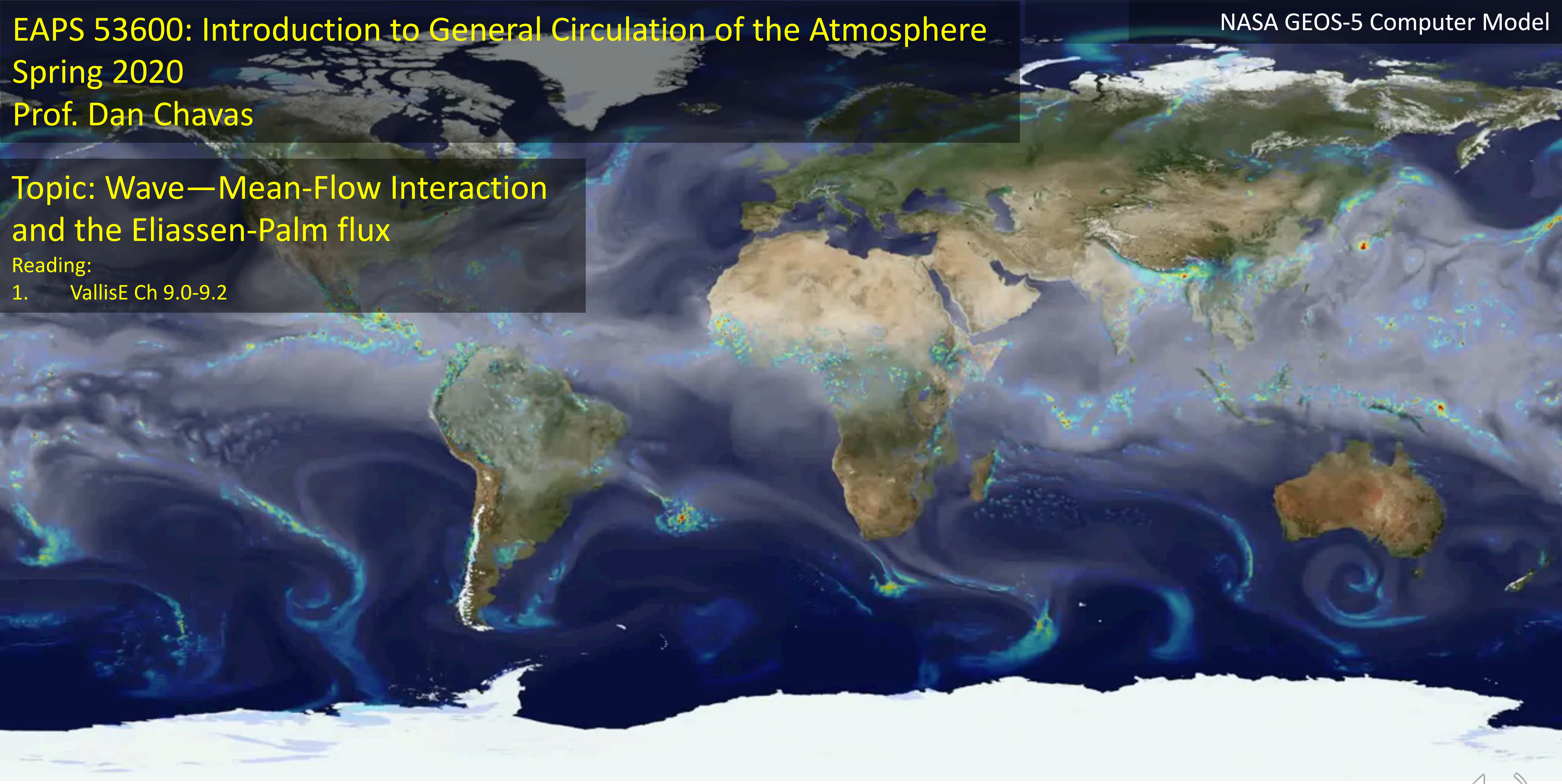
EAPS 53600: Introduction to General Circulation of the Atmosphere
Spring 2020
Prof. Dan Chavas



Topic: Wave—Mean-Flow Interaction and the Eliassen-Palm flux

Reading:

1. ValliseE Ch 9.0-9.2

White: total precipitable water (brigher white = more water vapor in column)
Colors: precipitation rate $(0 - 15\frac{mm}{hr}$, red=highest)

Source: https://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=30017

Learning outcomes for today:

- **Describe what “wave—mean-flow interaction” means**
- **Explain the distinction between the subtropical vs. mid-latitude jet stream**
- **Explain how eddies can change the mean PV**
- **Explain what the EP flux (wave activity flux) tells us about waves**

Why do we care about the *interaction* of waves with the mean flow?

Some “basic states” we’ve discussed thusfar.

Basic state = assume this background just magically exists.

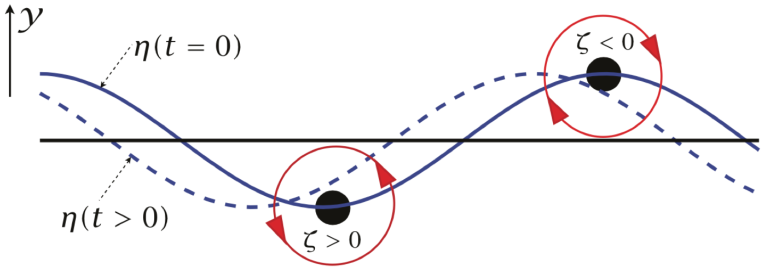


Fig. 6.6

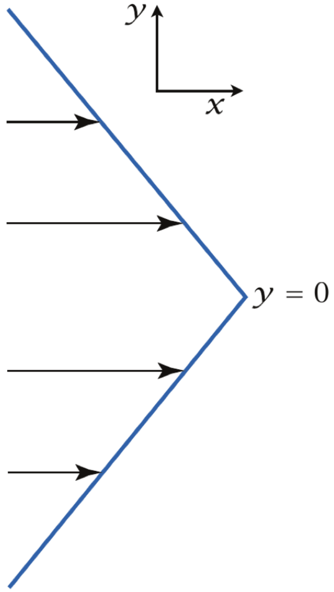


Fig. 8.4:

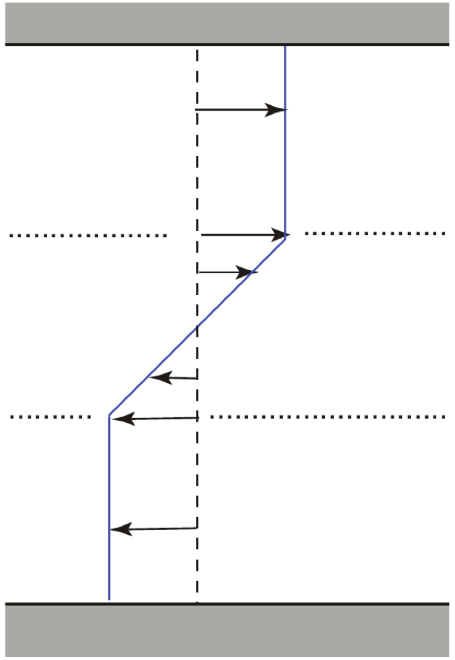
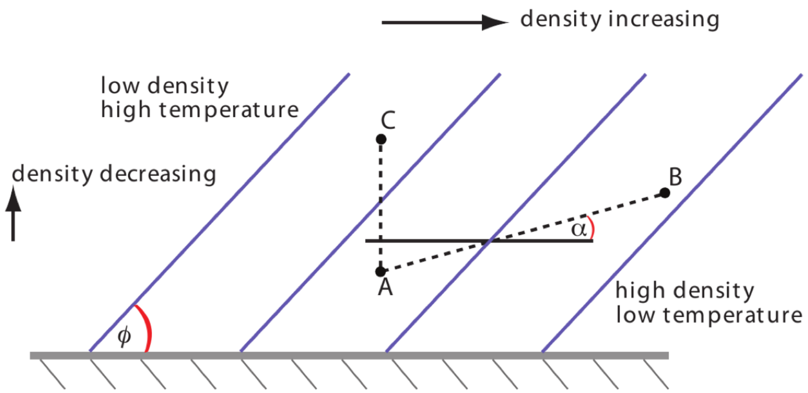


Fig. 8.3:

Fig. 8.6:



”Linear dynamics is mainly concerned with **waves and instabilities** that live on a **pre-determined background flow**.

But the real world isn't quite like that.

Rather, **the mean state is the *result*** of the combined effects of **thermal and mechanical forcing** (by radiation from the sun and, for the ocean, the winds) ***plus the action of the waves and instabilities themselves***.”

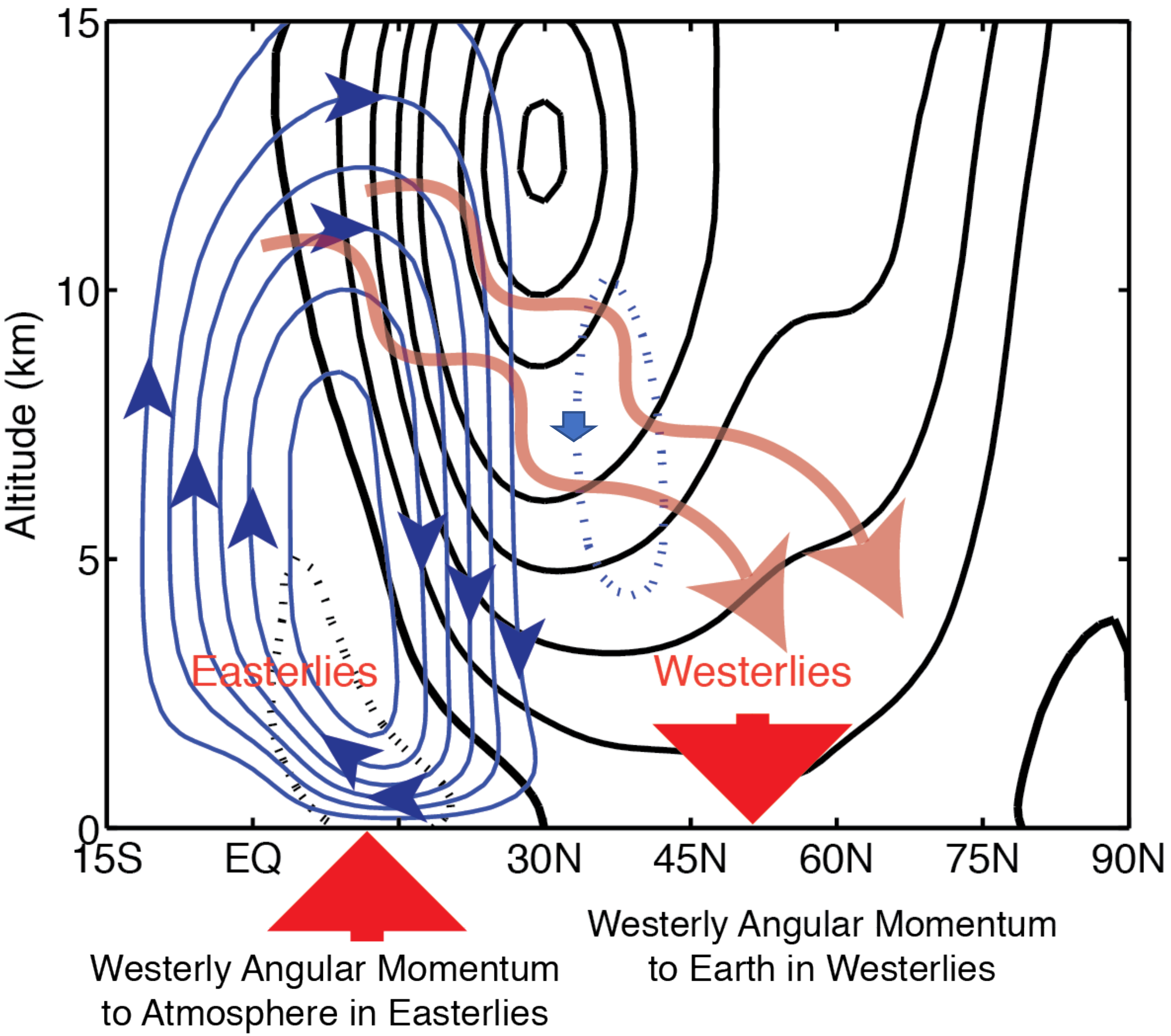
- ValliSE p170

Fig 6.17

The **mean state depends on** transport of energy and angular momentum by **circulations and eddies**.

However, the properties of these **circulations and eddies also *depend* on the mean state**.

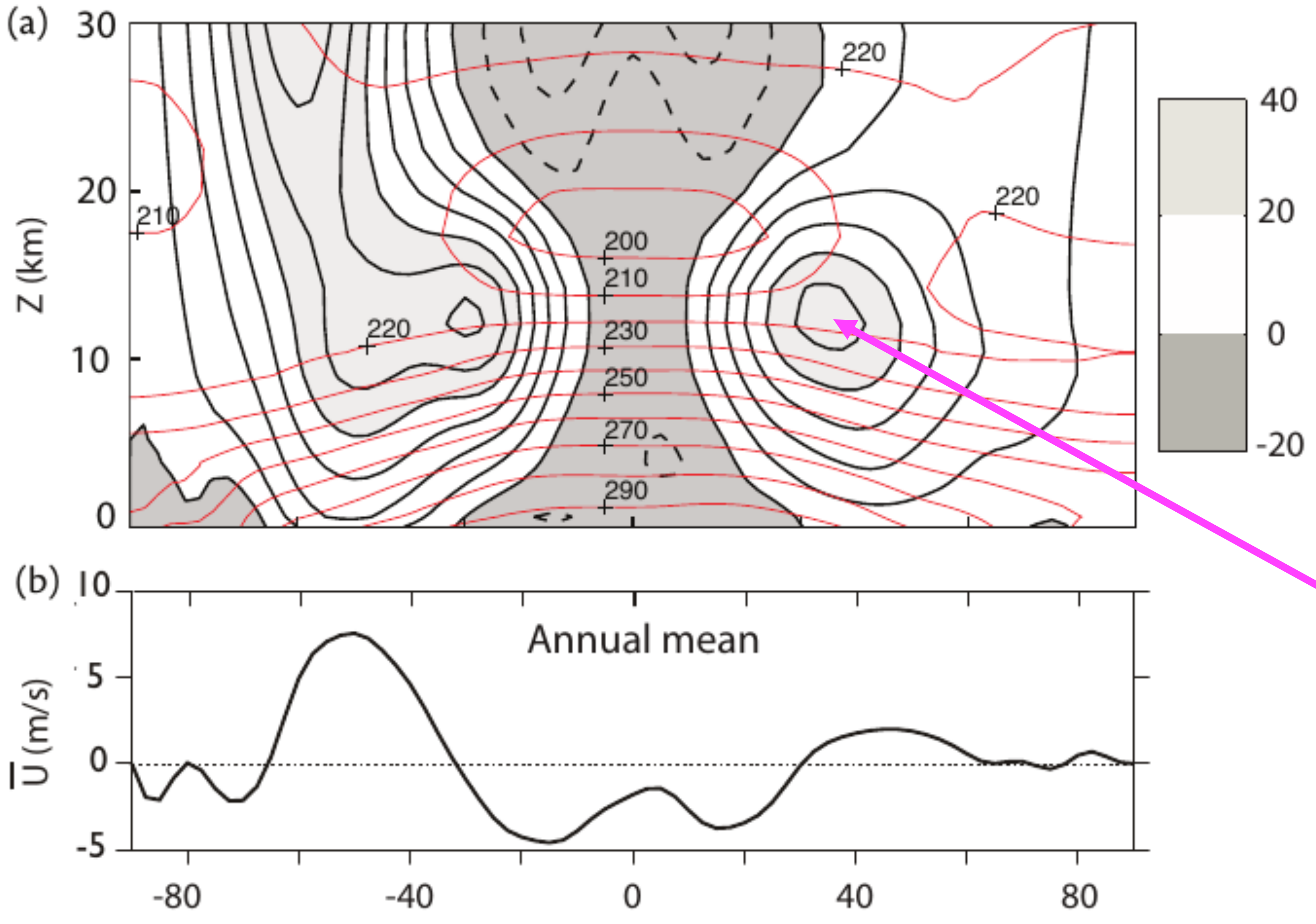
So there is a mutual interaction – **"eddy-mean flow interaction"** – that is fundamental, but is hard to deconvolve.



An extremely “simple” question an 8-year old might ask you:

Why is there a jet stream in the first place?

Fig. 11.1:

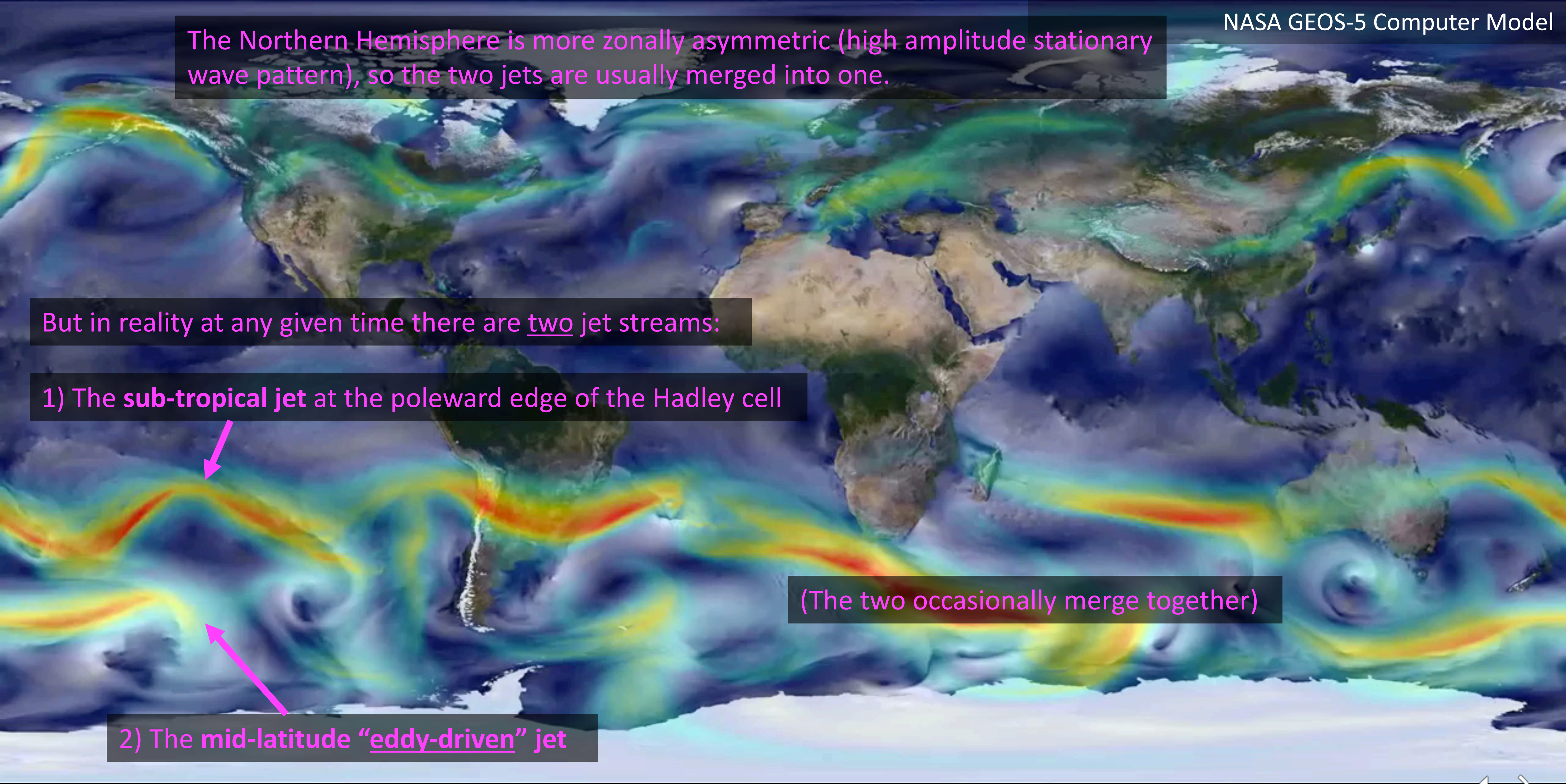


Annual-mean

Zonal-mean zonal wind (fill)

Temperature

Recall: this looks like one single jet in the zonal-mean...

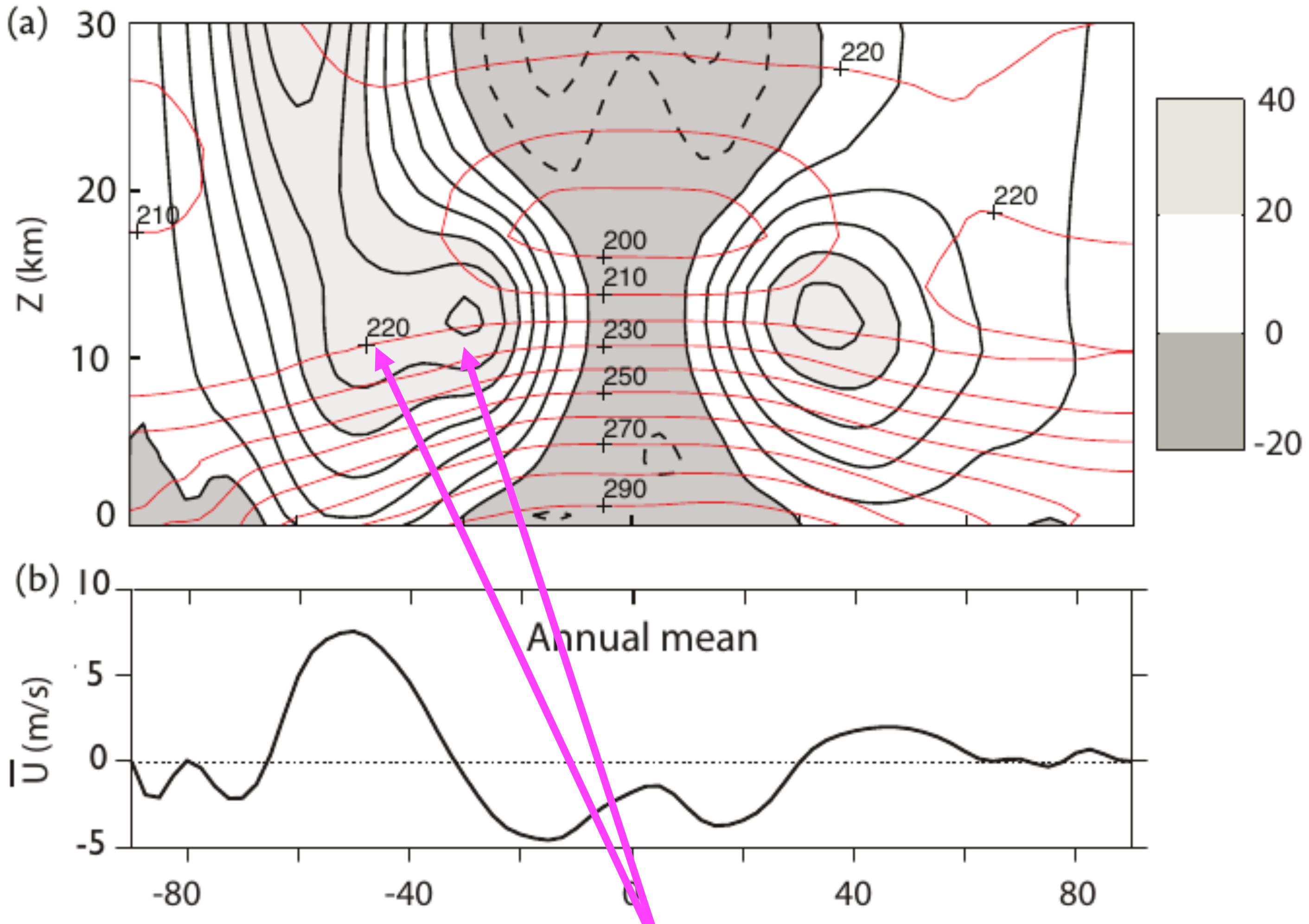


White shading: surface wind speed ($0 - 40\,\frac{m}{s}$, bright white = fastest)

Colors: upper-level (250 hPa) wind speed ($0 - 75\,\frac{m}{s}$, red = fastest).

Source: https://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=30017

Fig. 11.1:



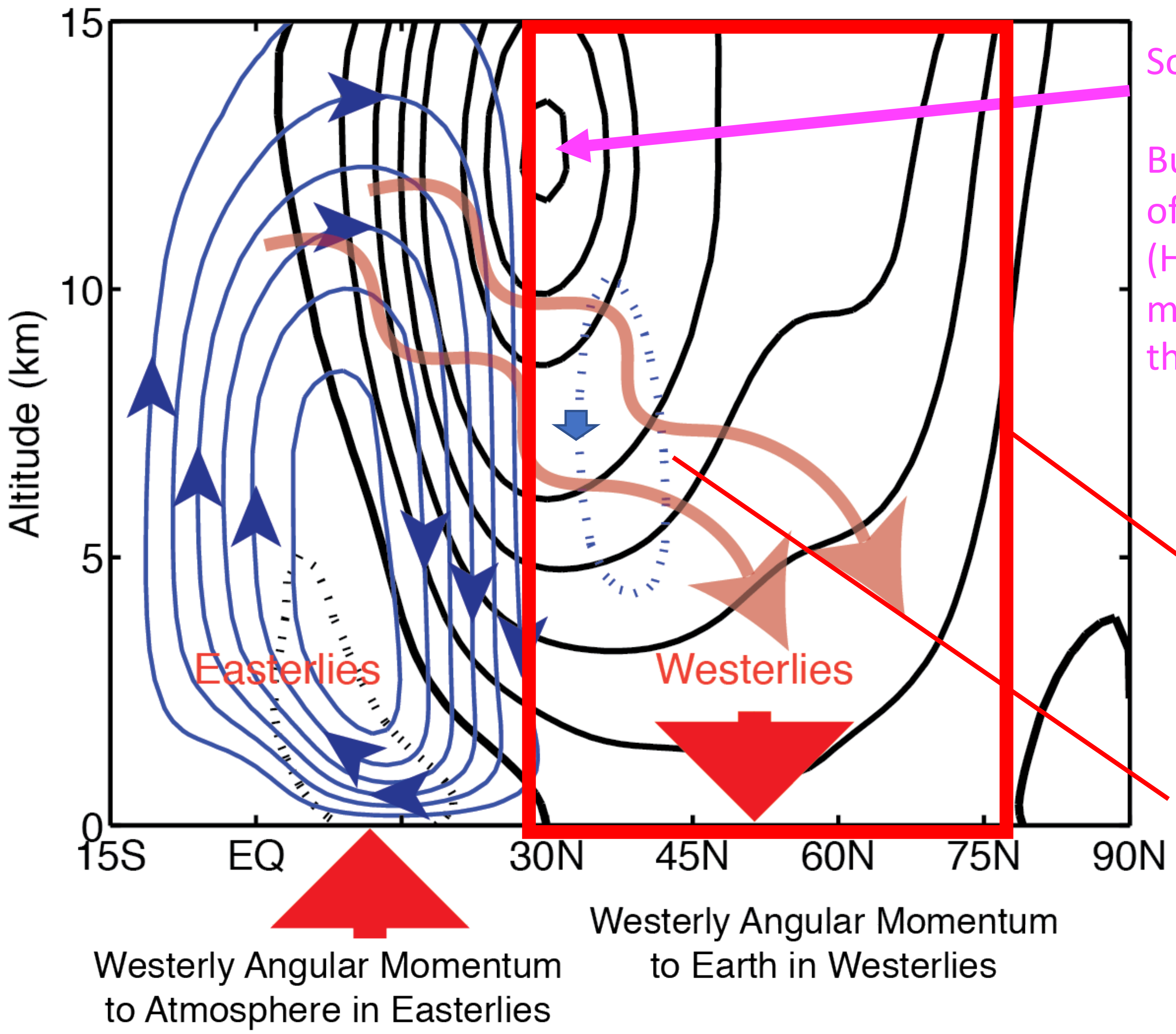
Annual-mean

Zonal-mean zonal wind (fill)

Temperature

There is actually a **hint of these two jets** in the zonal-mean in the Southern Hemisphere

Fig 6.17



So this looks like one jet.

But it's really the combination of the subtropical jet stream (Hadley cell angular momentum conservation) and the midlatitude jet stream.

Why does the midlatitude jet exist?

It's "eddy-driven" – it is maintained by the eddies themselves...

Also eddy-driven: the **Ferrell cell** overturning (thermally-indirect)

Do waves alter the mean flow?

How?

To answer these questions quantitatively, we need some new mathematical tools.

VallisE Ch 9 tackles this in the quasi-geostrophic system, Boussinesq fluid, for the zonal-mean.

General PV equation:

$$\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} = D$$

Local tendency of PV

Horizontal advection of PV

Sources/sinks of PV

Eddy PV

Linearized eddy QG PV equation:

$$\frac{\partial q'}{\partial t} + \bar{u}\frac{\partial q'}{\partial x} + v'\frac{\partial \bar{q}}{\partial y} = D', \quad (9.9)$$

Sources/sinks of eddy PV

Thermal wind balance:

$$f_0\frac{\partial \bar{u}}{\partial z} = -\frac{\partial \bar{b}}{\partial y}. \quad (9.11)$$

Before, we have assumed that the basic state $\overline{u}$ and $\overline{q}$ vary only in y and are fixed in time.

But we know that eddies transport things – momentum, heat (buoyancy) → and thus they transport potential vorticity itself. This would change the mean PV.

We need an equation for the mean PV, too: $\frac{\partial \overline{q}}{\partial t}$

Linearized <u>mean</u> QG PV equation:

$\overline{v} = 0$

$$\frac{\partial \overline{q}}{\partial t} + \frac{\partial}{\partial y}\overline{v'q'} = \overline{D}. \qquad (9.17)$$

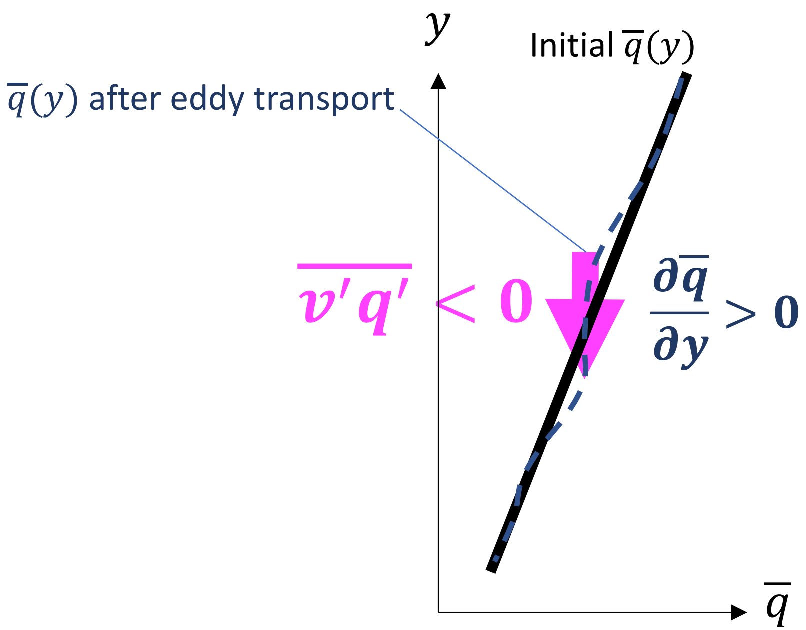
<u>Divergence</u> of eddy flux of PV

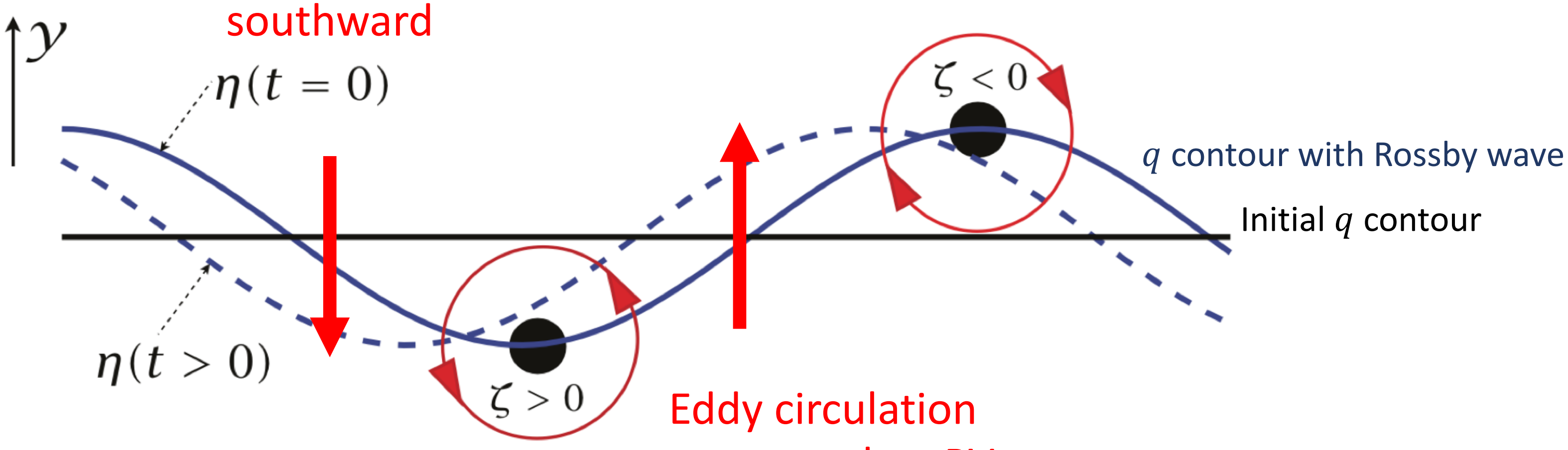
Sources/sinks of mean PV

What is this “eddy flux” term?

$v' < 0$
$q' > 0$
$\boldsymbol{v'q' < 0}$

Eddy circulation transports high PV southward





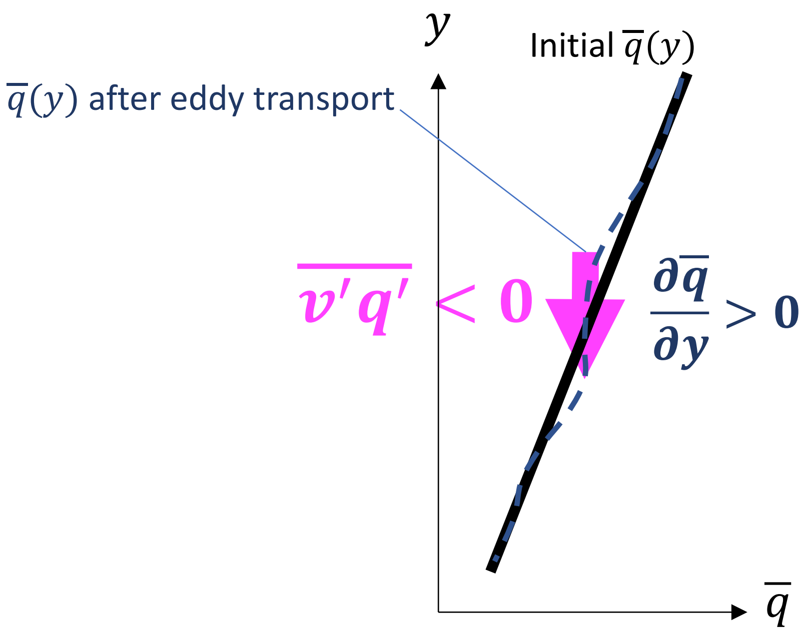
Eddy circulation transports low PV northward

$v' > 0$
$q' < 0$
$\boldsymbol{v'q' < 0}$

Net meridional flux of PV (here, it is southward)

This is "<u>down-gradient</u>".
i.e. from higher to lower values.

A rotating circulation in the presence of a mean gradient of a conserved quantity will transport that conserved quantity **down-gradient.** This is a <u>mixing</u> process, which will <u>smooth out</u> gradients locally.



Eddy PV flux	Eddy PV flux divergence (i.e. net eddy transport of PV)		Mean PV tendency
$\overline{v'q'} = 0$			
$\overline{v'q'} < 0$	$\frac{\partial}{\partial y}\left(\overline{v'q'}\right) > 0$	Eddy PV flux is **divergent** (eddies are removing PV)	$\frac{\partial \bar{q}}{\partial t} < 0$
	$\frac{\partial}{\partial y}\left(\overline{v'q'}\right) < 0$	Eddy PV flux is **convergent** (eddies are adding PV)	$\frac{\partial \bar{q}}{\partial t} > 0$
$\overline{v'q'} = 0$			

$$\frac{\partial \bar{q}}{\partial t} = -\frac{\partial}{\partial y}\left(\overline{v'q'}\right)$$

RHS is positive if there is the eddy PV transport is convergent
This acts to increase the mean PV.

Interior flow

Linearized eddy QG PV equation:

$$\frac{\partial q'}{\partial t} + \bar{u}\frac{\partial q'}{\partial x} + v'\frac{\partial \bar{q}}{\partial y} = D', \qquad (9.9)$$

Linearized mean QG PV equation:

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial}{\partial y}\overline{v'q'} = \bar{D}. \qquad (9.17)$$

Divergence of eddy flux of PV

Sources/sinks of mean PV

Top/bottom boundaries

(identical process as for PV above)

Linearized eddy buoyancy equation:

$$\frac{\partial b'}{\partial t} + \bar{u}\frac{\partial b'}{\partial x} + v'\frac{\partial \bar{b}}{\partial y} = S', \qquad (9.14)$$

Linearized mean buoyancy equation:

$$\frac{\partial \bar{b}}{\partial t} + \frac{\partial}{\partial y}\overline{v'b'} = \bar{S}. \qquad (9.18)$$

Divergence of eddy flux of buoyancy

Sources/sinks of mean buoyancy

Assume the system is inviscid (D=0) and unforced (S=0)

Linearized <u>mean</u> QG PV equation:

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial}{\partial y}\overline{v'q'} = 0, \qquad \text{(WM.1b)}$$

Definition of QG PV gradient:

$$\frac{\partial \bar{q}}{\partial y} = \beta - \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial}{\partial z}\left(\frac{f_0}{N^2}\frac{\partial \bar{b}}{\partial y}\right) = \beta - \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial \bar{u}}{\partial z}\right). \qquad \text{(WM.4)}$$

$\frac{\partial}{\partial y}(WM.1b) \rightarrow$

$$\frac{\partial}{\partial t}\frac{\partial \bar{q}}{\partial y} + \frac{\partial^2}{\partial y^2}\overline{v'q'} = 0$$

Plug in $(WM.4) \rightarrow$

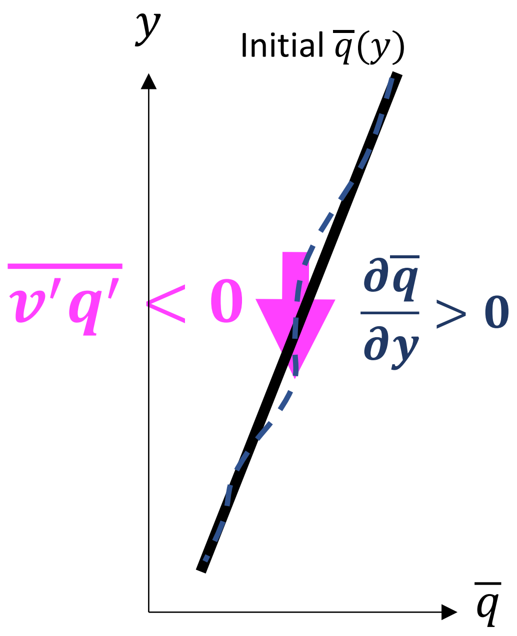
Note: $\frac{\partial}{\partial t}\beta = 0$

$$\left[\frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial}{\partial z}\right)\right]\frac{\partial \bar{u}}{\partial t} = \frac{\partial^2}{\partial y^2}\overline{v'q'}. \qquad \text{(WM.7)}$$

Acceleration of mean zonal flow

eddy PV fluxes

$$\left[\frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial}{\partial z}\right)\right]\frac{\partial \bar{u}}{\partial t} = \frac{\partial^2}{\partial y^2}\overline{v'q'}. \qquad \text{(WM7)}$$



Eddy PV flux	Eddy PV flux divergence (i.e. <u>net</u> eddy transport of PV)	*Meridional gradient* of eddy PV flux divergence	Mean zonal wind tendency
$\overline{v'q'} = 0$			
	$\frac{\partial}{\partial y}\left(\overline{v'q'}\right) > 0$		
$\overline{v'q'} < 0$		$\frac{\partial^2}{\partial y^2}\left(\overline{v'q'}\right) > 0$	$\frac{\partial \overline{u}}{\partial t} > 0$?
	$\frac{\partial}{\partial y}\left(\overline{v'q'}\right) < 0$		
$\overline{v'q'} = 0$			

Hmmm. So you're saying that Rossby waves might locally ***accelerate the zonal-mean flow***?

Interesting... (recall the "eddy-driven" jet)

We'll come back to this later.

An interactive system between the waves and the mean flow

Equation for how the **waves evolve**:

$$\frac{\partial q'}{\partial t} + \bar{u}\frac{\partial q'}{\partial x} + v'\frac{\partial \bar{q}}{\partial y} = 0, \qquad \text{(WM.1a)}$$

Equation for how the **mean evolves**:

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial}{\partial y}\overline{v'q'} = 0, \qquad \text{(WM.1b)}$$

Equation for how the **waves modify the mean flow**:

$$\left[\frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial}{\partial z}\right)\right]\frac{\partial \bar{u}}{\partial t} = \frac{\partial^2}{\partial y^2}\overline{v'q'}. \qquad \text{(WM.7)}$$

Equation relating **mean flow to mean buoyancy** (thermal wind balance):

$$f_0\frac{\partial \bar{u}}{\partial z} = -\frac{\partial \bar{b}}{\partial y}. \qquad (9.11)$$

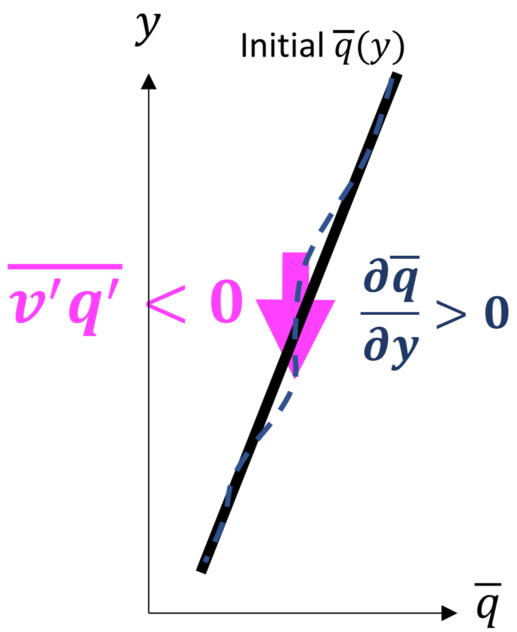
Note: in linearizing the equations, we have **removed** the interactions of eddies with other eddies (**“eddy-eddy interaction”**). This also assumes our eddies are small-amplitude and thus more wave-like than vortex-like.

Hence, we say this system accounts for “wave—mean-flow interaction”.

But what does this meridional PV flux mean? Can we have a little more physical insight?

Eliassen-Palm (EP) Flux
(a.k.a. wave activity flux)

The **eddy meridional PV flux** can be decomposed into two components



This can be written as the divergence of a 2D (y-z) flux vector...

$$\overline{v'q'} = \nabla \cdot \mathcal{F}. \qquad (9.28)$$

... called the **Eliassen-Palm (EP) flux** (a.k.a. **wave activity flux**):

$$\mathcal{F} = -\overline{u'v'}\,\mathbf{j} + \frac{f_0}{N^2}\overline{v'b'}\,\mathbf{k} \qquad (9.27)$$

Meridional flux of zonal momentum

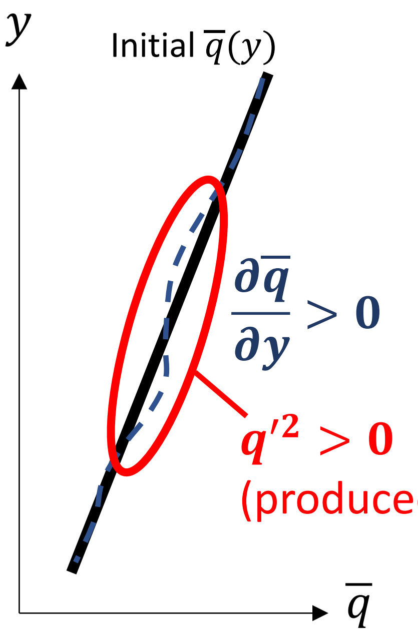
Meridional flux of buoyancy

Eliassen-Palm (EP) flux
(a.k.a. **wave activity flux**):

$$\mathcal{F} = -\overline{u'v'}\,\mathbf{j} + \frac{f_0}{N^2}\overline{v'b'}\,\mathbf{k} \qquad (9.27)$$

Eliassen-Palm (EP) relation:

$$\frac{\partial \mathcal{P}}{\partial t} + \nabla \cdot \mathcal{F} = \mathcal{D}, \qquad (9.29)$$



$$\mathcal{P} = \frac{\overline{q'^2}}{2\partial\overline{q}/\partial y}, \qquad \mathcal{D} = \frac{\overline{D'q'}}{\partial\overline{q}/\partial y}, \qquad (9.30a,b)$$

Pseudomomentum
a.k.a **wave activity** (density) ← *I find this term more intuitive*

Note: the quantity $Z = \frac{1}{2}q^2$ is called the **enstrophy**.

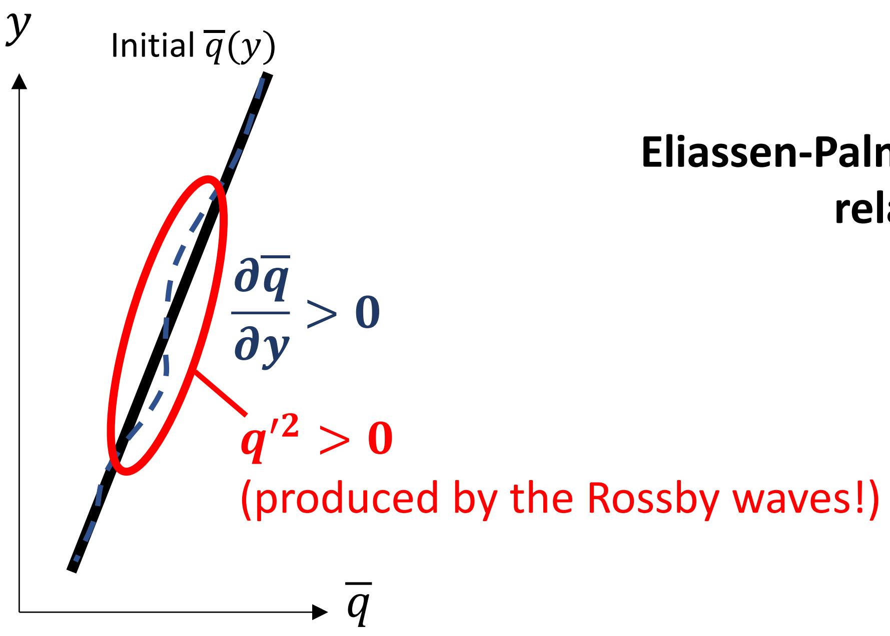
Thus, $\mathcal{P}$ measures the strength of the PV anomaly ($\overline{q'^2}$) relative to the strength of the background PV gradient $\frac{\partial q}{\partial y}$

Hence, $\mathcal{P}$ is a measure of the "strength" of the waves themselves.

Why? If you have a stronger PV gradient, it's easier for a wave to produce a PV anomaly.

Eliassen-Palm (EP) flux
(a.k.a. **wave activity flux**):

$$\mathcal{F} = -\overline{u'v'}\,\mathbf{j} + \frac{f_0}{N^2}\overline{v'b'}\,\mathbf{k} \tag{9.27}$$



Eliassen-Palm (EP) relation:

$$\frac{\partial \mathcal{P}}{\partial t} + \nabla \cdot \mathcal{F} = \mathcal{D}, \tag{9.29}$$

$$\mathcal{P} = \frac{\overline{q'^2}}{2\partial\overline{q}/\partial y}, \qquad \mathcal{D} = \frac{\overline{D'q'}}{\partial\overline{q}/\partial y}, \tag{9.30a,b}$$

This is a governing equation for the **conservation of wave activity**.

In other words, it tells you **how wave activity will propagate within the system** (meridionally or vertically).

For a system with no dissipation (D=0), if you integrate (9.29) over an area that contains all of your waves:

$$\frac{d}{dt}\underbrace{\int_A \mathcal{P}\,dA}_{\text{total wave activity}} = 0. \tag{9.31}$$

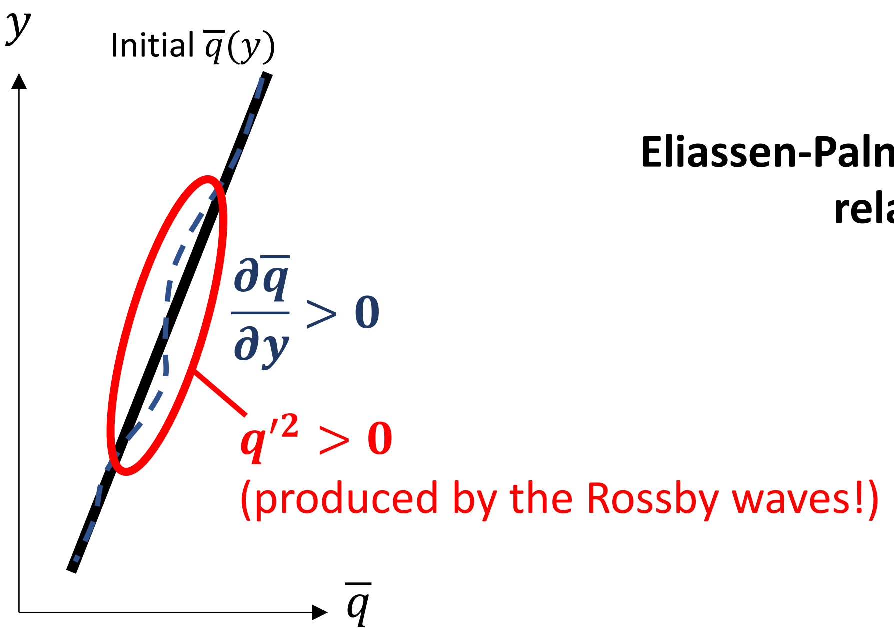
Eliassen-Palm (EP) flux
(a.k.a. **wave activity flux**):

$$\mathcal{F} = -\overline{u'v'}\,\mathbf{j} + \frac{f_0}{N^2}\overline{v'b'}\,\mathbf{k} \tag{9.27}$$

Eliassen-Palm (EP) relation:

$$\frac{\partial \mathcal{P}}{\partial t} + \nabla \cdot \mathcal{F} = \mathcal{D}, \tag{9.29}$$

$$\mathcal{P} = \frac{\overline{q'^2}}{2\partial\bar{q}/\partial y}, \qquad \mathcal{D} = \frac{\overline{D'q'}}{\partial\bar{q}/\partial y}, \tag{9.30a,b}$$



This is a governing equation for the **conservation of wave activity**.

In other words, it tells you **how wave activity will propagate within the system** (meridionally or vertically).

But if this wave activity $\mathcal{P}$ is associated with **Rossby waves**...
and it is a measure of their strength (though it is *not* an energy)...
Then shouldn't wave activity propagate with the Rossby wave group velocity?

Wave activity is moved around a system at the Rossby wave group velocity

(see VallisE p177 for derivation)

Eliassen-Palm (EP) flux
(a.k.a. **wave activity flux**):

$$\mathcal{F} = (\mathcal{F}^y, \mathcal{F}^z) = \mathbf{c}_g \mathcal{P}. \qquad (9.42)$$

Eliassen-Palm (EP) relation:

$$\boxed{\frac{\partial \mathcal{P}}{\partial t} + \nabla \cdot (\mathcal{P}\mathbf{c}_g) = 0,} \qquad (9.43)$$

$$\mathcal{P} = \frac{\overline{q'^2}}{2\partial \overline{q}/\partial y}$$

That feels obvious. So what's the point of this formulation?

The EP relation defines the specific measure of these waves that is conserved in the system.

We will be able to use the EP flux to directly link how wave activity moves around in the system to changes in the mean flow. (next lecture...)

Now go to Blackboard to answer a few questions about this topic!