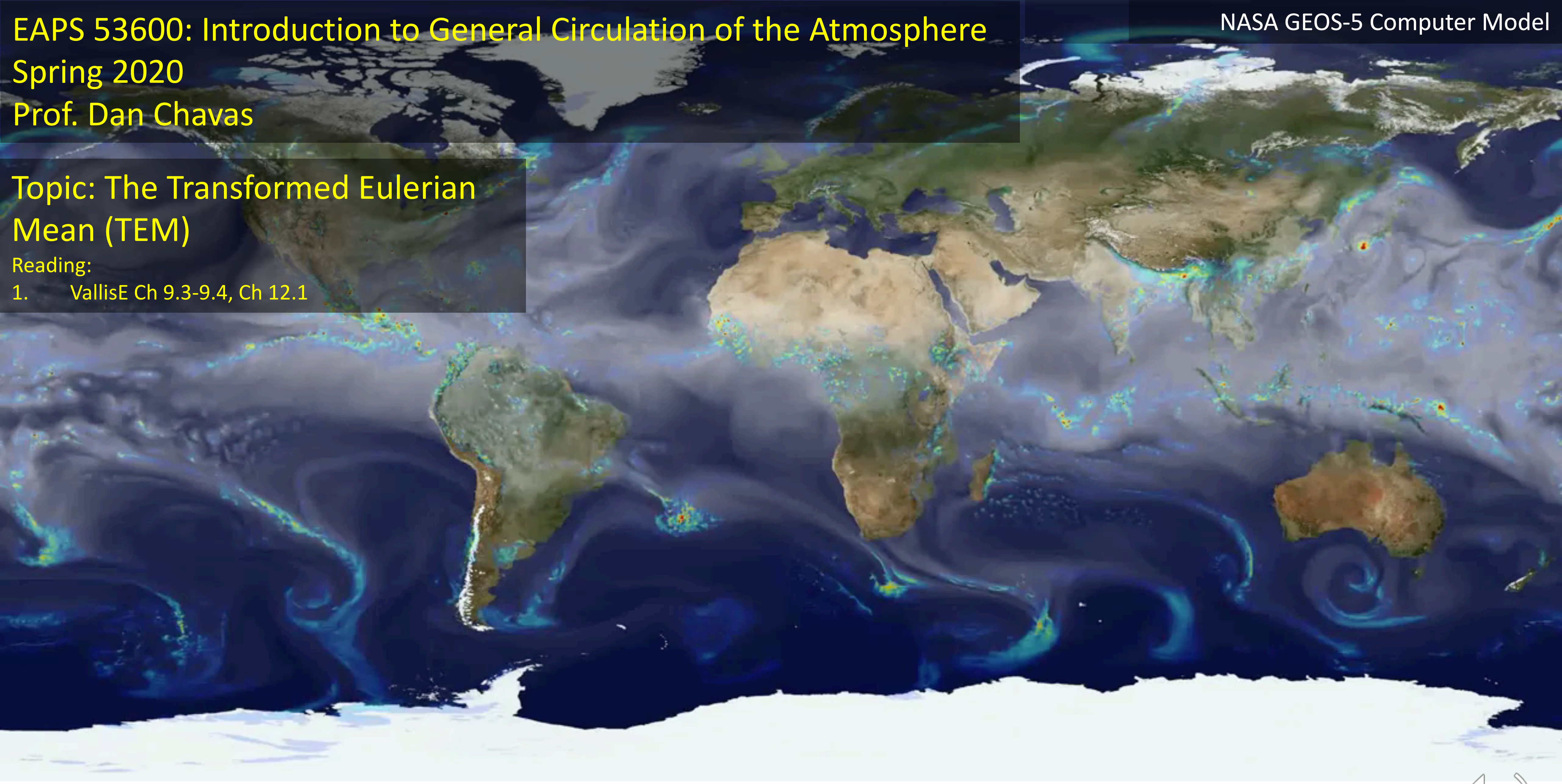
# EAPS 53600: Introduction to General Circulation of the Atmosphere
Spring 2020
Prof. Dan Chavas

## Topic: The Transformed Eulerian Mean (TEM)

Reading:
1. ValliisE Ch 9.3-9.4, Ch 12.1




White: total precipitable water (brigher white = more water vapor in column)
Colors: precipitation rate ($0-15\frac{mm}{hr}$, red=highest)

Source: https://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=30017

**Learning outcomes for today:**

- **Describe how the Transformed Eulerian Mean differs from a regular Eulerian mean**
- **Explain the relationship among wave activity, EP fluxes, EP flux divergence/convergence, and acceleration/deceleration of the zonal-mean flow**
- **Describe how these concepts are relevant to our real atmosphere**

**Eulerian mean**: the normal way you take an average, think about eddies

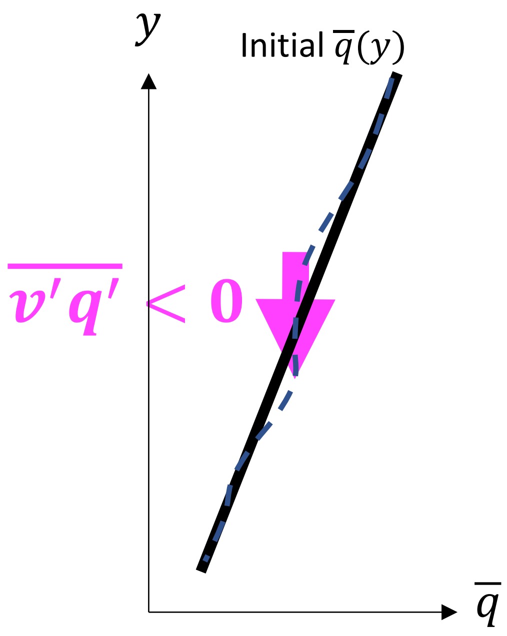
(average at a fixed location in time (e.g. annual mean) and/or space (e.g. zonal mean))

$$\underset{\text{Total}}{v} = \underset{\substack{\text{Mean}\\\text{(Eulerian)}}}{\overline{v}} + \underset{\substack{\text{Deviation}\\\text{(eddy)}}}{v'}$$

***Transformed* Eulerian mean (TEM)**: a modified framework that lets you **directly** link **eddy effects** to changes in **mean zonal flow**

- Moves eddy PV flux effects out of thermodynamic equation and entirely into momentum equation

# Last time: The **eddy meridional PV flux** can be written as the **EP flux divergence**




**Eddy meridional PV flux:** $\overline{v'q'} = \nabla \cdot \boldsymbol{\mathcal{F}}$ *(9.28)*

**Eliassen-Palm (EP) flux** (a.k.a. **wave activity flux**): $\boldsymbol{\mathcal{F}} = -\overline{u'v'}\hat{\boldsymbol{j}} + \frac{f_0}{N^2}\overline{v'b'}\hat{\boldsymbol{k}}$ *(9.27)*

Meridional flux of zonal momentum

Meridional flux of buoyancy

So the meridional eddy PV fluxes are related to both:

1) meridional fluxes in zonal wind ($u$)
2) meridional fluxes in buoyancy ($b$)

**Which of these do eddies actually *do*? Or is it both?**

Let's go back to the individual equations for each...

# Eddy PV flux is actually an eddy momentum flux + eddy buoyancy flux

Eulerian mean zonal-mean equations, Boussinesq fluid:

Zonal wind: $$\frac{\partial \bar{u}}{\partial t} + \cancel{\bar{v}\frac{\partial \bar{u}}{\partial y}} + \cancel{\bar{w}\frac{\partial \bar{u}}{\partial z}} - f\bar{v} = -\frac{\partial}{\partial y}\overline{u'v'} - \cancel{\frac{\partial}{\partial z}\overline{u'w'}} + \bar{F} \qquad (9.44a)$$

Friction

Buoyancy: $$\frac{\partial \bar{b}}{\partial t} + \cancel{\bar{v}\frac{\partial \bar{b}}{\partial y}} + \bar{w}\frac{\partial \bar{b}}{\partial z} = -\frac{\partial}{\partial y}\overline{v'b'} - \cancel{\frac{\partial}{\partial z}\overline{w'b'}} + \bar{S} \qquad (9.44b)$$

Heating

Purely ageostrophic (meridional overturning)

e.g. the Hadley cell is poleward near tropopause, equatorward near surface

Quasi-geostrophic:

neglect vertical eddy flux convergences (terms with w')

neglect ageostrophic velocities ($\overline{v}$ or $\overline{w}$) except when multiplied by $f_0$ or $N^2$

Note: $\frac{\partial \overline{b}}{\partial z} = N^2$

# Eddy PV flux is actually an eddy momentum flux + eddy buoyancy flux

Zonal wind:
$$\frac{\partial \bar{u}}{\partial t} = f_0 \bar{v} - \frac{\partial}{\partial y}\overline{u'v'} + \bar{F} \quad (9.45a)$$

Buoyancy:
$$\frac{\partial \bar{b}}{\partial t} = -N^2 \bar{w} - \frac{\partial}{\partial y}\overline{v'b'} + \bar{S} \quad (9.45b)$$

Thermal wind balance relates $\overline{u}$ and $\overline{b}$

$$f_0 \frac{\partial \overline{u}}{\partial z} = -\frac{\partial \overline{b}}{\partial y}$$

Recall: TWB combines

geostrophic balance (meridional): $f_0 \overline{u} = -\frac{\partial \overline{\phi}}{\partial y}$

hydrostatic balance (vertical): $\overline{b} = \frac{\partial \overline{\phi}}{\partial z}$

**Now we can see the individual eddy fluxes**. However, because of thermal wind balance...

- eddy momentum fluxes can change $\overline{u}$, which via TWB changes $\overline{b}$
- eddy buoyancy fluxes can change $\overline{b}$, which via TWB changes $\overline{u}$

**So we still don't actually know how the eddy fluxes change $\overline{u}$ and $\overline{b}$!**

# Eddy PV flux is actually an eddy momentum flux + eddy buoyancy flux

Zonal wind:

$$\frac{\partial \bar{u}}{\partial t} = \boxed{f_0 \bar{v} - \frac{\partial}{\partial y}\overline{u'v'}} + \bar{F} \qquad (9.45a)$$

Buoyancy:

$$\frac{\partial \bar{b}}{\partial t} = \boxed{-N^2 \bar{w} - \frac{\partial}{\partial y}\overline{v'b'}} + \bar{S} \qquad (9.45b)$$

Adiabatic cooling / subsidence warming

Diabatic heating

Thermal wind balance relates $\overline{u}$ and $\overline{b}$

$$f_0 \frac{\partial \overline{u}}{\partial z} = -\frac{\partial \overline{b}}{\partial y}$$

Recall: TWB combines

Geostrophic balance (meridional): $f_0 \overline{u} = -\frac{\partial \overline{\phi}}{\partial y}$

Hydrostatic balance (vertical): $\overline{b} = \frac{\partial \overline{\phi}}{\partial z}$

**In the extratropics:** these are the dominant balance and these are typically **small residuals**

**$\overline{S}$: small but important -- this drives the mean meridional overturning**

(note: in the tropics it creates the temperature gradients that drive the Hadley cell)

Can we **transform these equations** into a more useful form, that **accounts for this residual term $\overline{S}$**?

# Let's absorb the dominant thermodynamic balance into our equations

Zonal wind:

$$\frac{\partial \bar{u}}{\partial t} = f_0 \bar{v} - \frac{\partial}{\partial y}\overline{u'v'} + \bar{F} \quad (9.45a)$$

Buoyancy:

$$\frac{\partial \bar{b}}{\partial t} = \boxed{-N^2 \bar{w} - \frac{\partial}{\partial y}\overline{v'b'}} + \bar{S} \quad (9.45b)$$

Dominant balance:

$$\overline{w}_{bal} = \frac{\partial}{\partial y}\left(\frac{1}{N^2}\overline{v'b'}\right)$$

Define residual flow:

$$\overline{w}^* = \overline{w} - \overline{w}_{bal}$$

Mass continuity: relates $\overline{v}$ and $\overline{w}$

$$(\bar{v}, \bar{w}) = \left(-\frac{\partial \psi_m}{\partial z}, \frac{\partial \psi_m}{\partial y}\right) \quad (9.46)$$

$\psi_m$: Eulerian mean meridional streamfunction

where $\frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0$

Plug in $\overline{w}_{bal}$ to get $\overline{v}_{bal}$

$$\frac{\partial \overline{v}_{bal}}{\partial y} = -\frac{\partial}{\partial z}\frac{\partial}{\partial y}\left(\frac{1}{N^2}\overline{v'b'}\right) = \frac{\partial}{\partial y}\left(-\frac{\partial}{\partial z}\left(\frac{1}{N^2}\overline{v'b'}\right)\right)$$

$$\overline{v}_{bal} = -\frac{\partial}{\partial z}\left(\frac{1}{N^2}\overline{v'b'}\right)$$

Define residual flow:

$$\overline{v}^* = \overline{v} - \overline{v}_{bal}$$

Define a **residual mean meridional circulation ($\overline{v}^*, \overline{w}^*$)**:

$$\bar{v}^* = \bar{v} - \frac{\partial}{\partial z}\left(\frac{1}{N^2}\overline{v'b'}\right), \quad \bar{w}^* = \bar{w} + \frac{\partial}{\partial y}\left(\frac{1}{N^2}\overline{v'b'}\right) \quad (9.48)$$

where $\frac{\partial \overline{v}^*}{\partial y} + \frac{\partial \overline{w}^*}{\partial z} = 0$

(Mass continuity has to apply to the residual, too!)

with **residual streamfunction $\psi^*$**:

$$(\bar{v}^*, \bar{w}^*) = \left(-\frac{\partial \psi^*}{\partial z}, \frac{\partial \psi^*}{\partial y}\right) \quad (9.47b)$$

$$\psi^* \equiv \psi_m + \frac{1}{N^2}\overline{v'b'} \quad (9.47a)$$

$\psi^*$: Transformed Euleria[illegible]ean (residual) meridional streamfunction

# Let's absorb the dominant thermodynamic balance into our equations

## Eulerian mean

Zonal wind: $$\frac{\partial \bar{u}}{\partial t} = f_0\bar{v} - \frac{\partial}{\partial y}\overline{u'v'} + \bar{F}$$

Buoyancy: $$\frac{\partial \bar{b}}{\partial t} = -N^2\bar{w} - \frac{\partial}{\partial y}\overline{v'b'} + \bar{S}$$

Recall the eddy PV flux:

(9.45a) $$\overline{v'q'} = -\frac{\partial}{\partial y}\overline{u'v'} + \frac{\partial}{\partial z}\left(\frac{f_0}{N^2}\overline{v'b'}\right)$$

(9.45b) $$\boldsymbol{f_0\bar{v} - \frac{\partial}{\partial y}\overline{u'v'}}$$
$$= f_0\left(\bar{v}^* - \frac{\partial}{\partial z}\left(\frac{1}{N^2}\overline{v'b'}\right)\right) - \left(\overline{v'q'} - \frac{\partial}{\partial z}\left(\frac{f_0}{N^2}\overline{v'b'}\right)\right)$$
$$\boldsymbol{= f_0\bar{v}^* - \overline{v'q'}}$$

## Transformed Eulerian mean

Zonal wind: $$\frac{\partial \bar{u}}{\partial t} = f_0\bar{v}^* + \overline{v'q'} + \bar{F}$$

Buoyancy: $$\frac{\partial \bar{b}}{\partial t} = -N^2\bar{w}^* + \bar{S}$$

(9.50a,b)

Residual flow: $$\bar{v}^* = \bar{v} - \frac{\partial}{\partial z}\left(\frac{1}{N^2}\overline{v'b'}\right), \quad \bar{w}^* = \bar{w} + \frac{\partial}{\partial y}\left(\frac{1}{N^2}\overline{v'b'}\right)$$ (9.48)

We've effectively moved the eddy buoyancy flux divergence term ($\overline{v'b'}$) into the momentum equation and combined it with the zonal momentum flux divergence term ($\overline{u'v'}$).

The result? **A single eddy forcing term: the eddy PV flux, $\overline{v'q'}$**, that directly changes the zonal wind ($\frac{\partial \bar{u}}{\partial t}$)!

# Now let's link back to wave activity and the EP flux

$$\frac{\partial \bar{u}}{\partial t} = f_0 \bar{v}^* + \overline{v'q'} + \cancel{\bar{F}} \qquad (9.50a,b)$$

No external forcing

$$\frac{\partial \bar{u}}{\partial t} - f_0 \bar{v}^* = \nabla \cdot \mathcal{F} \qquad (9.73)$$

$$\frac{\partial P}{\partial t} + \nabla \cdot \boldsymbol{F} = \mathcal{D} \qquad (9.29)$$

Source/sink of wave activity

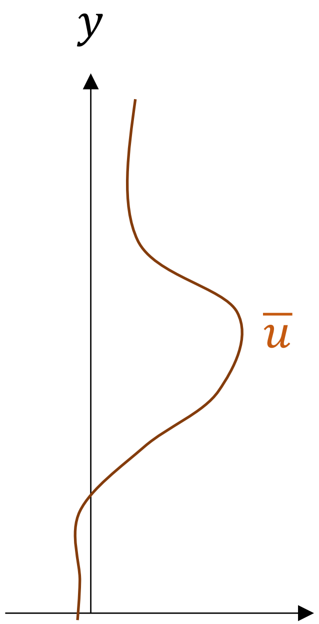
$$\frac{\partial}{\partial t}(\bar{u} + \mathcal{P}) - f_0 \bar{v}^* = \mathcal{D} \qquad (9.74)$$

Recall that the wave activity, $\mathcal{P}$, is also called "**pseudomomentum**".
This is why: **$\mathcal{P}$ acts dynamically like a zonal-mean zonal momentum per unit mass** (and it has units of m/s)

**We can use these equations to understand a lot about how the mean flow is affected by eddies. Let's think about some examples...**

## Case 1: No sources/sinks of wave activity ($\mathcal{D} = 0$), steady motionless waves ($\nabla \cdot \mathcal{F} = 0$)

$$\frac{\partial P}{\partial t} + \cancel{\nabla \cdot \boldsymbol{F}} = \cancel{\mathcal{D}} \qquad (9.29)$$




$$\frac{\partial}{\partial t}(\bar{u} + \mathcal{P}) - \cancel{f_0 \bar{v}^*} = \cancel{\mathcal{D}} \qquad (9.74)$$

For this case, there will be no residual circulation either ($\overline{v}^* = 0$)

Result:

$$\frac{\partial P}{\partial t} = 0 \longleftarrow$$ wave activity doesn't change

$$\frac{\partial \overline{u}}{\partial t} = 0 \longleftarrow$$ so the mean flow doesn't change, either

**This is called the "non-acceleration" result.**

The mean flow is not accelerated at all, regardless of where wave activity is found.

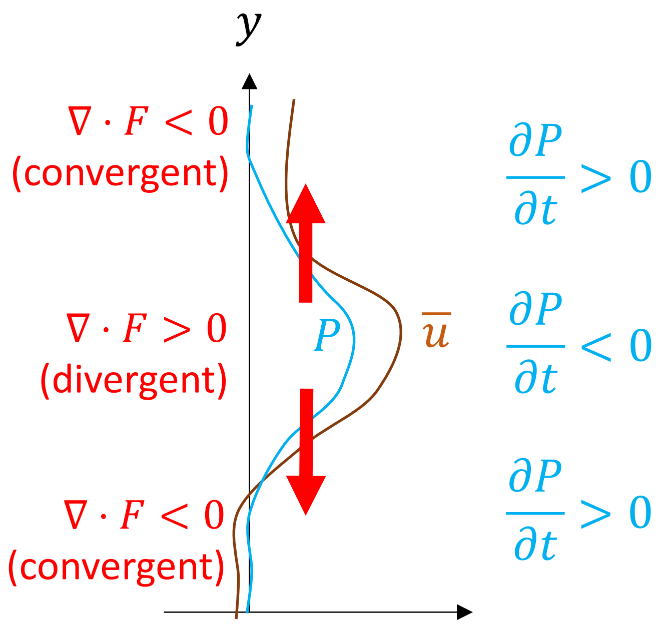
Ok, so waves alone aren't enough to change the mean flow. What if waves could move around?

# Case 2: No sources/sinks of wave activity ($\mathcal{D} = 0$), barotropic fluid ($\overline{v} = 0$)

$$\frac{\partial P}{\partial t} + \nabla \cdot \boldsymbol{F} = \cancel{\mathcal{D}} \qquad (9.29)$$

$$\frac{\partial}{\partial t}(\bar{u} + \mathcal{P}) - \cancel{f_0 \bar{v}^*} = \cancel{\mathcal{D}} \qquad (9.74)$$

Cannot have zonal-mean meridional flow if vertically uniform (barotropic)




Result:

$$\frac{\partial P}{\partial t} = -\nabla \cdot F$$ ← wave activity simply moves around (increases where EP flux converges)

$$\frac{\partial(\overline{u} + \mathcal{P})}{\partial t} = 0$$ ← If wave activity goes up, zonal-mean flow goes down, and vice versa.
(an exchange between zonal-mean momentum and pseudomomentum)

But this is not realistic:

**something has to generate (and dissipate) this wave activity in the first place...**

**Why does this matter?**

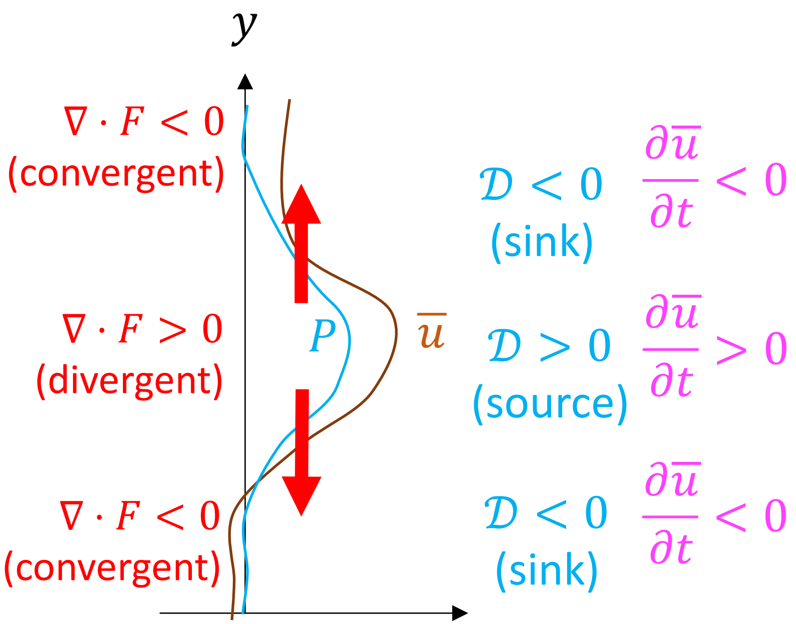
It means that **sources/sinks of wave activity** are **fundamental to changing the (barotropic) mean flow.**

# Case 3: steady wave activity, barotropic fluid ($\overline{v} = 0$)

$$\cancel{\frac{\partial P}{\partial t}} + \nabla \cdot \boldsymbol{F} = \mathcal{D} \qquad (9.29)$$

$$\frac{\partial}{\partial t}(\bar{u} + \cancel{\mathcal{P}}) - \cancel{f_0 \bar{v}^*} = \mathcal{D} \qquad (9.74)$$

Cannot have zonal-mean meridional flow if vertically uniform (barotropic)




Result:

$$\nabla \cdot F = \mathcal{D}$$

wave activity diverges away from where it is generated (source), and converges towards where it is dissipated (sink).

$$\frac{\partial \overline{u}}{\partial t} = \nabla \cdot F = \mathcal{D}$$

**The zonal flow will be accelerated where waves are generated, and decelerated where waves are dissipated!**

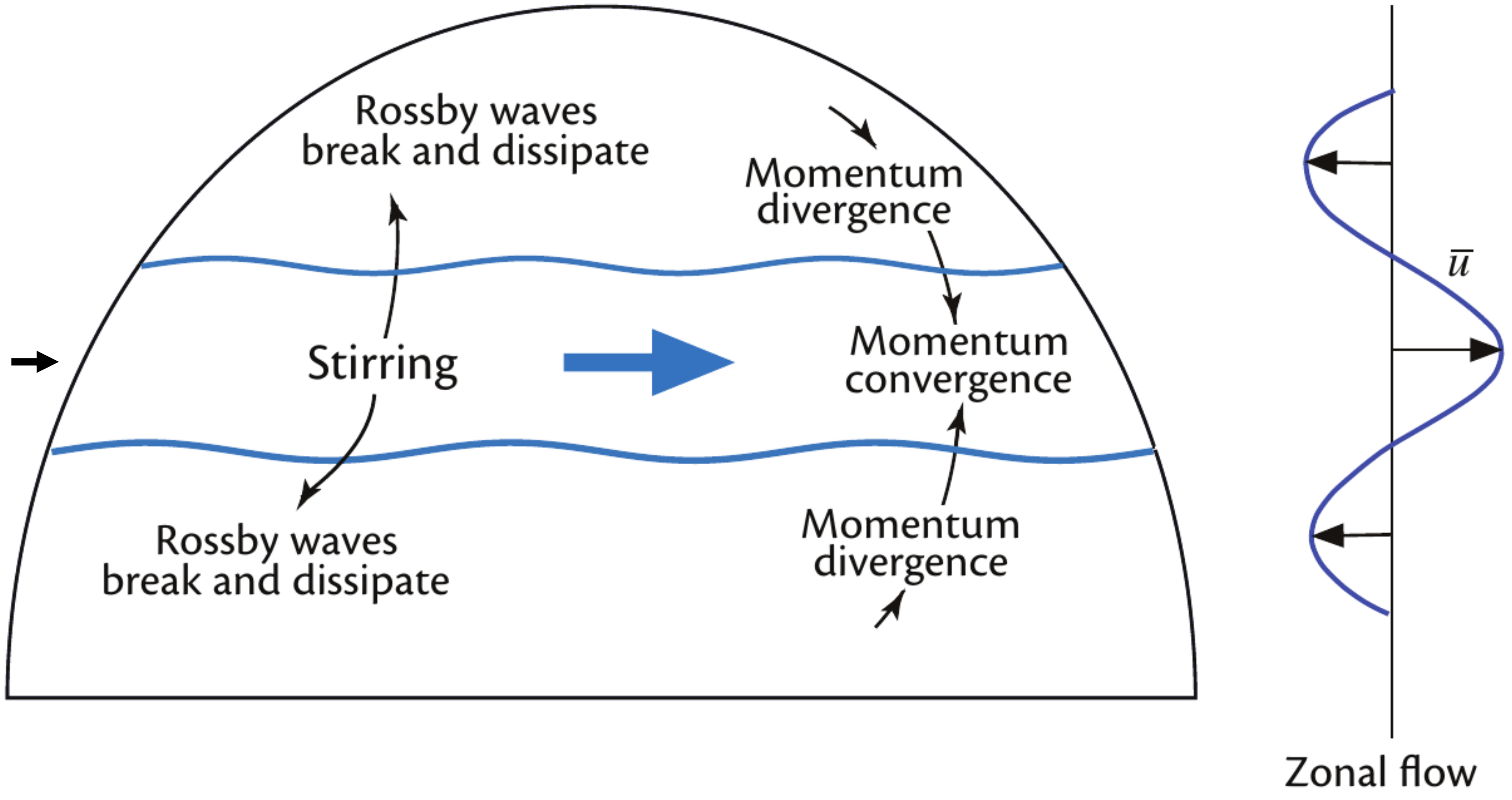
*Irreversibility* is essential to how waves affect the mean-flow.

**Ok, so how does this manifest itself in our actual atmosphere?**

Fig. 12.2:

Wave activity (i.e. waves whose amplitude is growing) is generated primarily by **baroclinic instability** of the jet stream at mid-latitudes.

This can be thought of as an external process that "stirs" the fluid, creating waves!




In this way, then waves are converging zonal momentum toward their source region.
This momentum is taken from lower/higher latitudes where the waves break and dissipate.

**In essence, then, baroclinic instability of the mid-latitude (barotropic) jet helps to sustain the jet's very existence.**
This instability tries to break down the zonal-mean jet into eddies *and* restore its zonal-mean state *simultaneousl*

**Can we apply these concepts to understand the large-scale circulation of our atmosphere?**

**Where do real eddies transport momentum vs. heat (buoyancy)?**

**How does the global momentum budget fit into this?**

**Where does the Ferrel cell come from?**

We’ll cover these topics in the next lecture.

**A couple of notes:**

- **A nice summary of the TEM equations is provided in the box VallisE p. 186**
- **Why would a meridional *buoyancy* flux ($\overline{v'b'}$) act like a force ($\frac{\partial \overline{u}}{\partial t}$)? This seems weird/magical.** (Recall: this shows up as the vertical component of the EP flux) See ValliSE Section 9.4.3 – the buoyancy flux acts like a form drag created by sloping interfaces that places a stress on the fluid (i.e. a sink of momentum). Transporting heat (buoyancy) acts to change the slope of pressure surfaces and thus changes this form drag.

Now go to Blackboard to answer a few questions about this topic!