White: total precipitable water (brighter white = more water vapor in column)
Colors: precipitation rate ($0 - 15 \text{ mm/hr}$, red=highest)
Learning outcomes for today:

• Describe how the Transformed Eulerian Mean differs from a regular Eulerian mean
• Explain the relationship among wave activity, EP fluxes, EP flux divergence/convergence, and acceleration/deceleration of the zonal-mean flow
• Describe how these concepts are relevant to our real atmosphere
**Eulerian mean**: the normal way you take an average, think about eddies (average at a fixed location in time (e.g. annual mean) and/or space (e.g. zonal mean))

\[ \nu = \bar{\nu} + \nu' \]

- Total
- Mean (Eulerian)
- Deviation (eddy)

**Transformed Eulerian mean (TEM)**: a modified framework that lets you **directly** link **eddy effects** to changes in **mean zonal flow**

- Moves eddy PV flux effects out of thermodynamic equation and entirely into momentum equation
Last time: The **eddy meridional PV flux** can be written as the **EP flux divergence**

\[ \overline{v'q'} = \nabla \cdot \mathcal{F} \]  
(9.28)

\[ \mathcal{F} = -\overline{u'v' \hat{j}} + \frac{f_0}{N^2} \overline{v'b' \hat{k}} \]  
(9.27)

**Eliassen-Palm (EP) flux**  
(a.k.a. **wave activity flux**):

- Meridional flux of zonal momentum
- Meridional flux of buoyancy

So the meridional eddy PV fluxes are related to both:

1) meridional fluxes in zonal wind \((u)\)
2) meridional fluxes in buoyancy \((b)\)

**Which of these do eddies actually do? Or is it both?**

Let’s go back to the individual equations for each...
Eddy PV flux is actually an eddy momentum flux + eddy buoyancy flux

Eulerian mean zonal-mean equations, Boussinesq fluid:

Zonal wind:
\[
\frac{\partial \bar{u}}{\partial t} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} - f\bar{v} = -\frac{\partial}{\partial y} \bar{u}' v' - \frac{\partial}{\partial z} \bar{u}' w' + \bar{F} \tag{9.44a}
\]

Buoyancy:
\[
\frac{\partial \bar{b}}{\partial t} + \bar{v} \frac{\partial \bar{b}}{\partial y} + \bar{w} \frac{\partial \bar{b}}{\partial z} = -\frac{\partial}{\partial y} \bar{v}' b' - \frac{\partial}{\partial z} \bar{w}' b' + \bar{S} \tag{9.44b}
\]

Quasi-geostrophic:

- neglect vertical eddy flux convergences (terms with \( w' \))
- neglect ageostrophic velocities (\( \bar{v} \) or \( \bar{w} \)) except when multiplied by \( f_0 \) or \( N^2 \)

Note: \( \frac{\partial \bar{b}}{\partial z} = N^2 \)
Eddy PV flux is actually an eddy momentum flux + eddy buoyancy flux

Zonal wind:
\[
\frac{\partial \bar{u}}{\partial t} = f_0 \bar{u} - \frac{\partial}{\partial y} \bar{u}'v' + \bar{F} \tag{9.45a}
\]

Buoyancy:
\[
\frac{\partial \bar{b}}{\partial t} = -N^2 \bar{u} - \frac{\partial}{\partial y} \bar{v}'b' + \bar{S} \tag{9.45b}
\]

Thermal wind balance relates \(\bar{u}\) and \(\bar{b}\)
\[
f_0 \frac{\partial \bar{u}}{\partial z} = -\frac{\partial \bar{b}}{\partial y}
\]

Recall: TWB combines
- geostrophic balance (meridional): \(f_0 \bar{u} = -\frac{\partial \bar{\phi}}{\partial y}\)
- hydrostatic balance (vertical): \(\bar{b} = \frac{\partial \bar{\phi}}{\partial z}\)

Now we can see the individual eddy fluxes. However, because of thermal wind balance...
- eddy momentum fluxes can change \(\bar{u}\), which via TWB changes \(\bar{b}\)
- eddy buoyancy fluxes can change \(\bar{b}\), which via TWB changes \(\bar{u}\)

So we still don’t actually know how the eddy fluxes change \(\bar{u}\) and \(\bar{b}\)!
Eddy PV flux is actually an eddy momentum flux + eddy buoyancy flux

Zonal wind:
\[
\frac{\partial \overline{u}}{\partial t} = f_0 \overline{v} - \frac{\partial}{\partial y} (\overline{u'} v') + \overline{F} \tag{9.45a}
\]

Buoyancy:
\[
\frac{\partial \overline{b}}{\partial t} = -N^2 \overline{\bar{w}} - \frac{\partial}{\partial y} (\overline{v' b'}) - \overline{\bar{S}} \tag{9.45b}
\]

Adiabatic cooling / subsidence warming

Diabatic heating

Thermal wind balance relates $\overline{u}$ and $\overline{b}$
\[
f_0 \frac{\partial \overline{u}}{\partial z} = -\frac{\partial \overline{b}}{\partial y}
\]

Recall: TWB combines
- Geostrophic balance (meridional): $f_0 \overline{u} = -\frac{\partial \overline{\Phi}}{\partial y}$
- Hydrostatic balance (vertical): $\overline{b} = \frac{\partial \overline{\Phi}}{\partial z}$

In the extratropics: these are the dominant balance and these are typically small residuals

$\overline{\bar{S}}$: small but important -- this drives the mean meridional overturning
(note: in the tropics it creates the temperature gradients that drive the Hadley cell)

Can we transform these equations into a more useful form, that accounts for this residual term $\overline{\bar{S}}$?
Let’s absorb the dominant thermodynamic balance into our equations

Zonal wind:

\[ \frac{\partial \bar{u}}{\partial t} = f_0 \bar{v} - \frac{\partial}{\partial y} \bar{u}' \bar{v}' + \bar{F} \]  

(9.45a)

Buoyancy:

\[ \frac{\partial \bar{b}}{\partial t} = -N^2 \bar{w} - \frac{\partial}{\partial y} \bar{v}' \bar{b}' + \dot{S} \]  

(9.45b)

Mass continuity:
relates \( \bar{v} \) and \( \bar{w} \)

\[ (\bar{v}, \bar{w}) = \left(- \frac{\partial \psi_m}{\partial z}, \frac{\partial \psi_m}{\partial y} \right) \]  

(9.46)

Define a residual mean meridional circulation \((\bar{v}^*, \bar{w}^*)\):

\[ \begin{align*}
\bar{v}^* &= \bar{v} - \frac{\partial}{\partial z} \left( \frac{1}{N^2} \bar{v}' \bar{b}' \right), \\
\bar{w}^* &= \bar{w} + \frac{\partial}{\partial y} \left( \frac{1}{N^2} \bar{v}' \bar{b}' \right)
\end{align*} \]  

(9.48)

Define residual flow:

\[ \bar{v}_{bal} = -\frac{\partial}{\partial z} \left( \frac{1}{N^2} \bar{v}' \bar{b}' \right) \]

\[ \bar{v}^* = \bar{v} - \bar{v}_{bal} \]

with residual streamfunction \( \psi^* \):

\[ \begin{align*}
(\bar{v}^*, \bar{w}^*) &= \left(- \frac{\partial \psi^*}{\partial z}, \frac{\partial \psi^*}{\partial y} \right) \\
\psi^* &\equiv \psi_m + \frac{1}{N^2} \bar{v}' \bar{b}'
\end{align*} \]  

(9.47a) (residual) meridional streamfunction

\[ \psi_m: \text{Eulerian mean meridional streamfunction} \]

\[ \psi^*: \text{Transformed Eulerian mean meridional streamfunction} \]
Let’s absorb the dominant thermodynamic balance into our equations

**Eulerian mean**

Zonal wind:
\[
\frac{\partial \bar{u}}{\partial t} = f_0 \bar{v} - \frac{\partial}{\partial y} u'v' + \bar{F}
\]

Buoyancy:
\[
\frac{\partial \bar{b}}{\partial t} = -N^2 \bar{w} - \frac{\partial}{\partial y} v'b' + \bar{S}
\]

**Transformed Eulerian mean**

Zonal wind:
\[
\frac{\partial \bar{u}}{\partial t} = f_0 \bar{v}^* + \bar{v}'q' + \bar{F}
\]

Buoyancy:
\[
\frac{\partial \bar{b}}{\partial t} = -N^2 \bar{w}^* + \bar{S}
\]

Residual flow:
\[
\bar{v}^* = \bar{v} - \frac{\partial}{\partial z} \left( \frac{1}{N^2} \bar{v}'b' \right), \quad \bar{w}^* = \bar{w} + \frac{\partial}{\partial y} \left( \frac{1}{N^2} \bar{v}'b' \right)
\]

Recall the eddy PV flux:
\[
\bar{v}'q' = -\frac{\partial}{\partial y} u'v' + \frac{\partial}{\partial z} \left( \frac{f_0}{N^2} v'b' \right)
\]

We’ve effectively moved the eddy buoyancy flux divergence term ($u'b'$) into the momentum equation and combined it with the zonal momentum flux divergence term ($u'v'$).

The result? A single eddy forcing term: the eddy PV flux, $\bar{v}'q'$, that directly changes the zonal wind ($\frac{\partial \bar{u}}{\partial t}$)!
Recall that the wave activity, $\mathcal{P}$, is also called “pseudomomentum”. This is why: $\mathcal{P}$ acts dynamically like a zonal-mean zonal momentum per unit mass (and it has units of m/s).
Case 1: No sources/sinks of wave activity ($\mathcal{D} = 0$), steady motionless waves ($\nabla \cdot \mathbf{F} = 0$)

\[
\frac{\partial P}{\partial t} + \nabla \cdot \mathbf{F} = \mathcal{D} \quad (9.29)
\]

\[
\frac{\partial}{\partial t}(\bar{u} + \mathcal{P}) - f_0 \bar{v}^* = \mathcal{D} \quad (9.74)
\]

For this case, there will be no residual circulation either ($\bar{v}^* = 0$)

Result:

\[
\frac{\partial P}{\partial t} = 0 \quad \text{wave activity doesn’t change}
\]

\[
\frac{\partial \bar{u}}{\partial t} = 0 \quad \text{so the mean flow doesn’t change, either}
\]

This is called the “non-acceleration” result. The mean flow is not accelerated at all, regardless of where wave activity is found.

Ok, so waves alone aren’t enough to change the mean flow. What if waves could move around?
Case 2: No sources/sinks of wave activity ($\mathcal{D} = 0$), barotropic fluid ($\bar{v} = 0$)

\[ \frac{\partial P}{\partial t} + \nabla \cdot F = \mathcal{D} \]  
(9.29)

\[ \frac{\partial}{\partial t} (\bar{u} + \mathcal{P}) - f_0 \sigma^* = \mathcal{D} \]  
(9.74)

Result:
- Wave activity simply moves around (increases where EP flux converges)
- If wave activity goes up, zonal-mean flow goes down, and vice versa.
   (an exchange between zonal-mean momentum and pseudomomentum)

But this is not realistic:
something has to generate (and dissipate) this wave activity in the first place...

Why does this matter?
It means that sources/sinks of wave activity are fundamental to changing the (barotropic) mean flow.
Case 3: steady wave activity, barotropic fluid ($\bar{v} = 0$)

\[ \frac{\partial P}{\partial t} + \nabla \cdot F = D \]  
(9.29)

\[ \frac{\partial (\bar{u} + P)}{\partial t} - f_0 \sigma = D \]  
(9.74)

Result:

\[ \nabla \cdot F = D \]

wave activity diverges away from where it is generated (source), and converges towards where it is dissipated (sink).

\[ \frac{\partial \bar{u}}{\partial t} = \nabla \cdot F = D \]

The zonal flow will be accelerated where waves are generated, and decelerated where waves are dissipated!

*Irreversibility* is essential to how waves affect the mean-flow.

Ok, so how does this manifest itself in our actual atmosphere?
Wave activity (i.e. waves whose amplitude is growing) is generated primarily by **baroclinic instability** of the jet stream at mid-latitudes. This can be thought of as an external process that “stirs” the fluid, creating waves!

Fig. 12.2:

- Wave activity is illustrated in the diagram with Rossby waves breaking and dissipating. 
- The diagram shows stirring, momentum divergence, and convergence.

In this way, then waves are converging zonal momentum toward their source region. This momentum is taken from lower/higher latitudes where the waves break and dissipate.

**In essence, then, baroclinic instability of the mid-latitude (barotropic) jet helps to sustain the jet’s very existence.** This instability tries to break down the zonal-mean jet into eddies and restore its zonal-mean state simultaneously.
Can we apply these concepts to understand the large-scale circulation of our atmosphere?

Where do real eddies transport momentum vs. heat (buoyancy)?

How does the global momentum budget fit into this?

Where does the Ferrel cell come from?

We’ll cover these topics in the next lecture.
A couple of notes:

• A nice summary of the TEM equations is provided in the box VallisE p. 186

• Why would a meridional buoyancy flux $(\nu' b')$ act like a force $(\frac{\partial \bar{u}}{\partial t})$? This seems weird/magical. (Recall: this shows up as the vertical component of the EP flux) See VallisE Section 9.4.3 – the buoyancy flux acts like a form drag created by sloping interfaces that places a stress on the fluid (i.e. a sink of momentum). Transporting heat (buoyancy) acts to change the slope of pressure surfaces and thus changes this form drag.
Now go to Blackboard to answer a few questions about this topic!