Decentralized Control Framework for Networked Control Systems

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Abstract— **The combination of decentralized and networked control where control loops are closed through a network is defined as Decentralized Networked Control System (DNCS). In this paper, we introduce a general framework that converts a generic decentralized control configuration of non-networked systems to the general setup of Networked Control Systems (NCS). An observer-based decentralized control method is used to illustrate the applicability of the proposed framework. First, a method for designing decentralized observer-based controllers for non-networked systems is discussed. Second, we introduce the general framework that maps the non-networked decentralized control scheme to the networked one. Third, we provide two methods to analyze the closed-loop system stability, given the proposed model. Perturbation bounds of the DNCS are derived. Finally, example and simulation results are shown and discussed.**

I. INTRODUCTION

The recent research efforts in the area of networked control systems have paved the way to better understand and interact with large-scale decentralized modern control systems. To mention a few, large-scale Networked Control Systems (NCS) can be found in many diverse applications, such as: transportation networks, smart-grids, digital communication systems, and robotics. Since communication networks are an essential component of these systems, the analysis of a networked version of decentralized control systems is becoming crucial. The objective of this paper is to introduce a general framework that converts a generic decentralized control configuration of non-networked systems to the general setup of an NCS.

A. Decentralized Control

The decentralized control methodology, in many cases, is intended to replace the complex, expensive, and impractical applications of centralized control. A main field of decentralized control is the large-scale interconnected systems. Transportation systems, communication networks, power systems, economic systems, manufacturing processes and many others, are examples where decentralized control is used. The main idea behind designing decentralized controllers is the use of local information to achieve global results.

In this paper we are considering the observer-based decentralized control design for large-scale interconnected systems where the feedback loops are closed through a network. The robust design of the decentralized control strategies has been introduced in [3], [4], [5]. In [6], the authors proposed an observer-based control algorithm for linear systems where the design uses low-order linear functional observers. The individual subsystem states are estimated in [7], [8] by using an observer where the separation principal needs information exchange between subsystems in order to be utilized. Observer-based control design for non-linear systems is introduced in [9], [10], [11], [12]. The key feature of the design proposed in [9] is that the separation principle of the linear systems case holds in their design for the non-linear system.

B. Networked Control Systems

The digital and computation progress spur the development of distributed control systems. These modern systems which include sensors and actuators that are controlled via a centralized or decentralized controllers, are connected by using a shared communication medium. This type of real-time networks are called networked control systems (NCS) [13]. Figure 1 shows an example of these distributed systems and illustrates the architecture of NCS.

NCS applications can be found in passenger cars, trucks and buses, aircraft and aerospace electronics, factory automation, industrial machine control, medical equipment, mobile sensor networks and many more [16]. However the NCS can potentially increase system reliability, reduce weight, space, power and wiring requirements, there are constraints that limit the applications. Examples of these limits are multiple-packet transmission, data packet dropouts and finite bandwidth that is, only one node can access the shared medium at a time. Conventional control theories having ideal assumptions, such as synchronization of the control or non-delayed sensing and actuation, have to be reevaluated to take the network effects in account before they are applied to NCS. Basically, the primary objective of NCS analysis and design is to efficiently use the finite bus capacity while maintaining good closed-loop control system performance [15].

C. Decentralized Networked Control Systems

It is noteworthy to mention that NCSs and decentralized control applications do often overlap, which adds to the significance of studying and analyzing Decentralized Networked Control Systems (DNCS). Basically, decentralized control is used when there is a large scale system (LSS) whose sub-systems have interconnections with existing constraints on data transfer between them. The problem of decentralized control can be viewed as designing local controllers for subsystems comprising a given system. Decentralized control is especially viable for systems whose subsystems are separated geographically. Unlike centralized control, the decentralized control can be robust

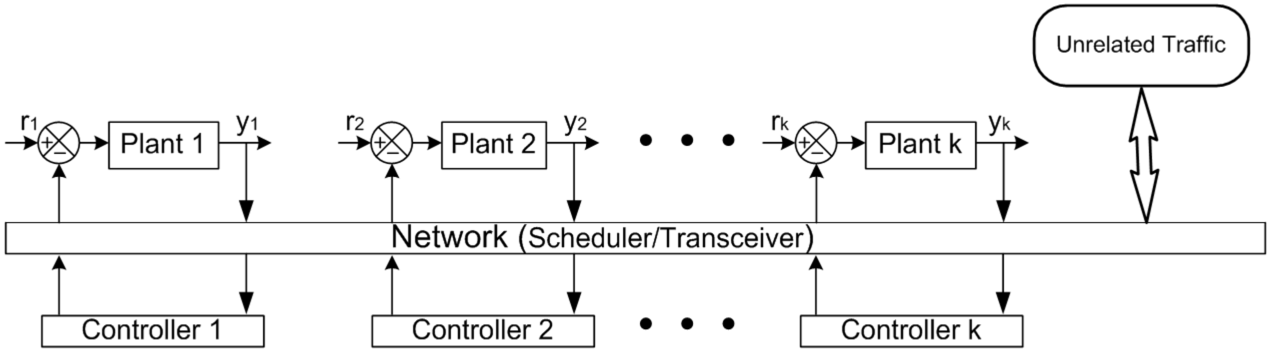

Fig. 1. General NCS Setup.

and scalable especially to the systems that are distributed over a large geographical area. The main feature of decentralized control is that it uses only local information to produce control laws.

It is very common to see systems which include sensors, actuators and controllers are connected through a shared communication medium. Some advantages of connecting the system components via network compared to traditional point-to-point control systems are modularity, flexibility of the system design, and simplicity of implementation such as reduced system wiring and configuration tools. Considering the benefits of decentralized control and the fact that modern control systems are increasingly becoming networked control systems, the area of decentralized networked control systems (DNCS) just emerged recently [22].

D. Paper Preliminaries and Organization

In order to introduce the proposed framework, a decentralized control design scheme of non-networked systems is chosen. We consider the observer-based control design in [6]. The authors considered the case when there is no communication network between the system's components. In this paper, we analyze the case where the control loops of the conventional decentralized controlled system are closed through a network. We adopt a design of the observed-based controller for the DNCS and then analyze the stability of the networked closed loop system. Two approaches to model the network effect are chosen to analyze the stability of the DNCS.

The paper contributions are as follows:

- Development of the general framework that converts a generic decentralized control configuration of non-networked systems to the general setup of NCS
- Analysis of the closed-loop system stability of the DNCS through two approaches,
- Derivation of the perturbation bounds of the networked system.

The remainder of this paper can be summarized as the following. Section II dedicated to the problem formulation. Section III is addressing the stability analysis and the perturbation bounds. In section IV we introduce two examples and show the simulation results. Conclusions and summary of the paper are given in the last section.

II. Problem Formulation

In this section, we start with a controller design method for the non-networked system from the literature of decentralized control, then we map the closed-loop non-networked system to it's equivalent configuration in networked dynamical systems. This would facilitate applying the stability analysis tools from the NCS literature.

A. Observer Based Control Design Formulation

In this paper, we are considering the observer based control design from [6]. We have a large-scale system where the plant dynamics are described as follow:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \sum_{i=1}^{N} \boldsymbol{B}_i \boldsymbol{u}_i \\ \boldsymbol{y}_i = \boldsymbol{C}_i \boldsymbol{x}, \; i = 1, 2, \ldots, N \end{cases} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector of the plant of the large-scale system, $\boldsymbol{u}_i \in \mathbb{R}^{m_i}$ is the input vector of the i$^{\text{th}}$ subsystem and $\boldsymbol{y}_i \in \mathbb{R}^{p_i}$ is the output vector of the i$^{\text{th}}$ subsystem. $\boldsymbol{A} \in \mathbb{R}^{n\times n}, \boldsymbol{B}_i \in \mathbb{R}^{n\times m_i}$, and $\boldsymbol{C}_i \in \mathbb{R}^{p_i \times n}$ are all real constant matrices. Let

$$\begin{cases} \boldsymbol{u} = \begin{bmatrix}\boldsymbol{u}_1^\top & \ldots & \boldsymbol{u}_N^\top\end{bmatrix}^\top, \boldsymbol{y} = \begin{bmatrix}\boldsymbol{y}_1^\top & \ldots & \boldsymbol{y}_N^\top\end{bmatrix}^\top \\ \boldsymbol{B} = \begin{bmatrix}\boldsymbol{B}_1 & \ldots & \boldsymbol{B}_N\end{bmatrix}, \boldsymbol{C} = \begin{bmatrix}\boldsymbol{C}_1^\top & \ldots & \boldsymbol{C}_N^\top\end{bmatrix}^\top. \end{cases}$$

Then the plant can be written in the following compact form:

$$\begin{aligned} \dot{\boldsymbol{x}}_p &= \boldsymbol{A}_p \boldsymbol{x}_p + \boldsymbol{B}_p \boldsymbol{u}_p \\ \boldsymbol{y} &= \boldsymbol{C}_p \boldsymbol{x}_p. \end{aligned}$$

We assume the following as in [6]:

Assumption 1: The triplet $(\boldsymbol{A}_p, \boldsymbol{B}_p, \boldsymbol{C}_p)$ is controllable and observable.

Assumption 2: The triplets $(\boldsymbol{A}_p, \boldsymbol{B}_i, \boldsymbol{C}_i)$ are stable if there exist decentralized fixed modes that are associated with the triplets.

Assumption 3: There exists a complete decentralized structure of the information of each subsystem (i.e., only the local output and control law of each subsystem are available).

Assumption 4: Global state feedback control exists such that $\boldsymbol{u} = -\boldsymbol{F}\boldsymbol{x}$, where $\boldsymbol{F} \in \mathbb{R}^{m\times n}$.
The global state feedback control gain $\boldsymbol{F}$ can be obtained by using any standard state feedback control method. Partitioning the global controller $\boldsymbol{u}$, we get,

$$\begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \\ \vdots \\ \boldsymbol{u}_N \end{bmatrix} = - \begin{bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \vdots \\ \boldsymbol{F}_N \end{bmatrix} \boldsymbol{x}.$$

In [6], they proposed the following decentralized controller:

$$\begin{aligned} \boldsymbol{u}_i = -\boldsymbol{F}_i\boldsymbol{x} &\approx -(\boldsymbol{K}_i\boldsymbol{L}_i + \boldsymbol{W}_i\boldsymbol{C}_i)\boldsymbol{x} \\ &\approx -\boldsymbol{K}_i\boldsymbol{z}_i - \boldsymbol{W}_i\boldsymbol{y}_i, \end{aligned}$$

where $\boldsymbol{z}_i \in \mathbb{R}^{o_i}$ is an estimate of the weighted plant state ($\boldsymbol{z}_i$ tracks $\boldsymbol{L}_i\boldsymbol{x}$) that has following dynamics:

$$\dot{\boldsymbol{z}}_i = \boldsymbol{E}_i\boldsymbol{z}_i + \boldsymbol{L}_i\boldsymbol{B}_i\boldsymbol{u}_i + \boldsymbol{G}_i\boldsymbol{y}_i, \tag{2}$$

where

$$\boldsymbol{E}_i \in \mathbb{R}^{o_i\times o_i}, L_i \in \mathbb{R}^{o_i\times n}, \boldsymbol{K}_i \in \mathbb{R}^{m_i\times o_i}, \boldsymbol{W}_i \in \mathbb{R}^{m_i\times p_i}$$
$$\text{and } \boldsymbol{G}_i \in \mathbb{R}^{o_i\times p_i}$$

are real matrices that represent the controller design parameters [6].

B. Mapping the DNCS to the NCS Setup

The general setup of a DNCS is shown in Figure 2. The state-space representation for the plant is:

$$\begin{cases} \dot{\boldsymbol{x}}_p = \boldsymbol{A}_p\boldsymbol{x}_p + \boldsymbol{B}_p\hat{\boldsymbol{u}} \\ \boldsymbol{y} = \boldsymbol{C}_p\boldsymbol{x}_p + \boldsymbol{D}_p\hat{\boldsymbol{u}}, \end{cases} \tag{3}$$

where

$$\begin{cases} \boldsymbol{B}_p = \begin{bmatrix} \boldsymbol{B}_{p_1} & \dots & \boldsymbol{B}_{p_N} \end{bmatrix}, \boldsymbol{C}_p = \begin{bmatrix} \boldsymbol{C}_{p_1}^\top & \dots & \boldsymbol{C}_{p_N}^\top \end{bmatrix}^\top \\ \boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_1^\top & \dots & \boldsymbol{y}_N^\top \end{bmatrix}^\top, \hat{\boldsymbol{u}} = \begin{bmatrix} \hat{\boldsymbol{u}}_1^\top & \dots & \hat{\boldsymbol{u}}_N^\top \end{bmatrix}^\top. \end{cases}$$

The controller state-space representation is given by:

$$\begin{cases} \dot{\boldsymbol{x}}_c = \boldsymbol{A}_c\boldsymbol{x}_c + \boldsymbol{B}_c\hat{\boldsymbol{y}} \\ \boldsymbol{u} = \boldsymbol{C}_c\boldsymbol{x}_c + \boldsymbol{D}_c\hat{\boldsymbol{y}}, \end{cases} \tag{4}$$

where,

$$\begin{cases} \boldsymbol{B}_c = \begin{bmatrix} \boldsymbol{B}_{c_1} & \dots & \boldsymbol{B}_{c_N} \end{bmatrix}, \boldsymbol{C}_c = \begin{bmatrix} \boldsymbol{C}_{c_1}^\top & \dots & \boldsymbol{C}_{c_N}^\top \end{bmatrix}^\top \\ \boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}_1^\top & \dots & \boldsymbol{u}_N^\top \end{bmatrix}^\top, \boldsymbol{y} = \begin{bmatrix} \hat{\boldsymbol{y}}_1^\top & \dots & \hat{\boldsymbol{y}}_N^\top \end{bmatrix}^\top. \end{cases}$$

To analyze the stability of the overall system under the proposed observer-based decentralized control design, we convert the DNCS setup to the general setup of the NCS,

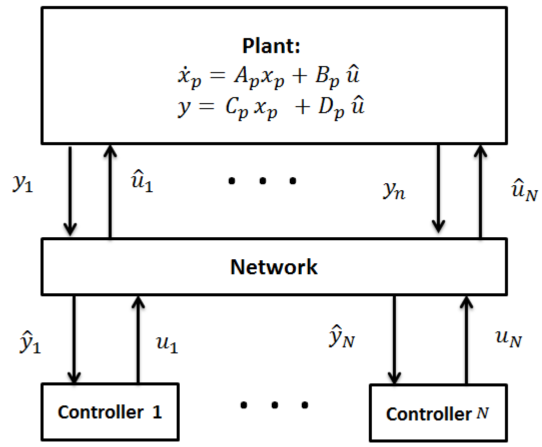

Fig. 2. DNCS State-Space Configuration (2).

as shown in Figure 3. The delayed versions of $\boldsymbol{u}$ and $\boldsymbol{y}$ are defined as: $\hat{\boldsymbol{u}} = \boldsymbol{u} - \boldsymbol{e}_{nu}$ and $\hat{\boldsymbol{y}} = \boldsymbol{y} - \boldsymbol{e}_{ny}$, where $\boldsymbol{e}_{nu}$ and $\boldsymbol{e}_{ny}$ are the delay error due to the presence of the network.

Now we map the decentralized controller to the typical NCS form of the controller.

$$\begin{aligned} \dot{\boldsymbol{z}}_i &= \boldsymbol{E}_i\boldsymbol{z}_i + \boldsymbol{L}_i\boldsymbol{B}_i\boldsymbol{u}_i + \boldsymbol{G}_i\hat{\boldsymbol{y}}_i \\ &= \boldsymbol{E}_i\boldsymbol{z}_i + \boldsymbol{L}_i\boldsymbol{B}_i(-\boldsymbol{K}_i\boldsymbol{z}_i - \boldsymbol{W}_i\hat{\boldsymbol{y}}_i) + \boldsymbol{G}_i\hat{\boldsymbol{y}}_i \\ &= (\boldsymbol{E}_i - \boldsymbol{L}_i\boldsymbol{B}_i\boldsymbol{K}_i)\boldsymbol{z}_i + (\boldsymbol{G}_i - \boldsymbol{L}_i\boldsymbol{B}_i\boldsymbol{W}_i)\hat{\boldsymbol{y}}_i. \end{aligned}$$

Let $\boldsymbol{x}_c = \boldsymbol{z}$, where $\boldsymbol{z} = \begin{bmatrix} \boldsymbol{z}_1^\top & \boldsymbol{z}_2^\top & \dots & \boldsymbol{z}_N^\top \end{bmatrix}^\top$, and introduce the following compact matrix notation:

$$\begin{cases} \boldsymbol{E} = \text{diag}(\boldsymbol{E}_1, \boldsymbol{E}_2, \dots, \boldsymbol{E}_N), \\ \boldsymbol{K} = \text{diag}(\boldsymbol{K}_1, \boldsymbol{K}_2, \dots, \boldsymbol{K}_N), \\ L = \begin{bmatrix} \boldsymbol{L}_1^\top & \boldsymbol{L}_2^\top & \dots & \boldsymbol{L}_N^\top \end{bmatrix}^\top, \\ \boldsymbol{B}_p = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \dots & \boldsymbol{B}_N \end{bmatrix}, \\ \boldsymbol{G} = \text{diag}(\boldsymbol{G}_1, \boldsymbol{G}_2, \dots, \boldsymbol{G}_N), \\ \boldsymbol{W} = \text{diag}(\boldsymbol{W}_1, \boldsymbol{W}_2, \dots, \boldsymbol{W}_N). \end{cases}$$

Therefore, we now have a compact form of the controller's dynamics:

$$\begin{cases} \dot{\boldsymbol{z}} = (\boldsymbol{E} - \boldsymbol{L}\boldsymbol{B}\boldsymbol{K})\boldsymbol{z} + (\boldsymbol{G} - \boldsymbol{L}\boldsymbol{B}\boldsymbol{W})\hat{\boldsymbol{y}} \\ \boldsymbol{u} = (-\boldsymbol{K})\boldsymbol{z} + (-\boldsymbol{W})\hat{\boldsymbol{y}} \end{cases} \tag{5}$$

Knowing that $\hat{\boldsymbol{y}} = \boldsymbol{y} - \boldsymbol{e}_{ny}$, we can map (5) to the standard NCS state-space form of the controller from (4):

$$\dot{\boldsymbol{x}}_c = \boldsymbol{A}_c\boldsymbol{x}_c + \boldsymbol{B}_c\hat{\boldsymbol{y}}, \qquad \hat{\boldsymbol{y}} = \boldsymbol{C}_p\boldsymbol{x}_p - \boldsymbol{e}_{ny},$$

then,

$$\dot{\boldsymbol{x}}_c = \boldsymbol{A}_c\boldsymbol{x}_c + \boldsymbol{B}_c\boldsymbol{C}_p\boldsymbol{x}_p - \boldsymbol{B}_c\boldsymbol{e}_{ny}, \tag{6}$$

where

$$\begin{cases} \boldsymbol{A}_c = \boldsymbol{E} - \boldsymbol{L}\boldsymbol{B}_p\boldsymbol{K} \quad , \quad \boldsymbol{B}_c = \boldsymbol{G} - \boldsymbol{L}\boldsymbol{B}_p\boldsymbol{W} \\ \boldsymbol{C}_c = -\boldsymbol{K} \quad , \quad \boldsymbol{D}_c = -\boldsymbol{W}. \end{cases}$$

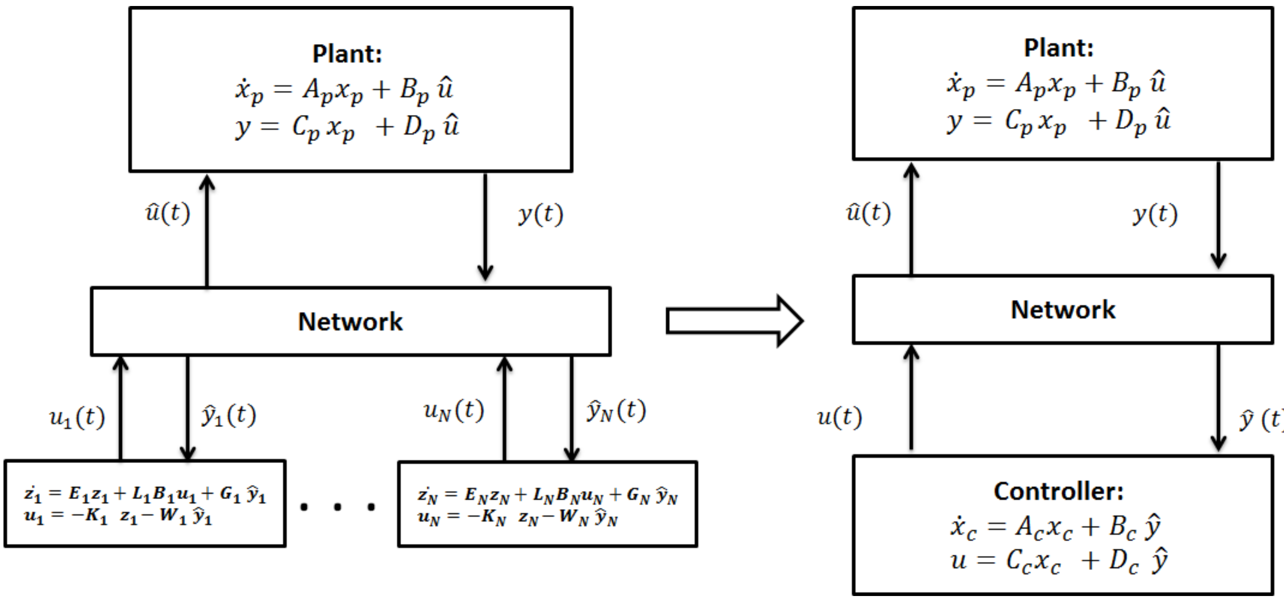

Fig. 3. Mapping DNCS to the Typical NCS Setup.

The plant state dynamics can be represented as:

$$\dot{\boldsymbol{x}}_p = \boldsymbol{A}_p \boldsymbol{x}_p + \boldsymbol{B}_p \boldsymbol{u} - \boldsymbol{B}_p \boldsymbol{e}_{nu}.$$

The controller's output $\boldsymbol{u}$ can be written as:

$$\begin{aligned} \boldsymbol{u} &= -\boldsymbol{K}\boldsymbol{z} - \boldsymbol{W}\hat{\boldsymbol{y}} \\ &= -\boldsymbol{K}\boldsymbol{x}_c - \boldsymbol{W}(\boldsymbol{y} - \boldsymbol{e}_{ny}) \\ &= -\boldsymbol{K}\boldsymbol{x}_c - \boldsymbol{W}\boldsymbol{C}_p\boldsymbol{x}_p + \boldsymbol{W}\boldsymbol{e}_{ny}. \end{aligned}$$

Recall that $\hat{\boldsymbol{u}} = \boldsymbol{u} - \boldsymbol{e}_{nu}$ and by substituting $\boldsymbol{u}$ in the plant state-space dynamics equation, we get:

$$\dot{\boldsymbol{x}}_p = (\boldsymbol{A}_p - \boldsymbol{B}_p\boldsymbol{W}\boldsymbol{C}_p)\boldsymbol{x}_p - \boldsymbol{B}_p\boldsymbol{K}\boldsymbol{x}_c + \boldsymbol{B}_p\boldsymbol{W}\boldsymbol{e}_{ny} - \boldsymbol{B}_p\boldsymbol{e}_{nu}. \tag{7}$$

III. Stability Analysis

In this section, we analyze the stability of the DNCS. We first find the dynamics of the network-induced error. After finding an expression for the networked-induced error, we then augment the error dynamics with the general state of the closed-loop system. The network-induced error is defined as: $\boldsymbol{e}_n = \begin{bmatrix} \boldsymbol{e}_{n_y}^\top & \boldsymbol{e}_{n_u}^\top \end{bmatrix}^\top$. Note that in our system $\boldsymbol{D}_p = \boldsymbol{O}$, thus $\boldsymbol{y} = \boldsymbol{C}_p\boldsymbol{x}_p$. Recall that $\hat{\boldsymbol{y}} = \boldsymbol{y} - \boldsymbol{e}_{ny}$. In addition,

$$\boldsymbol{u} = \boldsymbol{C}_c\boldsymbol{x}_c + \boldsymbol{D}_c\hat{\boldsymbol{y}}. \tag{8}$$

The error is:

$$\boldsymbol{e}_n = \begin{bmatrix} \boldsymbol{e}_{n_y} \\ \boldsymbol{e}_{n_u} \end{bmatrix} = \begin{bmatrix} \boldsymbol{y} - \hat{\boldsymbol{y}} \\ \boldsymbol{u} - \hat{\boldsymbol{u}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_p\boldsymbol{x}_p - \hat{\boldsymbol{y}} \\ \boldsymbol{C}_c\boldsymbol{x}_c + \boldsymbol{D}_c\hat{\boldsymbol{y}} - \hat{\boldsymbol{u}} \end{bmatrix}.$$

Note that $\hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{u}}$ are both piece-wise constant functions, thus: $\dot{\hat{\boldsymbol{y}}} = \boldsymbol{0}$, and $\dot{\hat{\boldsymbol{u}}} = \boldsymbol{0}$. Then,

$$\dot{e}_n = \begin{bmatrix} \boldsymbol{C}_p\dot{\boldsymbol{x}}_p \\ \boldsymbol{C}_c\dot{\boldsymbol{x}}_c \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_p\boldsymbol{A}_p\boldsymbol{x}_p + \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{u} - \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{e}_{n_u} \\ \boldsymbol{C}_c\boldsymbol{A}_c\boldsymbol{x}_c + \boldsymbol{C}_c\boldsymbol{B}_c\boldsymbol{C}_p\boldsymbol{x}_p - \boldsymbol{C}_c\boldsymbol{B}_c\boldsymbol{e}_{n_y} \end{bmatrix}.$$

Substituting (8) into the error dynamics we have (9).

Let $\boldsymbol{x}$ be the overall state of the closed loop system: $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_p^\top & \boldsymbol{x}_c^\top \end{bmatrix}^\top$. Let $\boldsymbol{w}$ be the general state vector that includes the network-induced error vector: $\boldsymbol{w} = \begin{bmatrix} \boldsymbol{x}^\top & \boldsymbol{e}_n^\top \end{bmatrix}^\top$. From (6)-(9), we can formulate the general state dynamics of the system as in (10).

Equation (10) combines the nominal closed-loop system and the perturbation that represents the network effect. To analyze the stability of the system, we consider two different approaches. In the two approaches, we separate the nominal system and the perturbation using two different methods. This is followed by deriving perturbation bounds for both methods.

A. The First Approach

Based on the general state dynamics in (10), the nominal closed-loop system can be found when the network effect is null. Therefore, we can separate the nominal system and the perturbation as in (11), where $\boldsymbol{S}$ represents the dynamics of the nominal closed-loop system and $\Delta\boldsymbol{S}$ represents the perturbation in the system dynamics. For stability analysis purposes, we introduce the matrix $\Delta\boldsymbol{C}$ which is used to guarantee that $(\boldsymbol{S} + \Delta\boldsymbol{C})$ is Hurwitz. We can now write the general system dynamics as:

$$\dot{\boldsymbol{w}} = (\boldsymbol{S} + \Delta\boldsymbol{C})\boldsymbol{w} + (\Delta\boldsymbol{S} - \Delta\boldsymbol{C})\boldsymbol{w} = \boldsymbol{S}_c\boldsymbol{w} + \Delta\boldsymbol{S}_c\boldsymbol{w}. \tag{12}$$

Theorem 1: For the DNCS in (3) and (4) and for any $\boldsymbol{Q} = \boldsymbol{Q}^\top \succ \boldsymbol{O}$, if the solution to the Lyapunov matrix equation

$$\boldsymbol{S}_c^\top\boldsymbol{P} + \boldsymbol{P}\boldsymbol{S}_c^\top = -2\boldsymbol{Q}, \;\; \boldsymbol{Q} = \boldsymbol{I}$$

is $\boldsymbol{P} = \boldsymbol{P}^\top \succ \boldsymbol{O}$, and if the norm of the perturbation matrix ($\Delta\boldsymbol{S}_c$) is upper bounded by:

$$\|\Delta\boldsymbol{S}_c\| \leq \frac{1}{\lambda_{max}(\boldsymbol{P})}$$

$$\dot{\boldsymbol{e}}_n = \begin{bmatrix} (\boldsymbol{C}_p\boldsymbol{A}_p + \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{D}_c\boldsymbol{C}_p)\boldsymbol{x}_p + \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{C}_c\boldsymbol{x}_c - \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{D}_c\boldsymbol{e}_{n_y} - \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{e}_{n_u} \\ \boldsymbol{C}_c\boldsymbol{B}_c\boldsymbol{C}_p\boldsymbol{x}_p + \boldsymbol{C}_c\boldsymbol{A}_c\boldsymbol{x}_c - \boldsymbol{C}_c\boldsymbol{B}_c\boldsymbol{e}_{n_y} \end{bmatrix}. \tag{9}$$

$$\dot{w} = \begin{bmatrix} \dot{\boldsymbol{x}}_p \\ \dot{\boldsymbol{x}}_c \\ \dot{\boldsymbol{e}}_{ny} \\ \dot{\boldsymbol{e}}_{nu} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{A}_p + \boldsymbol{B}_p\boldsymbol{D}_c\boldsymbol{C}_p & \boldsymbol{B}_p\boldsymbol{C}_c & -\boldsymbol{B}_p\boldsymbol{D}_c & -\boldsymbol{B}_p \\ \boldsymbol{B}_c\boldsymbol{C}_p & \boldsymbol{A}_c & -\boldsymbol{B}_c & \boldsymbol{O} \\ (\boldsymbol{C}_p\boldsymbol{A}_p + \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{D}_c\boldsymbol{C}_p) & \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{C}_c & -\boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{D}_c & -\boldsymbol{C}_p\boldsymbol{B}_p \\ \boldsymbol{C}_c\boldsymbol{B}_c\boldsymbol{C}_p & \boldsymbol{C}_c\boldsymbol{A}_c & -\boldsymbol{C}_c\boldsymbol{B}_c & \boldsymbol{O} \end{bmatrix}}_{\hat{\boldsymbol{A}}} \begin{bmatrix} \boldsymbol{x}_p \\ \boldsymbol{x}_c \\ \boldsymbol{e}_{ny} \\ \boldsymbol{e}_{nu} \end{bmatrix}. \tag{10}$$

$$\dot{\boldsymbol{w}} = \begin{bmatrix} \dot{\boldsymbol{x}}_p \\ \dot{\boldsymbol{x}}_c \\ \dot{\boldsymbol{e}}_{n_y} \\ \dot{\boldsymbol{e}}_{n_u} \end{bmatrix} = \underbrace{\left\{ \begin{bmatrix} \boldsymbol{A}_p + \boldsymbol{B}_p\boldsymbol{D}_c\boldsymbol{C}_p & \boldsymbol{B}_p\boldsymbol{C}_c & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{B}_c\boldsymbol{C}_p & \boldsymbol{A}_c & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \end{bmatrix} + \Delta\boldsymbol{C} \right\}}_{\boldsymbol{S}} \begin{bmatrix} \boldsymbol{x}_p \\ \boldsymbol{x}_c \\ \boldsymbol{e}_{n_y} \\ \boldsymbol{e}_{n_u} \end{bmatrix}$$

$$+ \underbrace{\left\{ \begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} & -\boldsymbol{B}_p\boldsymbol{D}_c & -\boldsymbol{B}_p \\ \boldsymbol{O} & \boldsymbol{O} & -\boldsymbol{B}_c & \boldsymbol{O} \\ (\boldsymbol{C}_p\boldsymbol{A}_p + \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{D}_c\boldsymbol{C}_p) & \boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{C}_c & -\boldsymbol{C}_p\boldsymbol{B}_p\boldsymbol{D}_c & -\boldsymbol{C}_p\boldsymbol{B}_p \\ \boldsymbol{C}_c\boldsymbol{B}_c\boldsymbol{C}_p & \boldsymbol{C}_c\boldsymbol{A}_c & -\boldsymbol{C}_c\boldsymbol{B}_c & \boldsymbol{O} \end{bmatrix} - \Delta\boldsymbol{C} \right\}}_{\Delta\boldsymbol{S}} \begin{bmatrix} \boldsymbol{x}_p \\ \boldsymbol{x}_c \\ \boldsymbol{e}_{n_y} \\ \boldsymbol{e}_{n_u} \end{bmatrix}. \tag{11}$$

then the DNCS is globally asymptotically stable.

Proof: Since $\boldsymbol{S}_c$ is stable, then for $\boldsymbol{Q} = \boldsymbol{I}$, the solution to the Lyapunov matrix equation:

$$\boldsymbol{S}_c^\top\boldsymbol{P} + \boldsymbol{P}\boldsymbol{S}_c^\top = -2\boldsymbol{Q},\ \boldsymbol{Q} = \boldsymbol{I}$$

is symmetric positive definite. Using the following candidate Lyapunov function, $V = \frac{1}{2}\boldsymbol{w}^\top\boldsymbol{P}\boldsymbol{w}$. Then,

$$\dot{V} = \boldsymbol{w}^\top\boldsymbol{P}\dot{\boldsymbol{w}} = \boldsymbol{w}^\top\boldsymbol{P}\boldsymbol{S}_c\boldsymbol{w} + \boldsymbol{w}^\top\boldsymbol{P}\Delta\boldsymbol{S}_c\boldsymbol{w}.$$

Notice that

$$\boldsymbol{w}^\top\boldsymbol{P}\boldsymbol{S}_c\boldsymbol{w} = \frac{1}{2}\boldsymbol{w}^\top\boldsymbol{S}_c^\top\boldsymbol{P}\boldsymbol{w} + \frac{1}{2}\boldsymbol{w}^\top\boldsymbol{P}\boldsymbol{S}_c\boldsymbol{w}$$

$$= \frac{1}{2}\boldsymbol{w}^\top\left(\boldsymbol{S}_c^\top\boldsymbol{P} + \boldsymbol{P}\boldsymbol{S}_c\right)\boldsymbol{w} = -\|\boldsymbol{w}\|^2.$$

In addition, we have:

$$\boldsymbol{w}^\top\boldsymbol{P}\Delta\boldsymbol{S}_c\boldsymbol{w} \le \|\boldsymbol{P}\Delta\boldsymbol{S}_c\|\|\boldsymbol{w}\|^2 = \|\boldsymbol{P}\|\|\Delta\boldsymbol{S}_c\|\|\boldsymbol{w}\|^2$$

$$= \lambda_{max}(\boldsymbol{P})\|\Delta\boldsymbol{S}_c\|\|\boldsymbol{w}\|^2.$$

Hence,

$$\dot{V} \le -\|\boldsymbol{w}\|^2 + \lambda_{max}(\boldsymbol{P})\|\Delta\boldsymbol{S}_c\|\|\boldsymbol{w}\|^2$$

$$= -\left(1 - \lambda_{max}(\boldsymbol{P})\|\Delta\boldsymbol{S}_c\|\right)\|\boldsymbol{w}\|^2.$$

For a valid Lyapunov candidate function, we should have $\dot{V} < 0$, thus:

$$\|\Delta\boldsymbol{S}_c\| \le \frac{1}{\lambda_{max}(\boldsymbol{P})}.$$

■

B. The Second Approach

In this approach we partition the augmented states in (10) as follows:

$$\dot{\boldsymbol{w}}(t) = \hat{\boldsymbol{A}}\boldsymbol{w}(t) = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}\boldsymbol{w}(t).$$

The state dynamics of the networked closed-loop system can be represented as: $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{11}\boldsymbol{x}(t) + \boldsymbol{A}_{12}\boldsymbol{e}_n(t)$, where

$$\boldsymbol{A}_{11} = \begin{bmatrix} \boldsymbol{A}_p + \boldsymbol{B}_p\boldsymbol{D}_c\boldsymbol{C}_p & \boldsymbol{B}_p\boldsymbol{C}_c \\ \boldsymbol{B}_c\boldsymbol{C}_p & \boldsymbol{A}_c \end{bmatrix},$$

and

$$\boldsymbol{A}_{12} = \begin{bmatrix} -\boldsymbol{B}_p\boldsymbol{D}_c & -\boldsymbol{B}_p \\ -\boldsymbol{B}_c & \boldsymbol{O} \end{bmatrix}.$$

Consider the time interval between transmissions: $t \in [t_i, t_{i+1}]$ where $i = 0, 1, 2, \ldots$, we get:

$$\hat{\boldsymbol{y}}(t) = \boldsymbol{y}(t_i) = \boldsymbol{C}_p\boldsymbol{x}_p(t_i)$$

and

$$\hat{\boldsymbol{u}}(t) = \boldsymbol{u}(t_i) = \boldsymbol{C}_c\boldsymbol{x}_c(t_i) + \boldsymbol{D}_c y(t_i)$$

$$= \boldsymbol{C}_c\boldsymbol{x}_c(t_i) + \boldsymbol{D}_c\boldsymbol{C}_p\boldsymbol{x}_p(t_i).$$

Let $\boldsymbol{g}(t, \boldsymbol{x}) = \boldsymbol{A}_{12}\boldsymbol{e}_n(t)$, then the system dynamics equation can be written as:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{11}\boldsymbol{x}(t) + \boldsymbol{g}(t, \boldsymbol{x}), \tag{13}$$

where $\boldsymbol{g}(t, \boldsymbol{x})$ is the perturbation caused by the network. Let $\boldsymbol{e}_x(t) = \boldsymbol{x}(t) - \boldsymbol{x}(t_i)$, then we can write the perturbation term as:

$$\boldsymbol{g}(t, \boldsymbol{x}) = \boldsymbol{A}_{12}e_n(t) = \underbrace{\boldsymbol{A}_{12}\begin{bmatrix} \boldsymbol{C}_p & \boldsymbol{O} \\ \boldsymbol{D}_c\boldsymbol{C}_p & \boldsymbol{C}_c \end{bmatrix}}_{\boldsymbol{D}}\left[\boldsymbol{x}(t) - \boldsymbol{x}(t_i)\right]$$

$$= \boldsymbol{D}\left[\boldsymbol{x}(t) - \boldsymbol{x}(t_i)\right] = \boldsymbol{D}\boldsymbol{e}_x(t).$$

Since the non-networked system is stable, then there exists a matrix $\boldsymbol{P} = \boldsymbol{P}^\top \succ \boldsymbol{O}$ such that the solution to the Lyapunov matrix equation:

$$\boldsymbol{A}_{11}^\top \boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{11} = -\boldsymbol{Q}$$

is symmetric positive definite ($\boldsymbol{P} = \boldsymbol{P}^\top \succ \boldsymbol{O}$). Let $\lambda_1 = \lambda_{min}(\boldsymbol{P})$ and $\lambda_2 = \lambda_{max}(\boldsymbol{P})$. In [21], Zhang et al. mentioned that an NCS is stable if the maximum allowable transfer interval (MATI) τ_m is upper bounded by:

$$\tau_m < \frac{\lambda_{min}(\boldsymbol{Q})}{16\lambda_2\sqrt{\frac{\lambda_2}{\lambda_1}}\|\boldsymbol{A}\|^2\left(1+\sqrt{\frac{\lambda_2}{\lambda_1}}\right)\sum_{i=1}^{p} i}.$$

Based on this τ_m upper bound and treating $\boldsymbol{g}(t, \boldsymbol{x})$ as a vanishing perturbation as in [19], we can introduce a bound to the perturbation that guarantees the stability of DNCS.

Theorem 2: For the perturbed general state of the system in (12), if the origin is a globally exponentially stable point of the non-networked system, and if τ_m satisfies:

$$1 - ||\boldsymbol{D}||\ ||\boldsymbol{A}_{11} + \boldsymbol{D}||^{-1}(e^{||\boldsymbol{A}_{11}+\boldsymbol{D}||\tau_m} - 1) > 0,$$

and the perturbation is upper bounded by

$$||\boldsymbol{e}_x(t)|| \le \gamma ||\boldsymbol{x}(t)||,$$

where

$$\gamma = \frac{||\boldsymbol{A}_{11}||\ ||\boldsymbol{A}_{11} + \boldsymbol{D}||^{-1}(e^{||\boldsymbol{A}_{11}+\boldsymbol{D}||\tau_m} - 1)e^{||\boldsymbol{A}_{11}+\boldsymbol{D}||\tau_m}}{1 - ||\boldsymbol{D}||\ ||\boldsymbol{A}_{11} + \boldsymbol{D}||^{-1}(e^{||\boldsymbol{A}_{11}+\boldsymbol{D}||\tau_m} - 1)},$$

then the origin is a globally exponentially stable equilibrium point of the DNCS.

Proof: The proof of the above theorem is very similar to the proof of Walsh et al. in [17]. ■

IV. Simulation Results

This section is dedicated to discuss our results from simulating the behavior of the proposed design of the DNCS. We first discuss two methods that we used to find a bound for the maximum allowable transfer interval τ_m. The first method considers the network effect as a perturbation as in Theorems 1 and 2. We used the MATI bound for the computation of the sufficiency condition of stability to the DNCS. This bound is used for stability analysis in general NCS systems. From the simulation results, we note that it is very conservative bound for a sufficiency condition of stability.

In the second method we used a less conservative bound from the literature. In [18], they derive the MATI bound by treating the network effect as a pure time delay. Figure 4 shows a high level description for a network modeled as a time delay.

With this modeling, the plant and controller dynamics can be rewritten as:

$$\begin{aligned}
\dot{\boldsymbol{x}}_c(t) &= \boldsymbol{A}_c\boldsymbol{x}_c(t) + \boldsymbol{B}_c\boldsymbol{C}_p\boldsymbol{x}_p(t-\tau_{sc}) \\
\dot{\boldsymbol{x}}_p(t) &= \boldsymbol{A}_p\boldsymbol{x}_p(t) + \boldsymbol{B}_p\boldsymbol{D}_c\boldsymbol{C}_p\boldsymbol{x}_p(t-\tau_{sc}-\tau_{ca}) \\
&\quad +\boldsymbol{B}_p\boldsymbol{C}_c\boldsymbol{x}_c(t-\tau_{ca}).
\end{aligned}$$

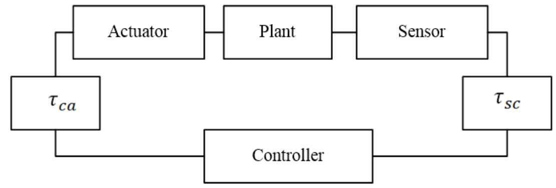

Fig. 4. The Network Effect Modeled as Pure Time Delay.

The main idea behind finding a bound on the maximum allowable transfer interval (MATI) or τ_m is to model the delayed state as a taylor series expansion:

$$\boldsymbol{x}(t-\tau) = \sum_{k=0}^{\infty}(-1)^n\frac{\tau^n}{n!}\boldsymbol{x}^{(n)}(t).$$

In [18], they applied the following approximation:

$$\boldsymbol{x}(t-\tau) \approx \boldsymbol{x}(t) - \tau\dot{\boldsymbol{x}}(t),$$

which leads to a significantly less conservative bound on τ_m as follows:

$$\tau_m < \frac{1}{\|\boldsymbol{B}_p[\boldsymbol{W}\boldsymbol{C}_p\ , \boldsymbol{K}]\|}. \tag{14}$$

A. Numerical Example

In this section we introduce a numerical examples to analyze the behavior of the proposed design of the observer-based controller of the DNCS. We also discuss the perturbation bounds that we derived in Theorems 1 and 2.

The following system appears in [20]:

$$\boldsymbol{A} = \begin{bmatrix} -3 & 0 & -0.6 & 1.5 & -0.30 \\ -0.3 & -6 & 0 & 0.6 & 1.5 \\ -1.2 & 1.5 & -9 & 0.3 & -3 \\ -2.25 & -0.6 & -2.4 & 2 & 0 \\ -0.6 & 1.5 & -1.5 & 1.5 & 3.75 \end{bmatrix}, \boldsymbol{B}_1 = \begin{bmatrix} 1 \\ 0 \\ 0.5 \\ 1 \\ -1 \end{bmatrix},$$

$$\boldsymbol{B}_2 = \begin{bmatrix} 0.2 \\ -0.1 \\ 1 \\ -2 \\ 0.3 \end{bmatrix}, \boldsymbol{C}_1 = \begin{bmatrix} 1 & 0.2 & -0.3 & 1 & 2 \\ 1 & 0 & 0 & 0 & -0.5 \end{bmatrix},$$

$$\boldsymbol{C}_2 = \begin{bmatrix} 0.5 & 0 & 0.1 & 0.7 & 0.9 \\ 0.6 & 0.4 & -0.4 & 0.5 & 0 \end{bmatrix}.$$

It is also an unstable system with two controllers. After computing the design parameters for the observer-based controller as in [6] (the global state feedback control gain matrix $\boldsymbol{F}$ is computed using the Continuous Algebraic Riccati Equation), we get the following decentralized control laws:

$$\boldsymbol{u}_1 = -\begin{bmatrix} -1.36 & -1.36 & 5.15 \end{bmatrix}\boldsymbol{z}_1 - \begin{bmatrix} -2.81 & 4.72 \end{bmatrix}\hat{\boldsymbol{y}}_1,$$

$$\boldsymbol{u}_2 = -\begin{bmatrix} 9.19 & 9.19 & -24.27 \end{bmatrix}\boldsymbol{z}_2 - \begin{bmatrix} -0.96 & -5.67 \end{bmatrix}\hat{\boldsymbol{y}}_2.$$

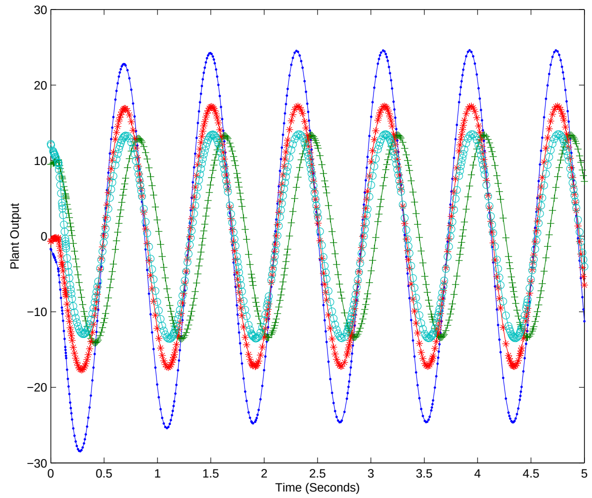

Fig. 5. Unstable Behavior of the System in Example 2 (for $\tau_m > 0.035324$ sec).

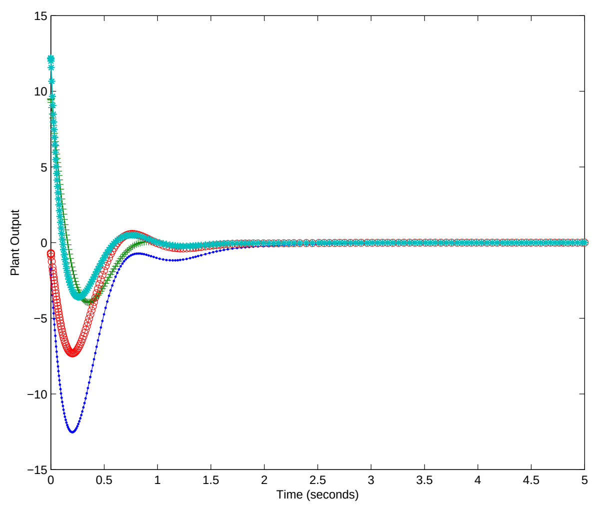

Fig. 6. Stable Behavior of the System of Example 1 (conservative bound, $\tau_m = 5.2753e^{-9}$ sec).

From the simulation results we note that the system becomes unstable for $\tau_m > 0.035324$ sec, as shown in Figure 5. Computing the bound of MATI by considering the Theorem 2, we find that $\tau_m = 5.2753e^{-9}$ sec, which is extremely conservative to guarantee the stable behavior of the DNCS as shown in Figure **??**. Nonetheless, when we use (14), we get $\tau_m = 0.0399$ sec, which is very close to the above bound of stability ($\tau_m < 0.0353247$ sec).

From Theorem 1, the sufficiency condition of stability is $\|\Delta \boldsymbol{S}_c\| \leq \frac{1}{\lambda_{max}(\boldsymbol{P})}$. From the simulation results, $\|\Delta \boldsymbol{S}_c\| = 99.3109$ and $\frac{1}{\lambda_{max}(\boldsymbol{P})} = 0.4216$. Again, we can see that the system is stable even with larger value of the norm of the perturbation which means that the perturbation bound of Theorem 1 is conservative as a sufficiency condition for stability.

In Theorem 2, the sufficiency condition of stability is $\gamma < \frac{1}{2\lambda_2}$. From the simulation results, $\gamma = 5.1640e^{-6}$ and $\frac{1}{2\lambda_2} = 0.2108$. This example also shows that the perturbation bound of Theorem 2 is satisfied, which is because the fact that in Theorem 2 the MATI bound that we used is very conservative as we mentioned before ($\tau_m < 5.2753e^{-9}$ sec).

V. Conclusions

This paper introduces a general framework that converts a generic decentralized control configuration of non-networked systems to the general setup of a Networked Control System. A design method from the literature of decentralized control for non-networked systems was chosen as a base for the design of a controller for the networked systems. The main idea of our design is to formulate the DNCS in the general form and then map the resulting system to the general form of the NCS. The network effect has been treated as a perturbation. Two methods to analyze the stability of the DNCS system are introduced. Perturbation bounds for stability of the DNCS systems have been derived. The maximum allowable transfer interval (MATI) is computed based on two different methods in the literature. The simulation results showed that if we used a conservative method to compute MATI, we get a less conservative results for the perturbation bound and vise versa. In the future, the results of this paper can be used to analyze the effect of the scheduling protocol on the MATI which is a critical factor in analyzing the stability of DNCSs.

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