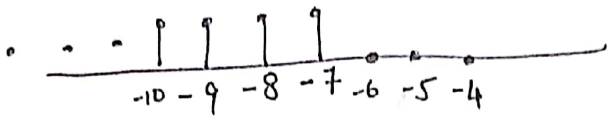
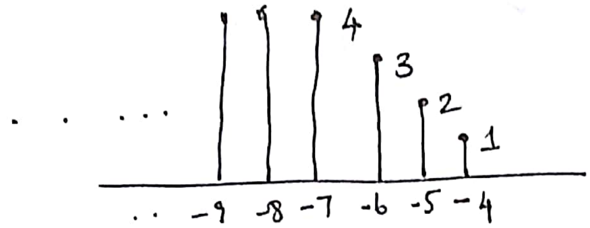
Problem 1

The output at time n for a system is obtained by accumulating the values of the input from $n+7$ to ∞.

10 a What is the impulse response? $h[n] = u[-n-7]$.



10. b What is $y[n]$ when $x[n] = u[n] - u[n-4]$, where $u[n]$ indicates unit step.



10 c Is this system (1 mark for each part if reason is correct, but solution is wrong)

2.5 i Causal?

No. Output depends on future inputs.

2.5 ii Memoryless?

No. Output depends on more than just present input.

2.5 iii Stable?

No. $x[n] = 1 \Rightarrow y[n] \to \infty$

2.5 iv Invertible? If invertible find the impulse response of the inverse system.

Yes.

$$y[n] = \sum_{i=n+7}^{\infty} x[i].$$

$$y[n-7] = \sum_{i=n}^{\infty} x[i]. = \sum_{i=n+1}^{\infty} x[i] + x[n].$$

$$\Rightarrow y[n-7] = y[n-6] + x[n] \Rightarrow x[n] = y[n-7] - y[n-6].$$

$\therefore$ Impulse response $h[n] = \delta[n-7] - \delta[n-6]$.

Problem 2

For the following questions, determine if the signals are periodic or not. If periodic, find their fundamental period.

5 a . e^{-7jt}

Periodic. $T = \frac{2\pi}{7}$

5 b . $e^{5jt} + je^{10jt}$

e^{j5t} $\quad$ je^{10jt}

Periodic. $T_1 = \frac{2\pi}{5}$ $\quad T_2 = \frac{2\pi}{10}$. $\Rightarrow T = \frac{2\pi}{5}$

5 c . e^{2t}

Not Periodic.

5 d . $e^{-2t}u(t)$

Not Periodic.

5 e . $\cos(7\pi n^3)$

Periodic. $N = 2$.

5 f . $\cos(10\pi t) + \cos(3t)$

$T_1 = \frac{2\pi}{10\pi} = \frac{1}{5}$ $\quad T_2 = \frac{2\pi}{3}$.

Cannot find LCM. ∴ Not periodic

Writing the correct. formula for E_∞ and $P_\infty \rightarrow 1.5$ each.

$E_\infty \rightarrow 2.5$

$P_\infty \rightarrow 2.5$

Problem 3

Calculate E_∞ and P_∞ for the following

7.5 a $x[n] = \sin(\frac{\pi}{4}n)$ $E_\infty = \infty$ $P_\infty = \frac{1}{2}$.

7.5 b $x(t) = e^{j(5t-\frac{\pi}{3})}$ $E_\infty = \cancel{1}\ \infty$ $P_\infty = 1$.

7.5 c $x[n] = u[n]$ $E_\infty = \infty$ $P_\infty = \frac{1}{2}$.

7.5 d $x[n] = \delta[n]$ $E_\infty = 1$ $P_\infty = 0$.

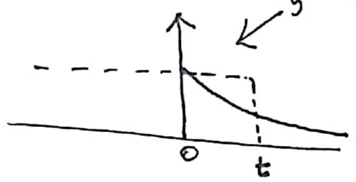
Problem 4

Find the convolution of the signals given below.

15 a i $x(t) = u(t)$

ii $h(t) = e^{-2t}u(t)$

5 marks.



$$y(t) = 0 \qquad t < 0.$$

$$y(t) = \int_0^t e^{-2\tau}\,d\tau \qquad t \geq 0.$$

$$= \frac{-1}{2}\left[e^{-2\tau}\right]_0^t$$

$$= \frac{1-e^{-2t}}{2} \qquad t \geq 0.$$

5 marks

$$y(t) = \frac{1-e^{-2t}}{2}\,u(t).$$ ← 5 marks.

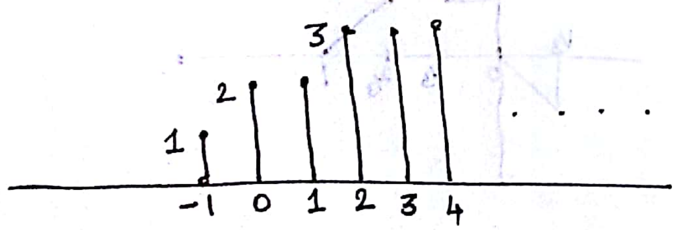
15 b i $x[n] = \delta[n] + \delta[n+1] + \delta[n-2]$

ii $h[n] = u[n]$

$$y[n] = x[n] * h[n]$$

$$= \left(\delta[n] + \delta[n+1] + \delta[n-2]\right) * \overset{u}{\cancel{h}}[n]$$

$$= u[n] + u[n+1] + u[n-2].$$



Problem 5

15 a) For the signal $x(t)$ in Figure 1, sketch $x(-3t-1)$

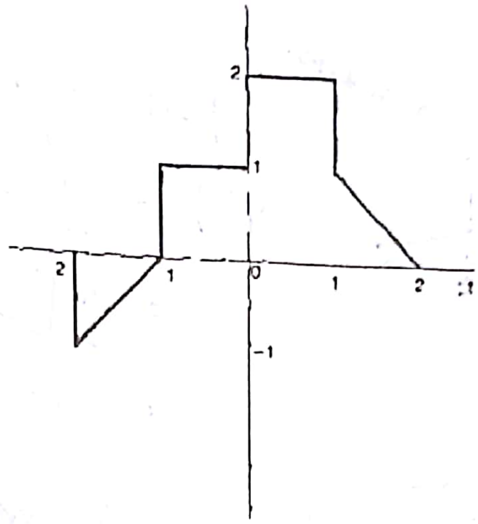
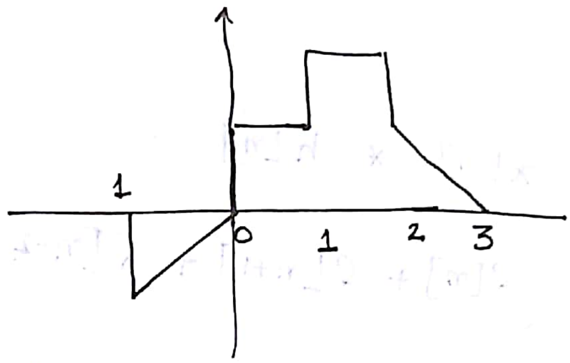


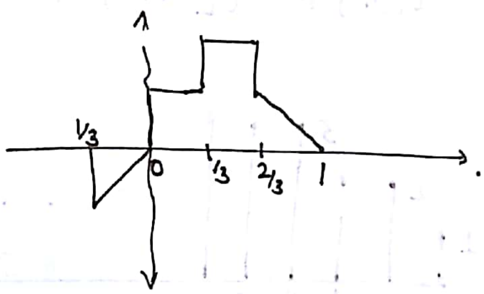
Figure 1: x(t)

$x(t-1)$.



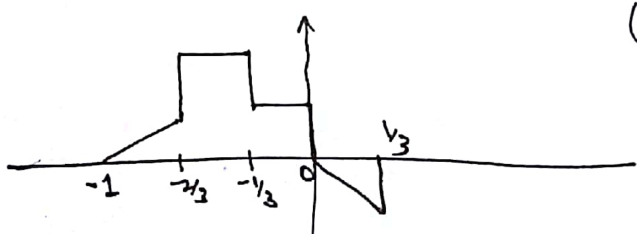
(5)

$x(3t-1)$.



(5)

$x(-3t-1)$.

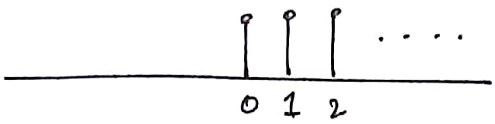


(5).

5 marks for each step.
Different steps also possible.

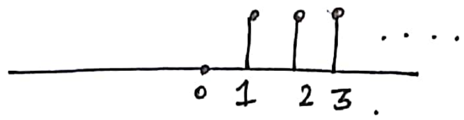
15. b) Sketch 2u[-n-1] where u[n] is a unit step function

u[n].



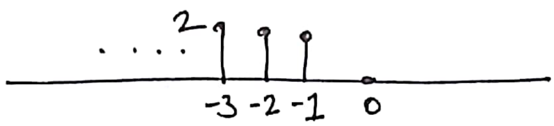
(5)

u[n-1].



(5)

2u[-n-1].



(5).

Different steps possible.