ECE - 301 - FALL 2018

HW - 1 SOLUTIONS.

Prof. Aly El Gamal.

1. a) ✶ $x_e(t) = \dfrac{x(t) + x(-t)}{2}$

$$x_e(-t) = \frac{x(-t) + x(t)}{2} = x_e(t).$$

$x_e(-t) = x_e(t)$. Therefore $x_e(t)$ is even.

✶ $x_o(t) = \dfrac{x(t) - x(-t)}{2}$

$$x_o(-t) = \frac{x(-t) - x(t)}{2} = -x_o(t).$$

$x_o(-t) = -x_o(t)$. Therefore $x_o(t)$ is odd.

b) (i) $x[n] = n^2 + \cos\left(\frac{\pi}{2}|n|\right)$

$$x[-n] = (-n)^2 + \cos\left(\frac{\pi}{2}|-n|\right)$$

$$= n^2 + \cos\left(\frac{\pi}{2}|n|\right) = x[n].$$

$x[-n] = x[n]$. Therefore $x[n]$ is even.

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(ii) $y(t) = \sin\left(\frac{3\pi}{4}t\right) + t^3$

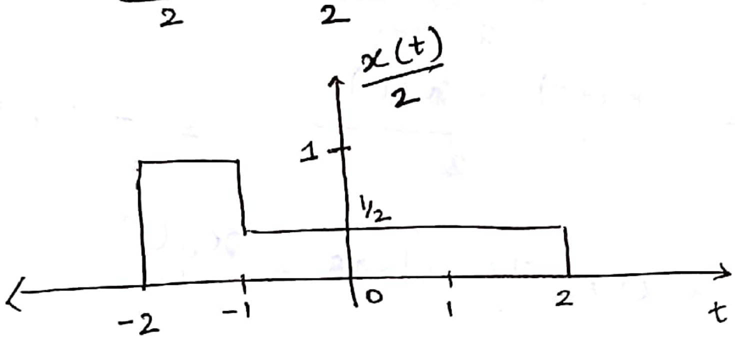
$$y(-t) = \sin\left(\frac{3\pi}{4}(-t)\right) + (-t)^3$$

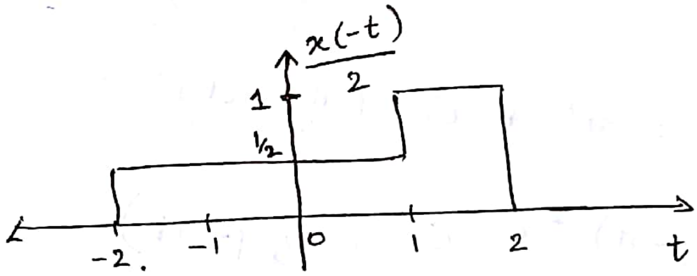
$$= -\sin\left(\frac{3\pi}{4}t\right) - t^3 = -\left[\sin\left(\frac{3\pi}{4}t\right) + t^3\right]$$

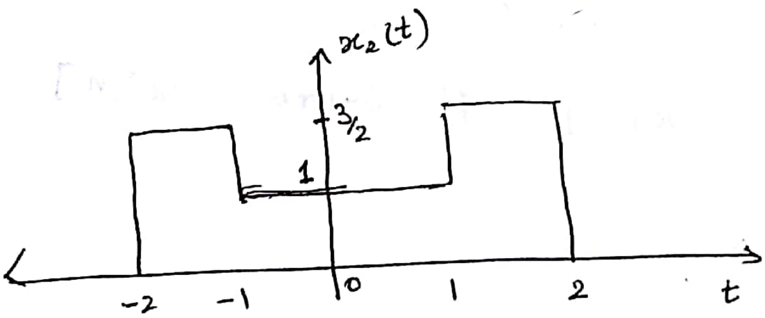
$$= -y(t).$$

$y(-t) = -y(t)$. Therefore $y(t)$ is odd.

c) I. (i) $x_e(t) = \frac{x(t)}{2} + \frac{x(-t)}{2}$



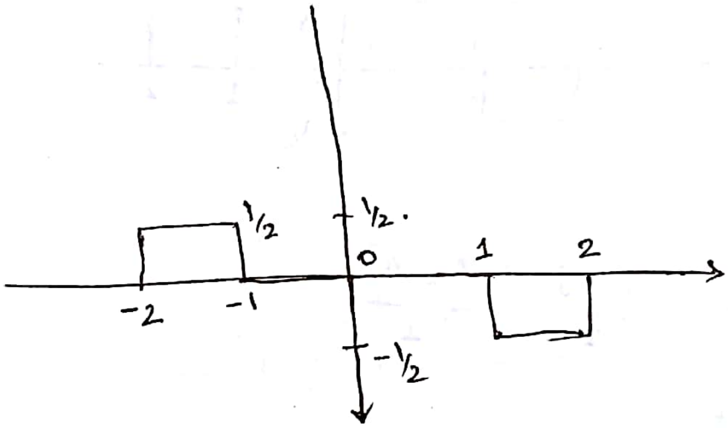




(ii) $x_o(t) = \dfrac{x(t) - x(-t)}{2}$.

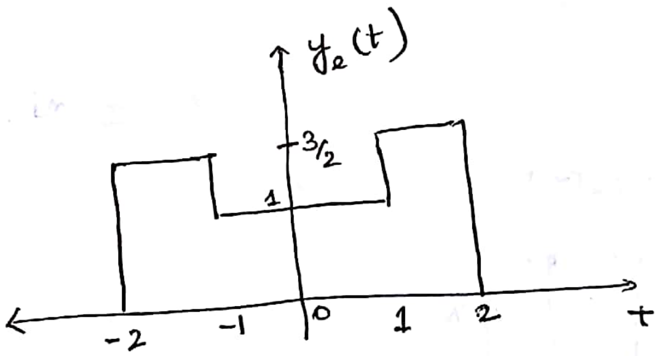
Refer to $\dfrac{x(t)}{2}$ and $\dfrac{x(-t)}{2}$ in (i).

Subtracting those 2 signals $\dfrac{x(t)}{2} - \dfrac{x(-t)}{2}$

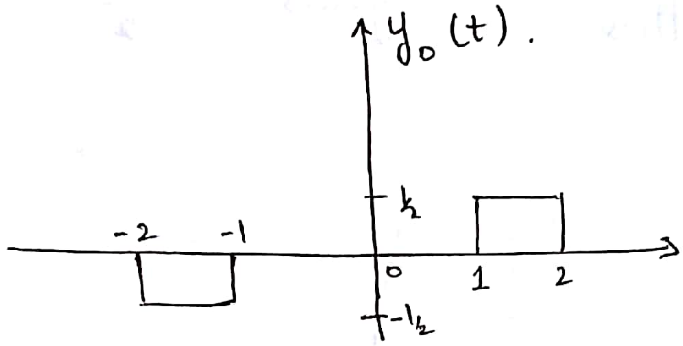


II. Notice $y(t) = x(-t)$.

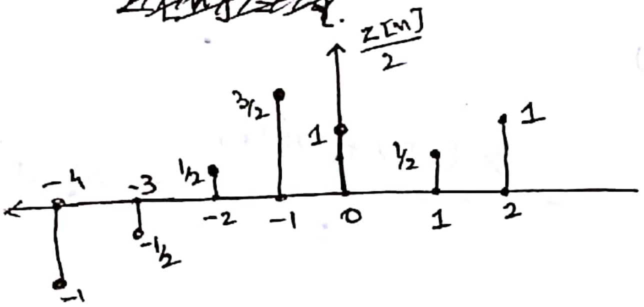
Now $y_e(t) = \dfrac{y(t) + y(-t)}{2} = \dfrac{x(-t) + x(t)}{2} = x_e(t)$.

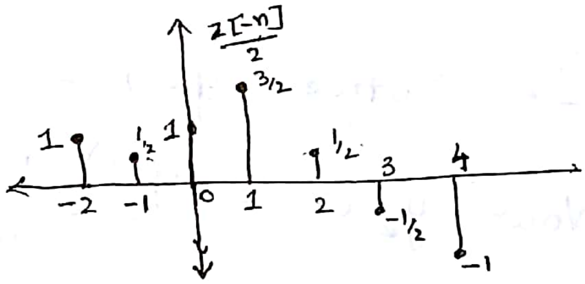


(ii) $y_o(t) = \dfrac{y(t) - y(-t)}{2} = \dfrac{x(-t) - x(t)}{2} = -x_o(t).$

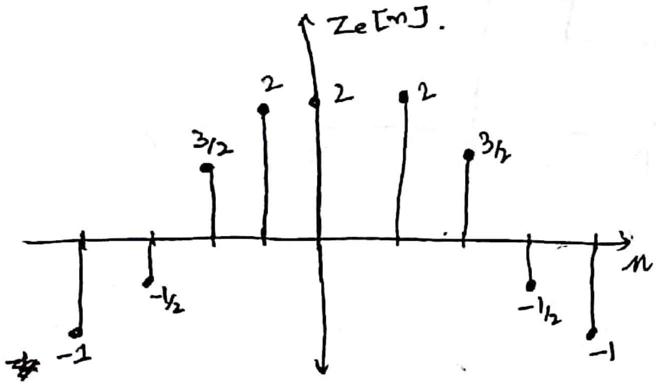


III. $y_e[n] = \dfrac{z[n]}{2} + \dfrac{z[-n]}{2}.$

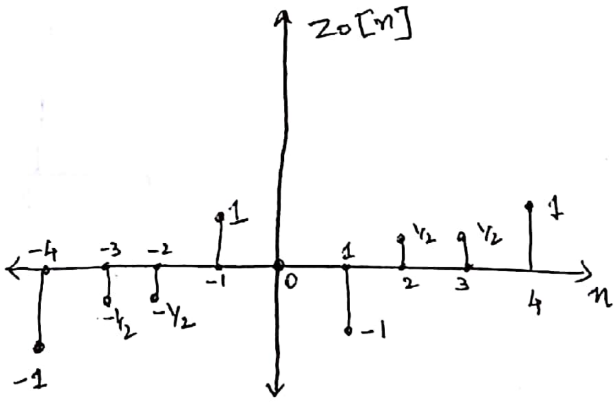




$Z_e[n]$

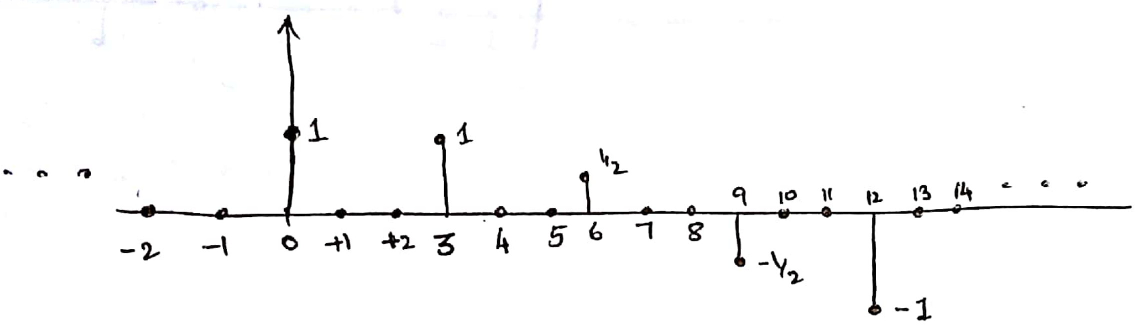


$Z_o[n]$.

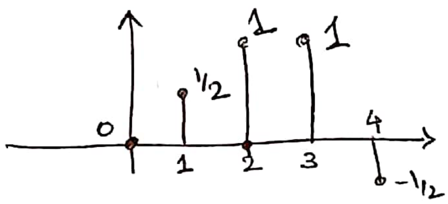


2.

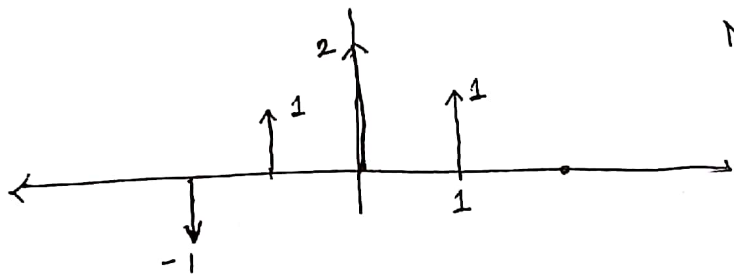
a) (i) $x[-n/3]u[n]$.



(ii) $x[-2n+5]$.



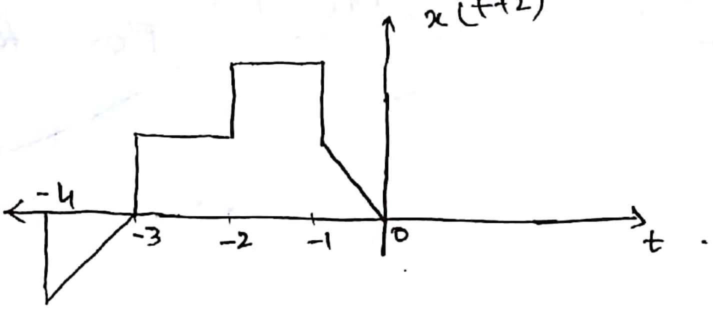
b) (i) $\sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT) \quad T=1$.



Notice: This is not the only correct solution.

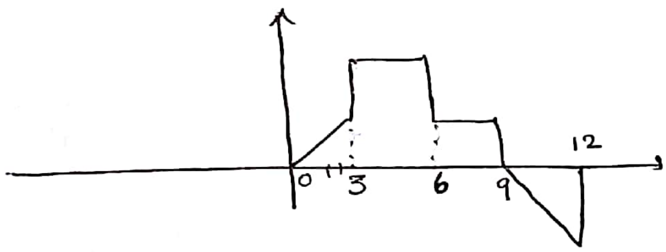
(ii) $x(-t/3+2)\ u(-t)$.

$x(t+2)$:

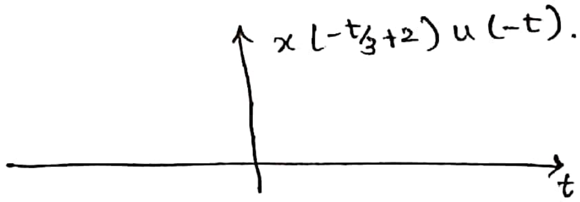


$x(-t+2)$.

$x(-t/3+2)$.



$x(-t/3+2)\ u(-t)$.



3. a) e^{j5t}. Here $\omega_0 = 5$

Periodic.

Period $= \frac{2\pi}{5}$

b). $j e^{5jt} = e^{j(5t)} \cdot e^{j\pi/2} = e^{j(5t+\pi/2)}$. ; $\omega_0 = 5$

Periodic.

Period $= \frac{2\pi}{5}$

c). e^{5t} Not periodic.

$e^{5(t+T)} = e^{5t} e^{5T}$

$e^{5t} \cdot e^{5T} = e^{5t}$ only if $T = 0$. No such $T \neq 0$ exists.

d) e^{-5t}. Not periodic.

Similar arguement as (C)

e). $\cos(3\pi n^3) + e^{j\frac{10\pi n}{5}}$

$\cos(3\pi n^3) = \{\ldots 1, -1, \underset{\substack{\uparrow \\ n=0}}{1}, -1, 1, -1, \ldots\}$.

Periodic with period $N = 2$.

$e^{j\frac{10\pi n}{5}}$: $\omega_0 = \frac{10\pi}{5}$ => Period : $\frac{\omega_0}{2\pi} = \frac{m}{N}$ => $1 = \frac{m}{N}$

$\therefore$ $N = 1$.

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$\therefore$ The period of the whole signal

$N = LCM(2,1) = 2.$

f. $\cos(5\pi t) + \cos(5t)$

$\cos 5\pi t$: Period : $T = \frac{2}{5}$. ← Rational

$\cos 5t$: Period : $T = \frac{2\pi}{5}$. ← Irrational.

Since one period is rational and other is irrational, we cannot find an LCM.
Therefore, the signal is not periodic.

4. a) $y(t) = x(t+2)$. Invertible

$y(t-2) = x(t)$.

b). $y(t) = \sin(2t)$.
Here the output is $\sin(2t)$, whatever the input signal is.
2 signals. $x(t) = 1 \Rightarrow y(t) = \sin(2t)$

$x(t) = \cos 2t. \Rightarrow y(t) = \sin(2t)$.

Not invertible.

c) $y[n] = \frac{1}{n} x[n]$

Notice $y[0] = \infty \quad \forall \; x[0] \neq 0$.

So consider

$x[n] = \{1, 1, 1\} \qquad y[n] = \{-1, \infty, 1\}$.

$x[n] = \{1, 2, 1\} \qquad y[n] = \{-1, \infty, 1\}$.

Not ~~an~~ invertible.

d) $y(t) = \int_0^t x(t)\, dt$

Consider 2 signals.

(i) $x(t) = \begin{cases} 1 & t \neq 0 \\ 0 & t = 0 \end{cases}$ (ii) $x(t) = 1 \;\; \forall t$.

For the inputs $y(t) = t$.

$\therefore$ Not invertible.

e) $y[n] = x[n]\, x[n-5]$.

Consider (i) $x[n] = 1, \forall n \qquad y[n] = 1$

(ii) $x[n] = -1, \forall n \qquad y[n] = 1$.

$\therefore$ Not invertible.

5. a) $E_\infty = \sum_{n=-\infty}^{\infty} \left| \sin \frac{\pi}{8} n \right|^2$

Note:
$\cos 2\theta = 1 - 2\sin^2\theta$

$$= \sum_{n=-\infty}^{\infty} \sin^2 \frac{\pi}{8} n = \sum_{n=-\infty}^{\infty} \frac{\left(1 - \cos \frac{2\pi}{8} n\right)}{2}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2} - \underbrace{\frac{1}{2} \sum_{n=-\infty}^{\infty} \cos \frac{2\pi}{8} n}_{\to 0}$$

$$E_\infty = \infty$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| \sin \frac{\pi}{8} n \right|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\sin^2 \frac{\pi}{8} n \right)$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1 - \cos \frac{2\pi}{8} n}{2}$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\frac{1}{2} - \frac{\cos \frac{2\pi}{8} n}{2} \right)$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1}{2} - \underbrace{\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{\cos \frac{2\pi}{8} n}{2}}_{\to 0}$$

$$= \lim_{N \to \infty} \frac{1}{\cancel{2N+1}} \cdot \frac{1}{2} \cdot \cancel{(2N+1)} = \frac{1}{2}$$

$$P_\infty = 1/2$$

b. $x(t) = e^{j(3t+\pi/4)}$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left| e^{j(3t+\pi/4)} \right|^2 dt$$

$$= \int_{-\infty}^{\infty} 1 . dt = \infty$$

$$E_\infty = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e^{j(3t+\pi/4)}|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1 . dt = \lim_{T\to\infty} \frac{1}{2T} . 2T = 1$$

$$P_\infty = 1.$$

c). $x[n] = \left(\frac{1}{3}\right)^n u[n]$.

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} \left(\left(\frac{1}{3}\right)^n u[n]\right)^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{2n} = \frac{1}{1-(1/3)^2} = \frac{9}{8}$$

Note: Sum of infinite GP

$a, ar, ar^2, \ldots$

$= \frac{a}{1-r}$

if $|r| < 1$.

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\left(\frac{1}{3}\right)^n u[n]\right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{3}\right)^{2n}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot \frac{1-\left(\frac{1}{3}\right)^{2(N+1)}}{1-\left(\frac{1}{3}\right)^2}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot \frac{1-\left(\frac{1}{3}\right)^{2(N+1)}}{\frac{8}{9}}$$

$$= \frac{\frac{9}{8}\lim_{N\to\infty}\left(1-\left(\frac{1}{3}\right)^{2(N+1)}\right)}{\lim_{N\to\infty} 2N+1} = \frac{9}{8}\cdot\lim_{N\to\infty}\frac{1}{2N+1} = 0$$

$P_\infty = 0$.

d.

$$x[n] = j\sin\left(\frac{\pi}{6}n\right).$$

Note:

- $|j| = 1$
- $\cos 2\theta = 1 - 2\sin^2\theta$

$$E_\infty = \sum_{n=-\infty}^{\infty} \left| j\sin\left(\frac{\pi}{6}n\right)\right|^2$$

$$= \sum_{n=-\infty}^{\infty} \sin^2 \frac{\pi}{6}n$$

$$= \sum_{n=-\infty}^{\infty} \frac{\left(1 - \cos \frac{2\pi}{6}n\right)}{2}$$

$= \infty$ [Similar to part (a)].

$P_\infty = \frac{1}{2}$ [Similar to (a)].