Discounting Models for Long-Term Decision Making

C. Robert Kenley
Donald C. Armstead
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The Origin of Discounting

• Samuelson showed in 1937 under certain assumptions that the value today of something received $k$ years from today is its value when received in year $k$ discounted by the factor $1/(1+i)^k$, where $i$ is a positive number

• He specifically made statements that
  – The model had no demonstrated descriptive validity with respect to empirical data
  – The model had no prescriptive validity as a method of determining social welfare

• Nevertheless, Samuelson’s constant rate discounting is now the method prescribed by the U.S. government and others for making life cycle cost comparisons
Samuelson showed in 1937 under certain assumptions that the value today of something received (positive number) or expended (negative number) \( k \) years from today is its value when received in year \( k \) discounted by the factor \( \frac{1}{(1+i)^k} \), where \( i \) is a positive number.

- Used for decision analysis in for-profit organizations.
- Prescribed by the U.S. government Office of Management and Budget (OMB) for making time preference comparisons on federal projects.

Define a discrete index for a set evenly-spaced time intervals: 
\[ \{0, 1, \ldots, T\} \]

Define a stream of flows over these time intervals: 
\[ \{C_0, C_1, \ldots, C_T\} \]

where positive flows are net inputs and negative flows are net outputs of value into a system.

The net present value is
\[
NPV = \sum_{t=0}^{T} \frac{C_t}{(1+r)^t} \\
= C_0 + \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_T}{(1+r)^T}
\]

Large closure costs occur late in the program over 100 years in the future.
How much should we invest up front to reduce closure costs?
Costs beyond 2049 are essentially ignored after discounting.

Implication is we would pay nothing today to reduce closure costs.
Research Approach

• Review literature in the area of long-term discounting
• Choose a model that has axiomatic rationale
• Choose a model that accurately reflects public preference
• Choose a model that can be implemented easily
A Live Experiment (Question 1)

• What is your preference: A or B?
A Live Experiment (Questions 2 & 3)

ALTERNATIVE A

ALTERNATIVE B

• Now what is your preference: A or B?

• What is your preference on 13 MAR 2103?
Definition: Preference Reversal

• When A is preferred to B at one time but B is preferred to A at another (later) time
• For instance, one might prefer 1 candy bar today to 2 candy bars tomorrow, but one would surely prefer 2 candy bars in one year and one day to 1 candy bar in a year
• Constant rate discounting does not allow preference reversal!
## Discounting Models

<table>
<thead>
<tr>
<th>Discounting Model Name</th>
<th>Time Perception Function $\alpha(t)$</th>
<th>Discount Factor $w_t$ at Time $t$</th>
<th>Preference Reversal Allowed?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Rate</td>
<td>$t$</td>
<td>$1/(1 + i)^t$</td>
<td>No</td>
</tr>
<tr>
<td>Exponential</td>
<td>$\frac{h}{\ln(1 + i)}t$</td>
<td>$1/e^{ht}$</td>
<td>No</td>
</tr>
<tr>
<td>Hyperbolic</td>
<td>$\frac{h}{g} \log_{1+i}(1 + gt)$</td>
<td>$1/(1 + gt)^{\frac{h}{g}}$</td>
<td>Yes</td>
</tr>
<tr>
<td>Relative</td>
<td>$h \log_{1+i}(1 + t)$</td>
<td>$1/(1 + t)^{h}$</td>
<td>Yes</td>
</tr>
<tr>
<td>Proportional</td>
<td>$\log_{1+i}(1 + gt)$</td>
<td>$1/(1 + gt)$</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Preference Implication of Constant Rate Discounting

• If the time difference between receipt of 1 candy bar and 4 candy bars is constant, the preferences remain the same
  – If 1 candy bar 1 day from today is equivalent to 4 candy bars 2 days from today, then
    • 1 candy bar 1 year from today is equivalent to 4 candy bars 1 year and 1 day from today
    • 1 candy bar 4 years from today is equivalent to 4 candy bars 4 years and 1 day from today
Preference Implication for Relative Discounting

- The preference relationship is **relative** in that if ratio of the delay is constant then the ratio of the equivalent rewards is constant
  - If 1 candy bar 1 day from today is equivalent to 4 candy bars 2 days from today, the 1 candy bar 1 year from today is equivalent to 4 candy bars 2 years from today
Preference Implication for Proportional Discounting

- The preference relationship is **proportional** in that the acceptable amount of delay is proportional to the amount of reward
  - A delay of 1 year for 1 candy bar is equivalent to a delay of 4 years for 4 candy bars
Research Summary

- Empirical studies all agree that proportional discounting best fits data
- Proportional discounting allows for preference reversals
- Proportional discounting can be derived from the same type of axiomatic rationale as constant discounting
- Constant discounting has an inherent 40-year maximum
Selecting A Proportional Discount Parameter

• At year 30, a proportional model with parameter $g = 7.2\%$ is equivalent to a constant discount rate of $3.9\%$ applied to cost and benefits incurred in year 30

$$\frac{1}{1 + 0.072 \times 30} = \frac{1}{1.039^{30}}$$

• $3.9\%$ is the OMB-approved rate for 30-year internal government project tradeoff analyses

• This provides an “anchor point” for proportional discounting to match the OMB guidance
- Discounted closure costs are no longer essentially zero
- Is is possible to trade current investments vs. reduced closure costs
## Comparison of Discount Rates

<table>
<thead>
<tr>
<th>Term (yr)</th>
<th>OMB Nominal Discount Rates (Feb 2002)</th>
<th>Consumer Nominal Interest Rates (Feb 2002)</th>
<th>Proportional Discount Rate (g = 0.072 with Inflation Added)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/12</td>
<td>-</td>
<td>0.130</td>
<td>0.094</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>0.117</td>
<td>0.090</td>
</tr>
<tr>
<td>3</td>
<td>0.041</td>
<td>-</td>
<td>0.087</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>0.075</td>
<td>0.085</td>
</tr>
<tr>
<td>5</td>
<td>0.045</td>
<td>0.061</td>
<td>0.083</td>
</tr>
<tr>
<td>7</td>
<td>0.048</td>
<td>-</td>
<td>0.080</td>
</tr>
<tr>
<td>10</td>
<td>0.051</td>
<td>-</td>
<td>0.076</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>0.064</td>
<td>0.075</td>
</tr>
<tr>
<td>30</td>
<td>0.058</td>
<td>0.069</td>
<td>0.058</td>
</tr>
</tbody>
</table>

- The consumer rates and proportional rate that matches the OMB rate at year 30 are reasonably well-matched
  - Both rates decrease with time
  - Consumer rate generally is a little higher (the lender has to make some money)
  - Increase in consumer rate from 15 to 30 years indicates the mortgage market is more driven by lender / investor liquidity needs than by borrower / consumer needs
- The 30-year OMB and consumer rates are close
  - The OMB rate is based on bond yields paid to investors
  - The consumer rate is based on a 30-year mortgage paid by consumers
  - The difference in the two rates is an estimate of the profit margin for the lender
Conclusion

• Hyperbolic discounting models provide an excellent alternative to constant discount rate discounting
  – They provide a better representation of public preference as shown by empirical evidence
  – Proportional discounting not only was the best fit in nearly every study conducted, but as a single parameter model, is just as simple to implement as is the OMB-approved constant discounting

• The 30-year mortgage rate and 30-year bond yield data indicate there may be a market-based method of setting hyperbolic discounting parameters
  – Additional research of market-based rates is needed to confirm this assertion