}$. We consider three cases.
First, suppose that both other players select the public good in the first stage. If $s_{i}<y_{2}^{*}$ then the agent prefers not to receive the public good, and if $s_{i} \geq y_{2}^{*}$ the agent is indifferent between selecting the public good or not in the first stage.

Second, suppose that exactly one other player selects the public good in the first stage. In this case, the agent prefers not to receive the public good (because $s_{i}<y_{1}^{*}$ by assumption).

Third, suppose that both other players select the private good in the first stage. In this case, neither player will select the public good in the second stage either because $y_{0}^{*}<y_{1}^{*}$ by assumption. Therefore the agent can never receive the public good and is indifferent.

In each case the agent is either indifferent or prefers not to select the public good in the first stage. Therefore, the agent will never select the public good in the first stage when $s_{i}<y_{1}^{*}$. There cannot exist an equilibrium with $y_{0}^{*}<y_{1}^{*}$.

The declining cutoff values clarify the events that must be conditioned on at each stage of the game. Consider the case where exactly one agent selected the public good in the first stage. The two remaining agents will then play a continuation game in the second stage where each agent should condition expectations on the remaining opponent not selecting the PG in the second stage. To see why, consider that an agent with $s_{i}<y_{1,1}^{*}$ will never prefer the PG. For an agent with $s_{i} \geq y_{1,1}^{*}$, they always prefer the PG in the event that the remaining opponent selects the PG in the second stage. But, conditional on this event, the agent is indifferent between selecting the PG or not in the second stage: if they do not select it, then they can simply select the PG in the third stage. Therefore, the
event where the opponent does not select the PG in the second stage is the critical event.
Rolling back to the first stage, similar reasoning applies. The agent should condition behavior on the event where both opponents do not select the PG in the first stage. If another agent does select the PG in the first stage, then the agent can always select, and receive, the public good in the second stage.

LEMMA 7: In the second stage, after observing exactly one other player select the public good in the first stage, the equilibrium cutoff for the aware agent in the second stage $y_{1}^{*}$ satisfies:

$$
\begin{equation*}
y_{1}^{*}=\frac{\chi_{R}-\chi_{H}+2 D_{0}+\left(1-\chi_{R}\right) p_{0}^{*}}{3-\chi_{H}} \tag{A2}
\end{equation*}
$$

## PROOF:

Conditioning on the hypothetical event that the remaining player not selecting the public good, $\mathbb{E}[R G]=1+D_{0}$ and $\mathbb{E}\left[P G \mid s_{i}\right]=s_{i}+\left[1-\frac{p_{0}^{*}}{2}+\chi_{R}\left(\frac{p_{0}^{*}}{2}-\frac{1}{2}\right)\right]+$ $\left[\frac{1-p_{1}}{2}+\frac{\chi_{H} p_{1}}{2}\right]$. Solving for $\mathbb{E}[R G]=\mathbb{E}\left[P G \mid s_{i}=y_{1}^{*}\right]$, substituting $p_{1}=1-y_{1}^{*}$ and $s_{i}=y_{1}^{*}$, yields the required solution.

LEMMA 8: In the first stage $y_{0}^{*}$ for the aware agent satisfies:

$$
\begin{equation*}
y_{0}^{*}=\frac{D_{0}-1+2 p_{1}^{*}-\chi_{H} p_{1}^{*}+\sqrt{\Delta}}{2\left(2-\chi_{H}\right)} \tag{A3}
\end{equation*}
$$

where $\Delta=\left(D_{0}-1+2 p_{1}^{*}-\chi_{H} p_{1}^{*}\right)^{2}-4\left(2-\chi_{H}\right)\left(-D_{0}-p_{1}^{*}+\chi_{H} p_{1}^{*}+D_{0} p_{1}^{*}+p_{1}^{* 2}-\chi_{H} p_{1}^{* 2}\right)$.
PROOF:
$\mathbb{E}[R G]=1+D_{0}$. The expected value of the public good depends on the response of the other players in the second stage. The probability of each other player selecting the public good in the second stage, conditioned on the player not selecting it in the first, is given by $\frac{p_{1}^{*}-p_{0}}{1-p_{0}}$ and the probability of the player selecting the private good is $\frac{1-p_{1}^{*}}{1-p_{0}}$.

Thus, $\mathbb{E}\left[P G \mid s_{i}\right]=\frac{\left(1-p_{1}^{*}\right)^{2}}{\left(1-p_{0}\right)^{2}}\left(1+D_{0}\right)+\frac{\left(p_{1}^{*}-p_{0}\right)^{2}}{\left(1-p_{0}\right)^{2}}\left(s_{i}+\left(1-p_{1}^{*}\right)+\left(1-p_{0}\right)+\chi_{H}\left(p_{0}+\right.\right.$ $\left.\left.p_{1}^{*}-1\right)\right)+\frac{\left(p_{1}^{*}-p_{0}\right)\left(1-p_{1}^{*}\right)}{\left(1-p_{0}\right)^{2}}\left(s_{i}+\frac{1-p_{1}^{*}}{2}+\frac{2-p_{0}-p_{1}^{*}}{2}+\chi_{H}\left(p_{1}^{*}+\frac{p_{0}}{2}-\frac{1}{2}\right)\right)$.

Setting $\mathbb{E}\left[P G \mid s_{i}=y_{0}^{*}\right]=\mathbb{E}[R G]$, substituting $y_{0}^{*}=1-p_{0}$, and solving for $y_{0}^{*}$ yields the required expression.

Given values for $\chi_{R}$ and $\chi_{H}$, equations A3 and A2 can be solved simultaneously using numerical methods. A solution always exists whenever $0 \leq \chi_{R} \leq \chi_{H} \leq 1$, but may not exist for parameters outside these bounds. The Nash equilibrium is found by setting $\chi_{R}=\chi_{H}=0$, and the cursed equilibrium by setting $\chi_{H}=\chi_{R}=$ 1.

## B. Counterfactual Simulations

This appendix uses the estimated preference parameters to run some illustrative counterfactual simulations. The simulations serve multiple purposes: they validate our modeling approach, illustrate the utility of decomposing failures of counterfactual thinking into components related to complexity and unawareness, and provide insight into the cause of deviations from equilibrium behavior documented in Section 3 of the main paper.

We present two simulations. The first simulation, the Baseline simulation, is intended to validate our model. The Baseline simulation takes, as a starting point, the estimated $\chi_{H}, \chi_{R}, \psi$ and $\lambda$ parameters for each of the 96 subjects in the $D_{0}=$ 30 and $D_{0}=70$ treatments. The second simulation, the Unawareness simulation, simulates a counterfactual world in which all subjects exhibit unawareness. In this case, we use the estimated values of $\chi_{H}, \chi_{R}$ and $\lambda$ but set $\psi=1$ for all subjects.

Each simulation consists of 1000 sub-simulations. Each sub-simulation consists of a complete recreation of the $D_{0}=30$ and $D_{0}=70$ treatments. That is, the 96 simulated subjects are randomly sorted into 8 matching groups of 12 subjects each, with 4 matching groups being assigned to each of the $D_{0}=30$ and $D_{0}=70$ treatments. Each matching group is then simulated to participate
in 20 static rounds and 40 dynamic rounds, with the matching group of 12 subjects randomly split into 4 groups of 3 subjects each round. For each subject, the cutoff strategies are a deterministic function of $\chi_{H}, \chi_{R}$ and $\psi$ and calculated as outlined in Appendix A. At each decision node the action choice is determined using Equation 4 of the main text by drawing a random value for $\epsilon_{i, r, t}$ from a logistic function and selecting the PG if the inequality is true. For each sub-simulation the aggregate rate of PG provision and the rate of PG over-provision (i.e. cases where the PG is provisioned despite the private good having a higher value) are recorded.

The results are presented in Figure B1. The rate of PG provision is shown in the top two panels, and the rate of PG over-provision in the bottom two panels. Each figure displays the equilibrium predictions, the observed data, and the outcomes of both the Baseline and Unawareness simulations.


Figure B1. Rate of PG provision (top panels) and over-provision (bottom panels). Left hand panels show the static treatment, and right hand panels show the dynamic treatment. The $D_{0}=30$ treatment is displayed with blue crosses, and the $D_{0}=70$ treatment with red circles.

The Baseline simulations provide a validation check of the structural model.

The model performs well, with the observed data falling within bootstrapped $95 \%$ confidence intervals for all eight target outcomes. Although the Baseline simulation is an in-sample test, consistency of the simulations with the data is not trivial. First, the simulations take all 96 subjects from the $D_{0}=30$ and $D_{0}=70$ treatments and rematch them across the two treatments. Thus, there is a possibility that uncontrolled treatment effects could derail the simulations. In addition, the structural model places substantial restrictions on the set of strategies that are coherent with the model given that it identifies five cutoff points per subject using only three preference parameters. If the structural model is mispecified, in the sense that it rules out strategies that subjects are actually using, then the simulations could miss the targets.

Some comments on the interpretation of the Unawareness simulations are in order given that manipulating subject awareness has no effect on behavior in the static treatment, a result which may appear counterintuitive. In the static treatment, behavior is governed solely by the cursedness parameter $\chi_{H}$. Thus, $\chi_{H}$ can be interpreted as capturing the extent of difficulties with contingent reasoning in the standard, simultaneous task. We then use the two parameters, $\chi_{R}$ and $\psi$, to decompose the cause of the difficulty of contingent reasoning.

The difference $\chi_{H}-\chi_{R}$ captures the change in the difficulties of contingent reasoning when moving from a hypothetical to realized contingent reasoning task. And $\chi_{R}$ captures the residual difficulty when dealing solely with realized contingent reasoning. Thus, we can interpret $\chi_{R}$ as partitioning the complexity of contingent reasoning into two pieces: the piece associated with the hypothetical problem, and the piece associated with the realized problem.

The awareness parameter, $\psi$, can then be interpreted as a distinct aspect of the contingent reasoning problem. Is the subject ex-ante aware that the hypothetical contingent reasoning problem is distinct from the dynamic contingent reasoning (i.e. that first stage behavior of others will generate a valuable signal, and that the signal may, in addition, be easier to decode than initial behavior)? Whether
the subject is aware of this distinction has no bearing on behavior in the static treatment given that the estimate of $\chi_{H}$ already fully incorporates the difficulties with hypothetical reasoning in the static treatment. Instead, we can think of $\psi$ as identifying whether there is a component of $\chi_{H}$ that is derived from unawareness.

The Awareness simulation can, therefore, reveal the effects of unawareness while holding the aggregate complexity of contingent reasoning constant. The results indicate that a population that is unaware about the future value of information has a higher rate of both PG provision and PG over-provision in the dynamic treatment. Thus, a behavioral mechanism designer who is concerned about minimizing over-provision rates might find it useful to emphasize the value of future information to participants, while a designer who is concerned with maximizing PG provision rates might wish to de-emphasize the value of future information.

## C. Supplementary results

This appendix contains some supplementary results. Table C1 reports the public good provision and overprovision rate, summarized in Result 3 at the end of Section 4.1. Table C2 provides a further breakdown of this information, splitting the dynamic treatment results into the first 20 and last 20 rounds. Figure C1 displays the CDF of the midpoint of the cutoff intervals for individual subjects in the static treatment and in the first stage of the dynamic treatment.
Finally, Table C3 presents the subject level parameter estimates for the structural model described in Section 5 of the main text.

Table C3-: Individual level structural parameter estimates. Values in square brackets are bootstrapped $95 \%$ confidence intervals. Values in parentheses are the proportion of bootstraps in which $\psi=1 . \lambda$ is the goodness of fit parameter, where higher values indicate a better model fit.

| ID | $\chi_{H}$ | $\chi_{R}$ | $\psi$ | $\lambda$ | ID | $\chi_{H}$ | $\chi_{R}$ | $\psi$ | $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.73 | 0.25 | 1 | 0.00 | 61 | 0.56 | 0.07 | 0 | 18.71 |
|  | [0.00, 0.81 ] | [0.00,0.43] | (0.49) |  |  | $[0.32,1.00]$ | [0.00,0.37] | (0.05) |  |
| 2 | 0.67 | 0.67 | 0 | 20.16 | 62 | $\begin{gathered} 0.83 \\ {[0.28,1.00]} \end{gathered}$ | $\begin{gathered} 0.39 \\ {[0.00,0.83]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.32) \end{gathered}$ | 7.74 |
|  | $[0.35,0.84]$ | [0.33,0.83] | (0.01) |  |  |  |  |  |  |
| 3 | 0.15 | 0.15 | 0 | 10.11 | 63 | $\begin{gathered} 0.00 \\ {[0.00,0.44]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.36]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.01) \end{gathered}$ | 23.71 |
|  | $[0.00,1.00]$ | [0.00,0.34] | (0.04) |  |  |  |  |  |  |
| 4 | 0.35 | 0.35 | 0 | 11.30 | 64 | $\begin{gathered} 0.67 \\ {[0.32,0.82]} \end{gathered}$ | $\begin{gathered} 0.34 \\ {[0.00,0.48]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 17.60 |
|  | [0.00, 0.82] | $[0.00,0.72]$ | (0.03) |  |  |  |  |  |  |
| 5 | 0.13 | 0.13 | 0 | 10.65 | 65 | 1.00 | 1.00 | 0 | 9.77 |
|  | [0.00,0.64] | [0.00,0.64] | (0.00) |  |  | [0.82,1.00] | [0.00, 1.00] | (0.43) |  |

Table C3-: Individual level structural parameter estimates. Values in square brackets are bootstrapped $95 \%$ confidence intervals. Values in parentheses are the proportion of bootstraps in which $\psi=1$. $\lambda$ is the goodness of fit parameter, where higher values indicate a better model fit.

| ID | $\chi_{H}$ | $\chi_{R}$ | $\psi$ | $\lambda$ | ID | $\chi_{H}$ | $\chi_{R}$ | $\psi$ | $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | $\begin{gathered} 0.00 \\ {[0.00,0.68]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.24]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.04) \end{gathered}$ | 1.22 | 66 | $\begin{gathered} 0.45 \\ {[0.11,0.92]} \end{gathered}$ | $\begin{gathered} 0.45 \\ {[0.00,0.61]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.19) \end{gathered}$ | 23.19 |
| 7 | $\begin{gathered} 0.96 \\ {[0.48,1.00]} \end{gathered}$ | $\begin{gathered} 0.96 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.36) \end{gathered}$ | 10.31 | 67 | $\begin{gathered} 0.40 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.40 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.02) \end{gathered}$ | 8.37 |
| 8 | $\begin{gathered} 0.00 \\ {[0.00,0.44]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.44]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 6.45 | 68 | $\begin{gathered} 0.06 \\ {[0.00,0.44]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.10]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 26.71 |
| 9 | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 5.18 | 69 | $\begin{gathered} 0.57 \\ {[0.32,0.98]} \end{gathered}$ | $\begin{gathered} 0.14 \\ {[0.00,0.86]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 9.41 |
| 10 | $\begin{gathered} 0.91 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.91 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.26) \end{gathered}$ | 5.18 | 70 | $\begin{gathered} 1.00 \\ {[0.80,1.00]} \end{gathered}$ | $\begin{gathered} 1.00 \\ {[0.61,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.63) \end{gathered}$ | 12.75 |
| 11 | $\begin{gathered} 0.71 \\ {[0.56,1.00]} \end{gathered}$ | $\begin{gathered} 0.71 \\ {[0.04,0.82]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.06) \end{gathered}$ | 19.07 | 71 | $\begin{gathered} 0.14 \\ {[0.00,0.54]} \end{gathered}$ | $\begin{gathered} 0.14 \\ {[0.00,0.51]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 13.33 |
| 12 | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 6.65 | 72 | $\begin{gathered} 0.58 \\ {[0.34,1.00]} \end{gathered}$ | $\begin{gathered} 0.58 \\ {[0.27,0.93]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.16) \end{gathered}$ | 15.85 |
| 13 | $\begin{gathered} 0.67 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.65 \\ {[0.00,0.81]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.41) \end{gathered}$ | 11.46 | 97 | $\begin{gathered} 0.37 \\ {[0.30,0.80]} \end{gathered}$ | $\begin{gathered} 0.07 \\ {[0.05,0.32]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 62,849,835.02 |
| 14 | $\begin{gathered} 0.76 \\ {[0.65,0.90]} \end{gathered}$ | $\begin{gathered} 0.64 \\ {[0.00,0.79]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 25.49 | 98 | $\begin{gathered} 0.65 \\ {[0.37,1.00]} \end{gathered}$ | $\begin{gathered} 0.65 \\ {[0.00,0.78]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.20) \end{gathered}$ | 7.39 |
| 15 | $\begin{gathered} 0.75 \\ {[0.13,1.00]} \end{gathered}$ | $\begin{gathered} 0.75 \\ {[0.04,0.99]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.53) \end{gathered}$ | 10.16 | 99 | $\begin{gathered} 0.58 \\ {[0.33,0.76]} \end{gathered}$ | $\begin{gathered} 0.46 \\ {[0.07,0.58]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.01) \end{gathered}$ | 19.06 |
| 16 | $\begin{gathered} 0.09 \\ {[0.00,0.64]} \end{gathered}$ | $\begin{gathered} 0.09 \\ {[0.00,0.64]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 12.39 | 100 | $\begin{gathered} 0.85 \\ {[0.54,1.00]} \end{gathered}$ | $\begin{gathered} 0.85 \\ {[0.10,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.17) \end{gathered}$ | 7.61 |
| 17 | $\begin{gathered} 0.00 \\ {[0.00,0.41]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.37]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 7.02 | 101 | $\begin{gathered} 0.93 \\ {[0.67,1.00]} \end{gathered}$ | $\begin{gathered} 0.29 \\ {[0.00,0.81]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.15) \end{gathered}$ | 17.22 |
| 18 | $\begin{gathered} 0.82 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.15 \\ {[0.00,0.88]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.97) \end{gathered}$ | 10.31 | 102 | $\begin{gathered} 1.00 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.41]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.57) \end{gathered}$ | 5.44 |
| 19 | $\begin{gathered} 0.30 \\ {[0.00,0.65]} \end{gathered}$ | $\begin{gathered} 0.30 \\ {[0.00,0.63]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.01) \end{gathered}$ | 14.39 | 103 | $\begin{gathered} 0.68 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.57 \\ {[0.00,0.76]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.38) \end{gathered}$ | 13.03 |
| 20 | $\begin{gathered} 0.14 \\ {[0.00,0.52]} \end{gathered}$ | $\begin{gathered} 0.14 \\ {[0.00,0.50]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 10.06 | 104 | $\begin{gathered} 0.70 \\ {[0.50,0.87]} \end{gathered}$ | $\begin{gathered} 0.70 \\ {[0.00,0.86]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 13.59 |
| 21 | $\begin{gathered} 0.72 \\ {[0.47,0.89]} \end{gathered}$ | $\begin{gathered} 0.28 \\ {[0.11,0.59]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.67) \end{gathered}$ | 0.00 | 105 | $\begin{gathered} 0.12 \\ {[0.00,0.42]} \end{gathered}$ | $\begin{gathered} 0.12 \\ {[0.00,0.34]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 15.18 |
| 22 | $\begin{gathered} 0.41 \\ {[0.26,1.00]} \end{gathered}$ | $\begin{gathered} 0.41 \\ {[0.00,0.52]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.07) \end{gathered}$ | 16.03 | 106 | $\begin{gathered} 0.26 \\ {[0.00,0.72]} \end{gathered}$ | $\begin{gathered} 0.04 \\ {[0.00,0.70]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 9.85 |
| 23 | $\begin{gathered} 0.76 \\ {[0.32,0.99]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.63]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.02) \end{gathered}$ | 7.93 | 107 | $\begin{gathered} 0.66 \\ {[0.18,0.81]} \end{gathered}$ | $\begin{gathered} 0.66 \\ {[0.18,0.81]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 15.60 |
| 24 | $\begin{gathered} 0.00 \\ {[0.00,0.68]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.68]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 10.03 | 108 | $\begin{gathered} 0.59 \\ {[0.32,0.75]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.55]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 19.43 |
| 25 | $\begin{gathered} 0.34 \\ {[0.00,0.78]} \end{gathered}$ | $\begin{gathered} 0.34 \\ {[0.00,0.62]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.95) \end{gathered}$ | 10.63 | 121 | $\begin{gathered} 1.00 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 1.00 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.39) \end{gathered}$ | 0.66 |
| 26 | $\begin{gathered} 0.24 \\ {[0.00,0.88]} \end{gathered}$ | $\begin{gathered} 0.24 \\ {[0.00,0.71]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.35) \end{gathered}$ | 13.84 | 122 | $\begin{gathered} 0.69 \\ {[0.25,1.00]} \end{gathered}$ | $\begin{gathered} 0.69 \\ {[0.21,0.98]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.42) \end{gathered}$ | 14.88 |
| 27 | $\begin{gathered} 0.53 \\ {[0.01,1.00]} \end{gathered}$ | $\begin{gathered} 0.18 \\ {[0.00,0.74]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.18) \end{gathered}$ | 8.97 | 123 | $\begin{gathered} 0.20 \\ {[0.00,0.47]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 8.47 |
| 28 | $\begin{gathered} 0.25 \\ {[0.00,0.89]} \end{gathered}$ | $\begin{gathered} 0.25 \\ {[0.00,0.60]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.05) \end{gathered}$ | 8.70 | 124 | $\begin{gathered} 0.53 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.53 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.95) \end{gathered}$ | 7.74 |
| 29 | $\begin{gathered} 1.00 \\ {[0.05,1.00]} \end{gathered}$ | $\begin{gathered} 0.33 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.39) \end{gathered}$ | 4.34 | 125 | $\begin{gathered} 0.60 \\ {[0.30,0.92]} \end{gathered}$ | $\begin{gathered} 0.60 \\ {[0.30,0.68]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.03) \end{gathered}$ | 26.43 |
| 30 | $\begin{gathered} 1.00 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 1.00 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.29) \end{gathered}$ | 3.94 | 126 | $\begin{gathered} 0.72 \\ {[0.06,1.00]} \end{gathered}$ | $\begin{gathered} 0.72 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.39) \end{gathered}$ | 5.96 |
| 31 | $\begin{gathered} 0.34 \\ {[0.00,0.97]} \end{gathered}$ | $\begin{gathered} 0.34 \\ {[0.00,0.65]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.07) \end{gathered}$ | 9.04 | 127 | $\begin{gathered} 0.65 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.06]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.78) \end{gathered}$ | 5.60 |
| 32 | $\begin{gathered} 0.95 \\ {[0.25,1.00]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.15]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.58) \end{gathered}$ | 14.97 | 128 | $\begin{gathered} 0.57 \\ {[0.18,0.80]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.41]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.78) \end{gathered}$ | 31.80 |
| 33 | $\begin{gathered} 0.36 \\ {[0.01,1.00]} \end{gathered}$ | $\begin{gathered} 0.36 \\ {[0.00,0.84]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.19) \end{gathered}$ | 8.01 | 129 | $\begin{gathered} 0.18 \\ {[0.00,0.49]} \end{gathered}$ | $\begin{gathered} 0.18 \\ {[0.00,0.47]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.02) \end{gathered}$ | 14.62 |
| 34 | $\begin{gathered} 0.31 \\ {[0.03,0.62]} \end{gathered}$ | $\begin{gathered} 0.19 \\ {[0.00,0.48]} \\ \hline \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 35.86 | 130 | $\begin{gathered} 0.06 \\ {[0.00,0.60]} \\ \hline \end{gathered}$ | $\begin{gathered} 0.06 \\ {[0.00,0.40]} \\ \hline \end{gathered}$ | $\begin{gathered} 0 \\ (0.01) \end{gathered}$ | 34.20 |

Table C3-: Individual level structural parameter estimates. Values in square brackets are bootstrapped $95 \%$ confidence intervals. Values in parentheses are the proportion of bootstraps in which $\psi=1 . \lambda$ is the goodness of fit parameter, where higher values indicate a better model fit.

| ID | $\chi_{H}$ | $\chi_{R}$ | $\psi$ | $\lambda$ | ID | $\chi_{H}$ | $\chi_{R}$ | $\psi$ | $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 | $\begin{gathered} 0.93 \\ {[0.32,1.00]} \end{gathered}$ | $\begin{gathered} 0.44 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.79) \end{gathered}$ | 13.07 | 131 | $\begin{gathered} 0.63 \\ {[0.28,1.00]} \end{gathered}$ | $\begin{gathered} 0.24 \\ {[0.00,0.58]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.05) \end{gathered}$ | 22.33 |
| 36 | $\begin{gathered} 0.56 \\ {[0.29,1.00]} \end{gathered}$ | $\begin{gathered} 0.48 \\ {[0.11,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.19) \end{gathered}$ | 16.16 | 132 | $\begin{gathered} 0.72 \\ {[0.50,0.85]} \end{gathered}$ | $\begin{gathered} 0.28 \\ {[0.23,0.51]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.55) \end{gathered}$ | 0.00 |
| 37 | $\begin{gathered} 0.00 \\ {[0.00,0.61]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.10]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.04) \end{gathered}$ | 8.06 | 133 | $\begin{gathered} 0.30 \\ {[0.00,0.80]} \end{gathered}$ | $\begin{gathered} 0.30 \\ {[0.00,0.80]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 11.14 |
| 38 | $\begin{gathered} 0.37 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.08 \\ {[0.00,0.72]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.27) \end{gathered}$ | 11.09 | 134 | $\begin{gathered} 0.00 \\ {[0.00,0.55]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.55]} \end{gathered}$ | $\begin{gathered} 1 \\ (1.00) \end{gathered}$ | 5.52 |
| 39 | $\begin{gathered} 0.37 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.27) \end{gathered}$ | 7.88 | 135 | $\begin{gathered} 0.00 \\ {[0.00,0.14]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.14]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 7.66 |
| 40 | $\begin{gathered} 0.01 \\ {[0.00,0.60]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.55]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.01) \end{gathered}$ | 7.83 | 136 | $\begin{gathered} 0.71 \\ {[0.42,0.91]} \end{gathered}$ | $\begin{gathered} 0.34 \\ {[0.00,0.76]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.01) \end{gathered}$ | 12.76 |
| 41 | $\begin{gathered} 0.46 \\ {[0.16,1.00]} \end{gathered}$ | $\begin{gathered} 0.09 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.05) \end{gathered}$ | 10.84 | 137 | $\begin{gathered} 0.18 \\ {[0.00,0.52]} \end{gathered}$ | $\begin{gathered} 0.18 \\ {[0.00,0.49]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.01) \end{gathered}$ | 12.13 |
| 42 | $\begin{gathered} 0.60 \\ {[0.20,1.00]} \end{gathered}$ | $\begin{gathered} 0.44 \\ {[0.00,0.67]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.07) \end{gathered}$ | 14.77 | 138 | $\begin{gathered} 0.57 \\ {[0.09,0.81]} \end{gathered}$ | $\begin{gathered} 0.57 \\ {[0.00,0.77]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.03) \end{gathered}$ | 6.43 |
| 43 | $\begin{gathered} 0.70 \\ {[0.13,1.00]} \end{gathered}$ | $\begin{gathered} 0.70 \\ {[0.10,1.00]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.02) \end{gathered}$ | 14.66 | 139 | $\begin{gathered} 0.00 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.67]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.85) \end{gathered}$ | 10.31 |
| 44 | $\begin{gathered} 0.70 \\ {[0.00,0.87]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.47]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.58) \end{gathered}$ | 15.74 | 140 | $\begin{gathered} 0.00 \\ {[0.00,1.00]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.54]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.99) \end{gathered}$ | 4.30 |
| 45 | $\begin{gathered} 0.81 \\ {[0.56,0.85]} \end{gathered}$ | $\begin{gathered} 0.18 \\ {[0.11,0.46]} \end{gathered}$ | $\begin{gathered} 1 \\ (0.87) \end{gathered}$ | 2,101,298.50 | 141 | $\begin{gathered} 0.65 \\ {[0.47,0.81]} \end{gathered}$ | $\begin{gathered} 0.65 \\ {[0.24,0.70]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 42.28 |
| 46 | $\begin{gathered} 1.00 \\ {[0.74,1.00]} \end{gathered}$ | $\begin{gathered} 0.23 \\ {[0.00,0.82]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.29) \end{gathered}$ | 14.64 | 142 | $\begin{gathered} 0.24 \\ {[0.01,0.62]} \end{gathered}$ | $\begin{gathered} 0.24 \\ {[0.01,0.62]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.00) \end{gathered}$ | 19.82 |
| 47 | $\begin{gathered} 0.22 \\ {[0.00,0.77]} \end{gathered}$ | $\begin{gathered} 0.22 \\ {[0.00,0.48]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.02) \end{gathered}$ | 18.33 | 143 | $\begin{gathered} 0.42 \\ {[0.00,0.73]} \end{gathered}$ | $\begin{gathered} 0.41 \\ {[0.00,0.60]} \end{gathered}$ | $\begin{gathered} 0 \\ (0.01) \end{gathered}$ | 11.81 |
| 48 | $\begin{gathered} 0.41 \\ {[0.00,0.86]} \end{gathered}$ | $\begin{gathered} 0.41 \\ {[0.00,0.66]} \\ \hline \end{gathered}$ | $\begin{gathered} 0 \\ (0.02) \end{gathered}$ | 13.37 | 144 | $\begin{gathered} 0.00 \\ {[0.00,0.41]} \\ \hline \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.23]} \\ \hline \end{gathered}$ | $\begin{gathered} 0 \\ (0.02) \\ \hline \end{gathered}$ | 4.98 |

Table C1-Realized PG provision and overprovision for all treatments.

| Private good base value $\left(D_{0}\right):$ | 0 | 30 | 70 |
| :--- | :---: | :---: | :---: |
| Public Good frequency: |  |  |  |
| Static | 0.835 | 0.645 | 0.283 |
| (standard error of mean) | $(0.012)$ | $(0.015)$ | $(0.015)$ |
| Dynamic | 0.836 | 0.672 | 0.359 |
| (standard error of mean) | $(0.008)$ | $(0.011)$ | $(0.011)$ |
| Loss frequency (PG value < private good value): |  |  |  |
| Static | 0.152 | 0.165 | 0.094 |
| (standard error of mean) | $(0.012)$ | $(0.012)$ | $(0.009)$ |
| Dynamic | 0.158 | 0.178 | 0.123 |
| (standard error of mean) | $(0.008)$ | $(0.009)$ | $(0.008)$ |

Table C2-Realized PG provision and overprovision for 20-period ranges in all treatments.

| Private good base value $\left(D_{0}\right):$ | 0 | 30 | 70 |
| :--- | :---: | :---: | :---: |
| Public Good frequency: |  |  |  |
| Static actual (all periods 1-20) | 0.835 | 0.645 | 0.283 |
| (standard error of mean) | $(0.012)$ | $(0.015)$ | $(0.015)$ |
| Dynamic actual (all periods 1-40) | 0.836 | 0.672 | 0.359 |
| (standard error of mean) | $(0.008)$ | $(0.011)$ | $(0.011)$ |
| Dynamic actual (periods 1-20) | 0.857 | 0.728 | 0.373 |
| (standard error of mean) | $(0.011)$ | $(0.014)$ | $(0.016)$ |
| Dynamic actual (periods 21-40) | 0.816 | 0.617 | 0.346 |
| (standard error of mean) | $(0.013)$ | $(0.016)$ | $(0.015)$ |
| Loss frequency (PG value < private good value): |  |  |  |
| Static actual (all periods 1-20) | 0.152 | 0.165 | 0.094 |
| (standard error of mean) | $(0.012)$ | $(0.012)$ | $(0.009)$ |
| Dynamic actual (all periods 1-40) | 0.158 | 0.178 | 0.123 |
| (standard error of mean) | $(0.008)$ | $(0.009)$ | $(0.008)$ |
| Dynamic actual (periods 1-20) | 0.169 | 0.188 | 0.125 |
| (standard error of mean) | $(0.012)$ | $(0.013)$ | $(0.011)$ |
| Dynamic actual (periods 21-40) | 0.147 | 0.169 | 0.122 |
| (standard error of mean) | $(0.011)$ | $(0.012)$ | $(0.011)$ |



Figure C1. Cumulative Density Functions of the midpoint of the cutoff intervals (SEe Section 3B OF The main paper). Vertical Lines denote equilibrium predictions for Static Nash equiLibrium, Dynamic Nash equilibrium and Cursed equilibrium in navy, maroon and dark green, RESPECTIVELY.

## D. Experiment Instructions

This section consists of the experimental instructions. Differences between the dynamic and static treatments are highlighted in bold. Note that the numerical examples included in the instructions were the same across both treatments, and also the same for all subjects.

## D1. The instructions

## EXPERIMENT INSTRUCTIONS PART ONE

## Overview

This is an experiment in the economics of decision-making. The amount of money you earn depends partly on the decisions that you make and thus you should read the instructions carefully. The money you earn will be paid privately to you, in cash, at the end of the experiment. A research foundation has provided the funds for this study.

There are two parts to this experiment. These instructions pertain to Part 1A of the experiment. Once Part 1 is complete, the instructions for Part 2 will be distributed. Part 1 of the experiment is divided into many decision "periods." For Part 1, you will be paid your earnings in one, randomly selected, period. The period for which you will be paid shall be announced at the end of the experiment. Each decision you make is therefore important because it has a chance to affect the amount of money you earn.

In each decision period you will be grouped with two other people, who are sitting in this room, and the people who are grouped together will be randomly determined each period. You will be in a "matching group" of twelve people. You will only ever be matched with other people in the same "matching group" as yourself, which means that there are at most eleven other people you could be matched with each period.

You will make decisions privately, that is, without consulting other group mem-
bers. Please do not attempt to communicate with other participants in the room during the experiment. If you have a question as we read through the instructions or any time during the experiment, raise your hand and an experimenter will come by to answer it.

Your earnings in Part 1 of the experiment are denominated in experimental dollars, which will be exchanged at a rate of 10 experimental dollars $=1$ U.S. dollar at the end of the experiment.

## Your Decisions

Part 1A of the experiment consists of 40 periods. ( 20 periods for static treatment)

In each period, you will choose whether to receive earnings from the group project or you may instead choose to receive earnings from your private project. You will receive earnings either from the group or the private project, and never from both projects. Everyone in your group each period will make a similar decision. If you choose the group project, you only will receive earnings from the group project if at least one other person in your group chooses the group project. If you are the only one choosing the group project, then you receive earnings from the private project instead. The details of your earnings for these decisions are described below.

## Group Project

In each period, a random number will be selected by the computer for you from a uniform distribution between 0 and 100. The uniform distribution means that the 101 possible values $0,1,2, \ldots, 99,100$ are equally likely. We will call this random number your signal. Each other member of your group will also get a signal randomly selected by the computer from this same distribution. We will call the signals of the three group members S1, S2 and S3. All signals are drawn independently, which means that no drawn signal can have any influence on any other signal draws. During each period, you will not observe the signals of the other members. Similarly, other members of the group will not observe any signal
other than their own.
If you choose the group project, and if at least one other member of your group also chooses the group project, then you receive earnings that are equal to the sum of the signals of all three members of your group. We will call the sum of the signals of the three members of your group the value of the group project, or $V$ :
$\mathrm{V}=\mathrm{S} 1+\mathrm{S} 2+\mathrm{S} 3$
So, for example, if your signal is 50 and the other members of your group get signals of 25 , and 86 , then the sum of all three signals is:
$\mathrm{V}=50+25+86=161$
Thus, in this case, if you chose the group project and at least one other member of your group also choses the group project, then you would get 161 experimental dollars for that period.

If you choose the group project, but no other members of your group also choose the group project, then you receive earnings from your private project instead for that period. (In other words, if less than two of the three members of your group (including yourself) choose the group project, then all group members receive earnings from their private projects that period.) These private project earnings are described next.

## Private Project

In each period, the baseline value of your private project is 70 . This number is predetermined (i.e. not random) and is the same for all three members of your group. In each period, in addition, two random numbers will be selected by the computer for you from a uniform distribution between 0 and 100 . We will call these two draws D2 and D3. These two numbers are drawn independently and will determine your earnings from the private project. (Other group members will receive their own random numbers, independently drawn, for their private projects.)

Like the group project, your private project value (P) comes from the sum of
three values:
$\mathrm{P}=70+\mathrm{D} 2+\mathrm{D} 3$
You will only know the baseline value of 70 before you make your decision. So, for example, if the other two drawn values that you did not learn are D2=6 and $\mathrm{D} 3=46$, then your earnings from the project would be

$$
\mathrm{P}=70+6+46=122
$$

You will receive these private project earnings if either (1) you choose the private project or (2) you are the only person in your group who chooses the group project.


Figure D1. Decision Screen

Note that at the time of your choice, you only observe your own signal (S1, S2 or S3) of the group project value and the baseline number, 70, that determines part of the value of your private project. This is illustrated in your decision screen shown in Figure D1.

Three Choice Stages
You and other group members will have an opportunity to choose the group project in 3 sequential stages each period. If you choose the group project in an early stage you cannot switch to choose the private project instead in a later stage. But if you choose the private project in an early stage you can switch your choice to the group project in a later stage.

In Stage 1, everyone will make a first choice between the group and private project before learning the decisions of other group members.

In Stage 2, everyone will learn how many group members chose the group project in Stage 1, and those who have not yet chosen the group project may then switch to the group project. This is illustrated in Figure D2.

In Stage 3, everyone will learn how many group members chose the group project in Stages 1 and 2, and those who have not yet chosen the group project may then switch to the group project.

```
Decision Page -- Round 21 of 40
Stage 2 of 3
    Summary
    The Group Project has value V=S1+S2+S3
    The Private Project has value P=D1+D2+D3.
For all players in your group D1 =30
Your individual value of S1 for this round is S1 = 69
The number of group members who have already chosen the
group project so far is: 2
Please indicate whether you choose the group project:
Choose the group project?
......... .
```



```
.......... .
Next
```

Figure D2. Second Stage to Choose the Group Project

Note: The above Three Choice Stages subsection was included for only the dynamic treatment. In the static treatment this was replaced with the following paragraph, and Figure D2 was omitted.
You and the others in your group, will make your decisions at the same time. In other words, everyone in your group makes their choice before learning the choices of other group members.

## End of the Period

After all members of your group have made their choices, you will learn the values of the group project $(\mathrm{V})$ and the private project $(\mathrm{P})$, and your earnings in experimental dollars for the period. You will also learn how many other members of your group chose the group project, and the other two D2 and D3 draws that determine the private project value P .

As illustrated in Figure D3, your computer will also display at the end of the period a summary of the results from all previous periods in this part of the experiment, in a table you can scroll through if desired.

```
Results
The value of your private project is: 30 +67+50=107
The value of the group project is 23+35+67 = 15%
The number of people in your group that chose the group project (including yourself was: 2 out of 3
You chose: Group Project
You recevived: Group Project
The payoff for you this round is: IS points
Nex
Round history
\begin{tabular}{llllllll} 
Round \(\#\) & S1 & D1 & Group project Value & Private project value & Your Choice & Other group members' choices & Project selected \\
1 & 35 & 2 & & Group Project & \begin{tabular}{l} 
Group Project \\
Private Project
\end{tabular} & Group Project \\
\hline
\end{tabular}
```

Figure D3. Results Screen

Remember that you will be randomly and anonymously re-matched into new
groups of three at the start of each period. Also remember that signals for the group project and the D2 and D3 draws for the private project are randomly and independently drawn for each member of your group.

Of the 60 periods in Part 1, one will be randomly selected for payment. All participants will be paid their earnings converted to US dollars for the randomly selected period, plus a $\$ 5.00$ show-up payment. You will not find out which period you will be paid for until the end of the experiment, so you should treat each period as something for which you might get paid. You will not be paid for the periods that are not randomly selected for payment.

Summary of Part 1A
In each period:

- The value of the group project is given by $\mathrm{V}=\mathrm{S} 1+\mathrm{S} 2+\mathrm{S} 3$. You will observe your own signal, but not the signals of the other members of your group.
- The value of the private project is given by $\mathrm{P}=70+\mathrm{D} 2+\mathrm{D} 3$. You will observe the baseline value 70 but not D2 or D3.
- Static treatment only: You and others in your group make your choice for the group project or private project at the same time, before learning the choices made by any other group members.
- Dynamic treatment only: You may choose the group project in one of three stages. After choosing the group project in a period you cannot switch back to choose the private project. But if you do not choose the group project in the first stage, then you may do so in the second stage. If you do not choose the group project in the first or second stage, you may do so in the third stage. At each stage, you will observe how many of your group members chose the group project in a prior stage.
- If you chose the group project, and at least one other member of your group also chose the group project, then you will earn V (the value of the group project).
- If you do not choose the group project, or you are the only member of your group who chose the group project, then you will earn P (the value of the private project).
- At the start of each period, you will be randomly and anonymously matched into groups of three. At the start of each later period, you will be randomly and anonymously re-matched into new groups of three and you never learn the identities of the other group members in any period. It is possible, but unlikely, that you may be grouped with the same people in two consecutive periods.

Are there any questions before we begin the experiment?

## Distributed separately at the end of the session:

 EXPERIMENT INSTRUCTIONS PART TWOPart 2 will consist of two periods of decisions. You will be paid, in experimental dollars, for the sum of your earnings in both periods. At the end of the experiment we will convert the experimental dollars you earn in part 2 to U.S. dollars at an exchange rate of 50 experimental dollars equals $\$ 1$.

In each period, the computer will randomly draw an integer from a pre-specified interval. The interval with either be from 0 to 99 , or from 20 to 129. Each number in the interval will be equally likely to be chosen. In each period, you will be required to submit a bid to the computer.

If your bid is greater than or equal to the random number, you will receive 100 experimental dollars, plus 1.5 times the random number, minus your bid. If your bid is less than the random number you will receive 100 experimental dollars.

If, for example, you bid 42:

- Suppose the value of the random number is 36 . Then your payoff will be $100+1.5^{*} 36-42=112$.
- Suppose the value of the random number is 20 . Then your payoff will be $100+1.5^{*} 20-42=88$.
- Suppose the value of the random number is 67 . Then your payoff will be 100.

Your results from each period will be displayed on the screen.
Are there any questions before we begin part 2?

## REFERENCES

Calford, Evan M., and Timothy N. Cason. Forthcoming. "Contingent Reasoning and Dynamic Public Goods Provision." American Economic Journal: Microeconomics.


[^0]:    * Calford: John Mitchell Fellow, Research School of Economics, Australian National University, Canberra, Australia, Evan.Calford@anu.edu.au. Cason: Department of Economics, Purdue University, Indiana, USA, cason@purdue.edu. We thank William Brown for excellent research assistance, Gary Charness, Anujit Chakraborty, Caleb Cox, Simon Siegenthaler, Manu Vespa, the editor and three anonymous referees for valuable comments, as well as audiences at the ESA and M-BEES conferences and ANU, Innsbruck, UC-Davis, Melbourne, Oklahoma, Virginia Tech, New Zealand and WEBEAS eSeminars. Purdue University provided funding for experimental subject payments.
    ${ }^{1}$ For a definition of the game and notation, the reader is referred to the main paper.

[^1]:    ${ }^{2}$ Given our symmetry assumption, the agent holds the same beliefs regarding the behavior of each of the other two players.
    ${ }^{3}$ Note that these values are the expected payoffs associated with the given actions, while the values $\mathbb{E}[P]$ and $\mathbb{E}\left[V_{i}\right]$ in the main text are the payoffs associated with the outcomes of receiving the public or private goods.

[^2]:    ${ }^{4}$ This rules out equilibria of the following variety: all agents select the private good for the first two stages, and then play the static equilibrium in the third stage. This assumption is also justified by observed behavior in the experiment, which indicates that subjects did not universally delay their choice of the public good. For example, 143 out of the 144 participants chose the public good at least once in the first stage of the dynamic treatment.

[^3]:    ${ }^{5}$ As discussed below, an agent with $\psi=0$ will condition on the event that the remaining player does not select the public good in the second stage. However, because this reasoning requires the agent to think ahead to the third stage, an unaware agent does not perform this inference.

