Exam 2 Solns.

1. $$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT).$$

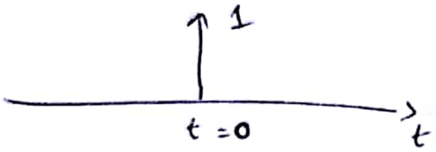
a). $$a_k = \cancel{\Sigma}\,\frac{1}{T}\int_T x(t)\, e^{-j\omega_0 k t}\, dt \qquad \omega_0 = \frac{2\pi}{T}$$

$$= \frac{1}{T}\int \delta(t)\, e^{-j\frac{2\pi}{T}t\cdot k}\, dt = \frac{1}{T}$$

b) $$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi\, a_k\, \delta(\omega - k\omega_0)$$

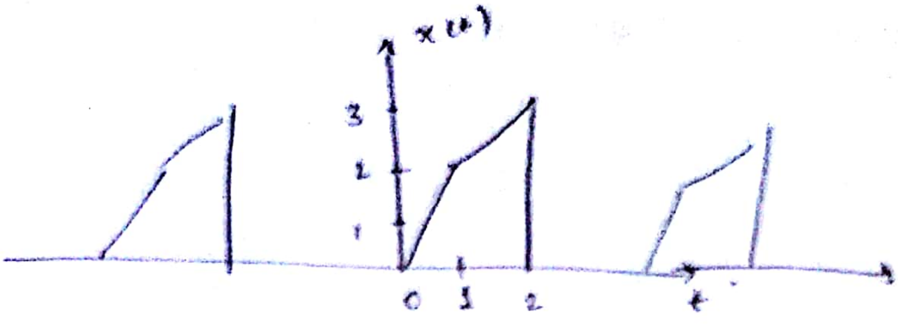
$$= \sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\, \delta(\omega - k\omega_0).$$

c).



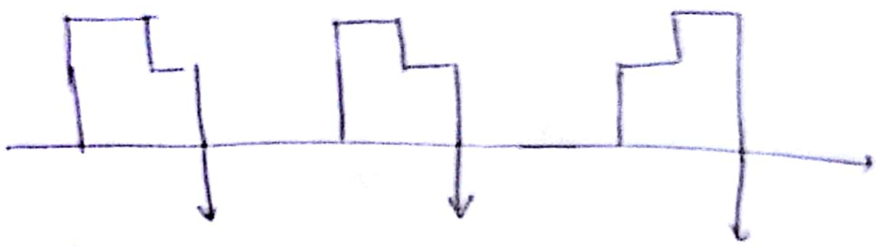
d). $$X(j\omega) = 1$$

2.



$x(t)$.

$\frac{dx(t)}{dt}$



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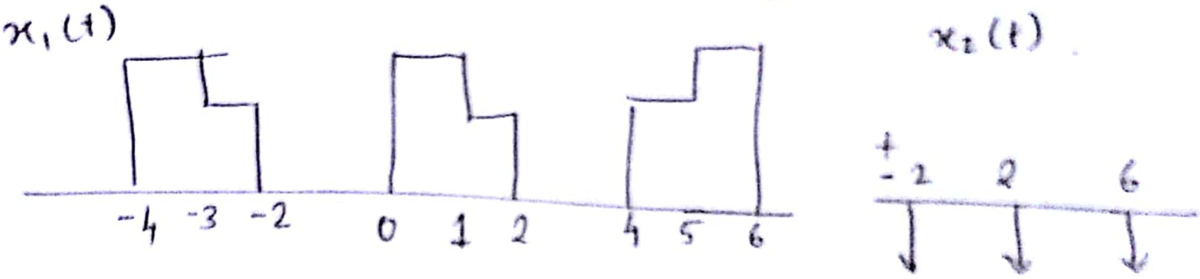
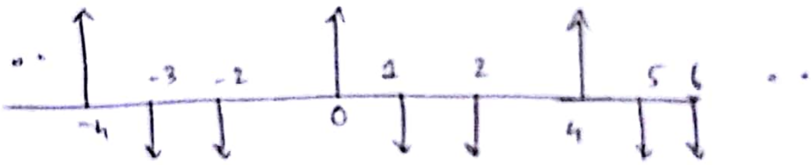


$\frac{dx_1(t)}{dt} =$



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$$a_k = \frac{1}{4} + e^{-j\frac{2\pi}{4}k1} \cdot \frac{1}{4} + e^{-j\frac{2\pi}{4}k2} \cdot \frac{1}{4}$$

$$a_k = \frac{1}{4}\left(1 + e^{-j\frac{\pi}{2}k} + e^{-j\pi k}\right)$$

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$$\frac{dx(t)}{dt} \stackrel{FS}{\longleftrightarrow} b_k = \frac{a_k}{j\omega} + \frac{1}{4} e^{-j\pi k}$$

$$= \frac{1}{4j\omega}\left(1 + e^{-j\frac{\pi}{2}k} + e^{-j\pi k}\right) + \frac{1}{4} e^{-j\pi k}.$$

$$\Rightarrow x(t) \stackrel{FS}{\longleftrightarrow} C_k = \frac{1}{4(j\omega)^2}\left(1 + e^{-j\frac{\pi}{2}k} + e^{-j\pi k}\right) + \frac{1}{4j\omega} e^{-j\pi k}$$

$$k \neq 0.$$

$$C_0 = \frac{1}{4} \int_0^4 x(t)\, dt$$

$$= \frac{1}{4}\left[\frac{1}{2} \times 1 \cdot 2 + \frac{1}{2} \times \left(3 \times 3 - 2 \times 2\right)\right]$$

$$= \frac{1}{4}\left[1 + \frac{1}{2} \times 5\right] = \frac{1}{4}\left[1 + \frac{5}{2}\right] = 7/8.$$

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi C_k\, \delta(\omega - k\omega_0) \qquad \omega_0 = \frac{2\pi}{4}.$$

3 .a) $\text{Even}\{x(t)\} = \dfrac{x(t) + x(-t)}{2}$.

$x(t) \leftrightarrow X(e^{j\omega})$

$x(-t) \leftrightarrow X(\cancel{e}\,-j\omega)$. $-①$

Given $x(t)$ is real.

$\Rightarrow x(t) = x^*(t)$

$\Rightarrow X(j\omega) = X^*(-j\omega)$. $-②$.

Combining ① & ② . we get $x(-t) \leftrightarrow X^*(j\omega)$.

$\therefore \text{Even}\{x(t)\} \leftrightarrow \dfrac{X(j\omega) + X^*(j\omega)}{2} = \text{Real}\{X(j\omega)\}$.

b) Similar to (a) $\text{Odd}\{x(t)\} \leftrightarrow \text{Im}\{X(j\omega)\}$.

c). $\displaystyle\sum_{k=-\infty}^{\infty} a_k e^{jk\frac{14\pi}{T}} = \sum_{k=-\infty}^{\infty} a_k \cdot e^{jk\cdot\frac{2\pi}{T}\cdot 7} = x(7)$.

d). $\displaystyle\sum_{k=-\infty}^{\infty} (-1)^k a_k = \sum_{k=-\infty}^{\infty} e^{j\pi k} a_k$

$\displaystyle = \sum_{k=-\infty}^{\infty} e^{\frac{j2\pi}{T}\cdot k\cdot\frac{T}{2}} a_k = x(T/2)$.

4. $x(t) \longleftrightarrow X(e^{j\omega})$

$$\int_{-\infty}^{\infty} e^{j\omega_0 t} x(t) e^{-j\omega t} = \int_{-\infty}^{\infty} x(t) e^{-j(\omega-\omega_0)t}$$

$$= X(e^{j(\omega-\omega_0)}).$$

5. F18 HW4 Q.2.

$$a_k = \begin{cases} \frac{1}{4j} e^{j\frac{\pi}{4}}, & k = 8, 9 \\ \frac{-1}{4j} e^{-j\frac{\pi}{4}} & k = 4, 3 \\ 0 & o\,w. \end{cases}$$

$N = 12.$