Fall 2018 - Final Exam
ECE 301: Signals and Systems

Prof. Aly El Gamal

Last Name :

First Name :

PUID :

Instructions.

- Write your full name, PUID on this page
- This is a 120 minutes exam containing 10 questions, 30 points each totaling 300 points.
- Please write clearly and legibly
- Your solutions must include detailed steps and/or explanations. Do not simply state the answer
- Three A-4 sized crib sheet front and back is allowed.
- There should be 12 pages including the cover page.

Question	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
5	
10	
Total	

Problem 1

1a) For $x(t)$ in Figure 1, sketch the following signals:

a $\sum_{n=-\infty}^{\infty} x\left(\frac{2n}{3}T\right)\delta(t-nT)$, with $T=1$

b $x\left(-\frac{t}{3}+2\right)u(-t)$

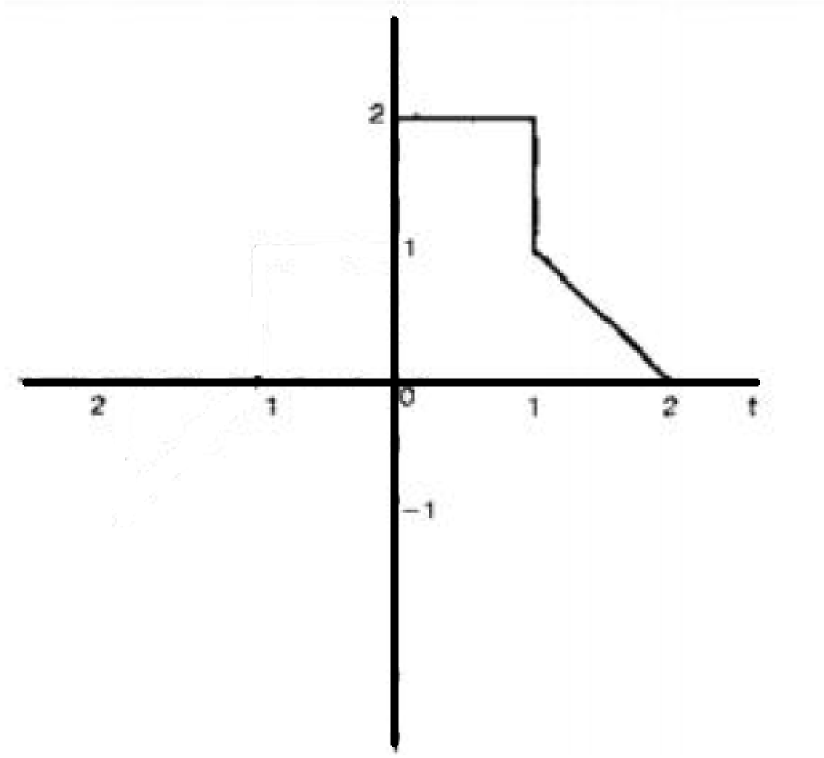

Figure 1: x(t)

1b). Consider the continuous time signal $x(t) = e^{kt}e^{jt}$, $k \in \mathbf{R}$. For what values of k is this signal periodic? Justify your answer.

1c). Determine if the following system is invertible. If it is, construct the inverse system. If it is not, find two input signals to the system that have same output.

$$y[n] = (x[n-1])^2$$

Problem 2

2a) For the discrete-time system described below with input $x[n]$ and output $y[n]$, determine if it is :
1. Linear or Non linear, 2. Causal or Non causal, 3. Time Variant or Time Invariant, 4. Stable or Unstable.

$$y[n] = 5(n-1)x[n] + 2$$

.

2b)

a Find $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt$

b Find $\int_{-\infty}^{\infty} x(\tau)\delta(t+t_0-\tau)d\tau$

c Find $\int_{-\infty}^{\infty} x(\tau)\delta(t+\tau)d\tau$

2c) A discrete-time LTI system with input $x_1[n]$ (shown in Figure 2a) produces the output $y_1[n]$ (shown in Figure 2b). Find the output produced by this system for the input $x_2[n]$ (shown in Figure 2c).

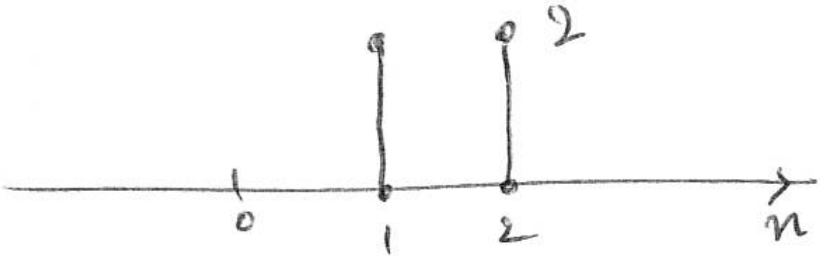

Fig a : $x_1[n]$

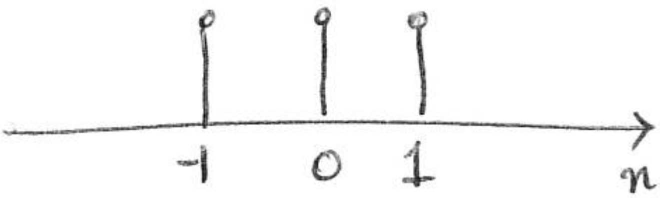

Fig b : $y_1[n]$

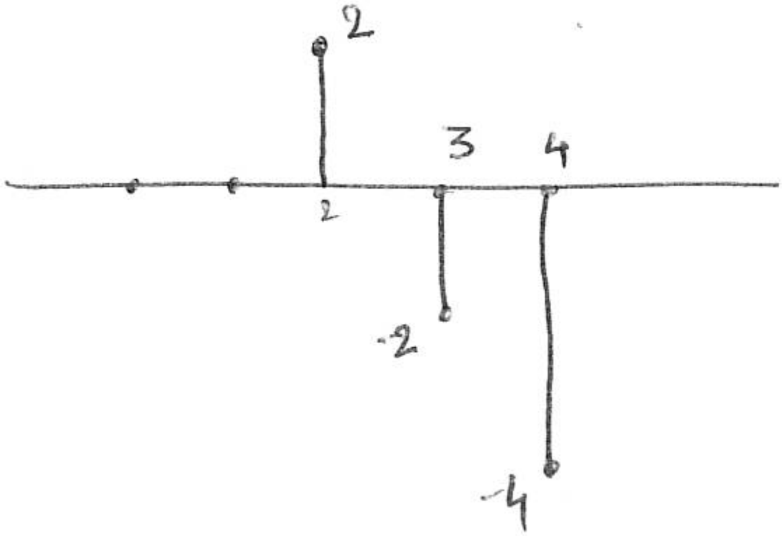

Fig c: $x_2[n]$

Problem 3

Solve the following linear constant coefficient differential equation assuming initial rest, using **two** different methods. First, solve it by finding the homogeneous and particular solutions. Then, solve it by using the frequency domain representation.

$$\frac{dy(t)}{dt} + 4y(t) = x(t)$$
$$x(t) = 2e^{-5t}u(t)$$

Problem 4

Let x[n] be a discrete-time periodic signal with fundamental period $N = 30$. The discrete Fourier Series coefficients of x[n] are a_k as given below:

$$a_k = \begin{cases} 5, k = 0 \\ -3, 1 \leq k \leq 9 \\ 1, 10 \leq k \leq 19 \\ 2, 20 \leq k \leq 24 \\ 0, 25 \leq k \leq 29 \end{cases}$$

a Find $\sum_{k=60}^{89} x[n]$

b Calculate the average power of the signal.

c Find x[240]

d find x[126]

e Calculate Fourier Series Coefficients of x[n+3]

Problem 5

5a) From the Fourier Transform shown in Figure 3, evaluate the following

a $E = \int\limits_{-\infty}^{\infty} |x(t)|^2$

b $D = \frac{d}{dt}x(t)|_{t=0}$

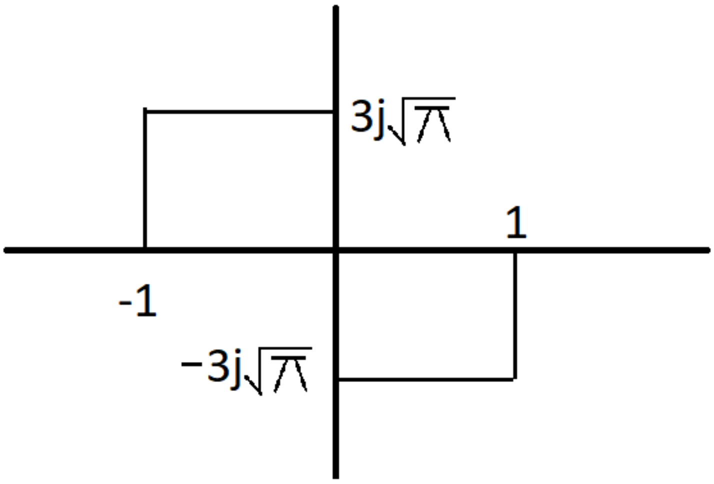

Figure 3: $X_2(j\omega)$

5b) What are two practical issues regarding the implementation of an ideal Low Pass Filter? Give an example approximation of the Low Pass Filter (show both time and frequency domain representations).

Problem 6

6a) Find the Inverse Fourier Transform of $\frac{\sin^2(2\omega)}{\omega^2}$.

6b) Consider an LTI system described by the following differential equation:

$$\frac{dy(t)}{dt} + ay(t) = x(t).$$

a Find $H(j\omega)$

b Find $h(t)$.

c Comment on the functionality of the system when a is real and $a > 0$.

d What impact does changing the value of a have on the system?

Problem 7

a Write the synthesis and analysis equations for Continuous Time Fourier Series(CTFS) and Discrete Time Fourier Transform(DTFT).

b Consider the equations for DTFS. Let a_k be the DTFS coefficients for the discrete periodic signal x[n]. We know that the sequence a_k is periodic. So, find the Fourier Series coefficients for a_k.

c In the Continuous Time Fourier Series equations, if we switch the time and frequency variables, what is the condition under which there is duality?

d What are the Fourier Series coefficients for the periodic signal $X(e^{j\omega})$(which is the DTFT of a discrete time signal $x[n]$) if we switch time and frequency?

Problem 8

Given the following details about a discrete-time signal x[n] with DTFT $X(e^{j\omega})$, find x[n].

a $x[n] = 0$ for $n > 0$

b $x[0] < 0$

c $\text{Im}(X(e^{j\omega})) = \sin(4\omega) - \sin(8\omega)$

d $\frac{1}{2\pi} \int\limits_{-\infty}^{\infty} |X(e^{j\omega})|^2 d\omega = 11$

Problem 9

Find the most efficient downsampling and/or upsampling scheme for the signal $x[n]$ whose Fourier Transform $X(e^{j\omega})$ is as shown in Figure 4. Show the steps involved clearly in frequency domain, and describe what kind of changes the steps introduce for the time domain signal.

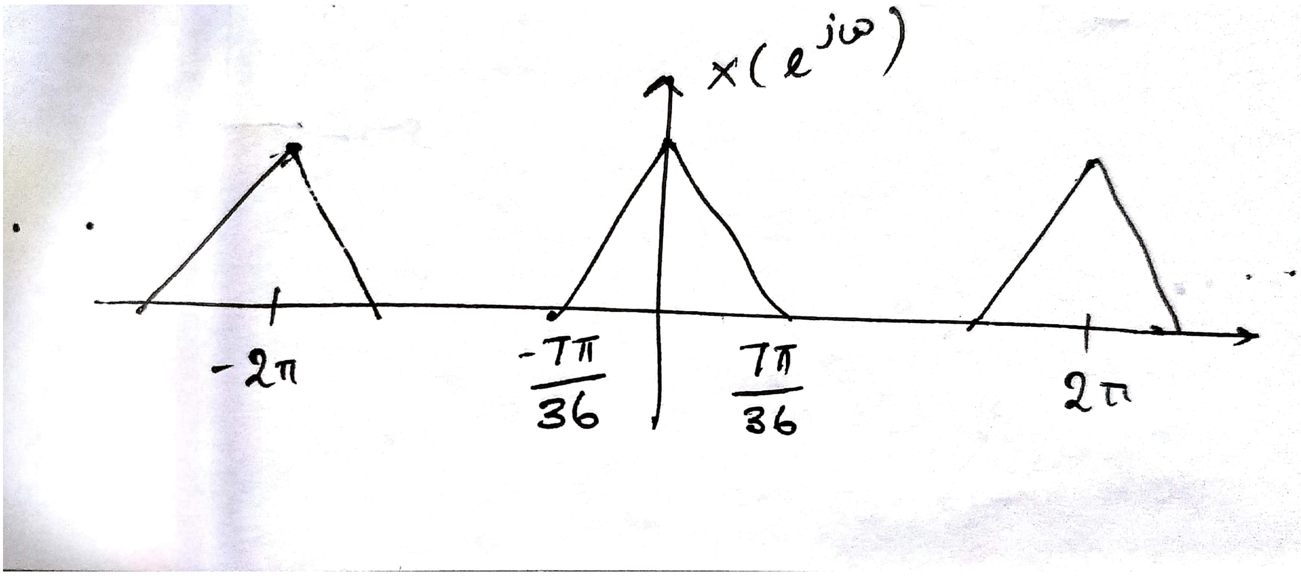

Figure 4

Problem 10

Find the Lapalce transform of the continuous-time signal $x(t) = e^{-2t}u(t) + e^{-t}\cos(3t)u(t)$, and answer the following.

a What are the poles of system?

b What is the Region of Convergence (RoC)?

c Does the Fourier Transform of this signal exist?