HW-2 solns.

1. a) $y[n] = x^2[n]$.

(i) $2x[n] \rightarrow$ [System] $\rightarrow 4x^2[n]$

Not Linear

(ii). Depends only on present value [or past]

$\therefore$ Causal

(iii). $x[n] \rightarrow$ [System] $\rightarrow y[n] = x^2[n]$.

$x_1[n] = x[n-N] \rightarrow$ [System] $\rightarrow y_1[n] = x^2[n-N] = y[n-N]$

$\therefore$ Time invariant.

(iv) $|x[n]| \le B \rightarrow |y[n]| \le B^2$

$\therefore$ Stable.

b) $y[n] = 5(n-1)\,x[n] + 2$.

~~(i) y[n] depends on n~~

~~$\therefore$ Cannot~~

(i) $x[n] \rightarrow$ [System] $\rightarrow y[n] = 5(n-1)x[n] + 2$

$a\,x[n] \rightarrow$ [System] $\rightarrow y_1[n] = 5a(n-1)\,x[n] + 2$
$\neq a y[n]$.

$\therefore$ Not linear

(ii). Depends only on present $x[n]$

$\therefore$ Causal

(iii). $y[n]$ depends on n

$\therefore$ ~~Ti~~ Time Variant.

(iv). $x[n] = 1 \Rightarrow \lim_{n \to \infty} y[n] = \infty$

$\therefore$ Not stable.

c) $y(t) = x(t+1) \cos(6\pi(t-1))$.

(i). $x(t) \longrightarrow$ [System] $\longrightarrow y(t) = x(t+1) \cos(6\pi(t-1))$.

$a x_1(t) \longrightarrow$ [System] $\longrightarrow y_1(t) = a\, x(t+1) \cos(6\pi(t-1))$

$= a\, y(t)$.

$x_1(t) + x_2(t) \longrightarrow$ [System] $\longrightarrow y_2(t) = (x_1(t+1) + x_2(t+1)) \cos(6\pi(t-1))$

$= x_1(t+1) \cos(6\pi(t-1)) + x_2(t+1) \cos(6\pi(t-1))$

$\therefore$ Linear.

(ii) $y(t)$ Depends on $x(t+1)$ [Future values].

$\therefore$ Not Causal

(iii) $y(t)$ has $\cos(6\pi(t-1))$ which depends on t

$\therefore$ Time variant.

(iv) Let $|x(t)| \leq B$

We know $|\cos(6\pi(t-1))| \leq 1$.

$\Rightarrow \quad |y(t)| \leq B$

$\therefore$ Stable.

d) $y(t) = e^{-10t} x(t)$.

(i) $a x_1(t) + b x_2(t) \rightarrow$ System $\rightarrow y'(t) = e^{-10t}(a x_1(t) + b x_2(t))$

$= a x_1(t) e^{-10t} + b x_2(t) e^{-10t}$

$\therefore$ Linear.

(ii) Depends only on present values

$\therefore$ Causal

(iii) e^{-10t} depends on 't'

$\therefore$ Time variant

(iv) $x(t) = 1$.

$\lim_{t \to -\infty} y(t) = \infty$

e) $y(t) = e^{-10jt} x(t)$.

(i) Similar to (d). Linear

(ii). Depends only on present values of x

$\therefore$ Causal

(iii). Similar to (d). Time Variant

(iv). Let $|x(t)| \leq B$.

$|e^{-10jt}| = 1$.

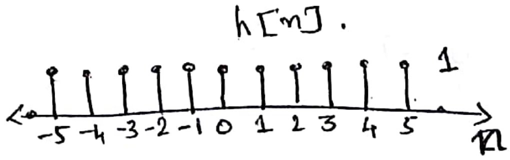
$\Rightarrow |y(t)| \leq B$.

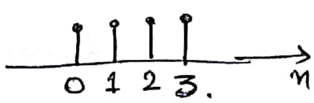
$\therefore$ Stable.

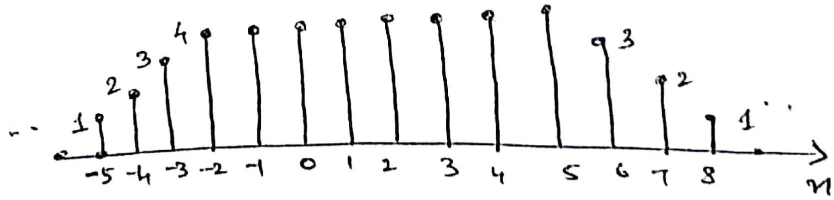
2. $y[n] = \sum_{i=n-5}^{n+5} x[i]$.

$y[n] = \sum_{i=-5}^{5} x[n+i]$.

a) $x[n] = \delta[n] \Rightarrow h[n] = \sum_{i=-5}^{5} \delta[n+i]$.



b) $x[n] =$ 

$y[n] =$ 

c) (i) $y[n]$ depends on future values

$\therefore$ Not Causal.

(ii) Not memoryless.

Uses more than just the present value of x $x[n]$.

(iii). Let $|x[n]| \leq B$

$\Rightarrow |y[n]| \leq 11 B$

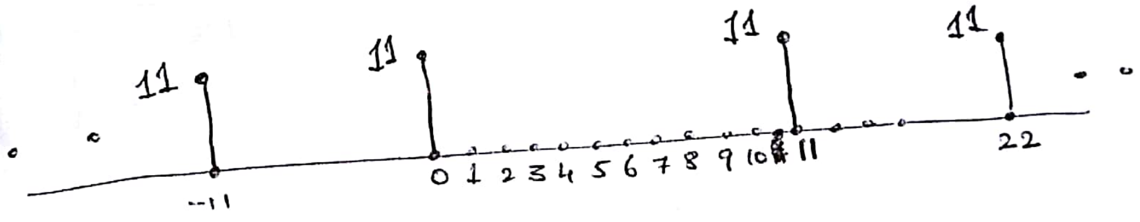
$\therefore$ stable

(iv). Consider

$x[n] = 1 \quad \forall n.$

$y[n] = 11 \quad \forall n.$

now consider the signal.

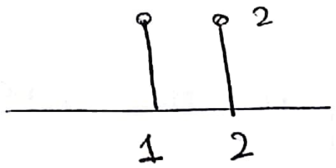


$y[n] = 11 \quad \forall n.$

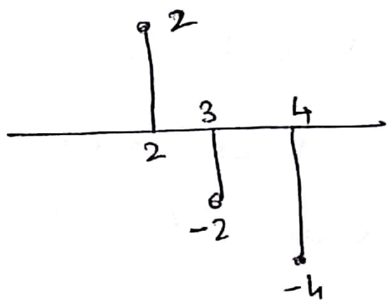
$\therefore$ System is not invertible.

3.

a). $x_1[n]$



$x_2[n]$

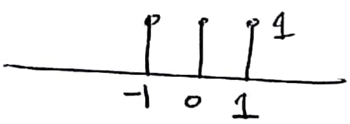


Notice: $x_2[n] = x_1[n-1] - 2x_1[n-2]$.

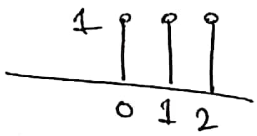
System is LTI

$\therefore \; y_2[n] = y_1[n-1] - 2y_1[n-2]$.

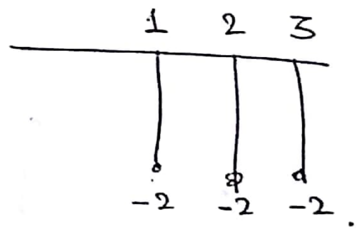
$y_1[n]$



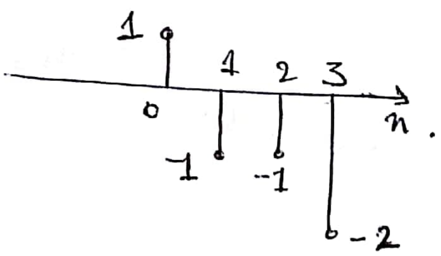
$y_1[n-1]$



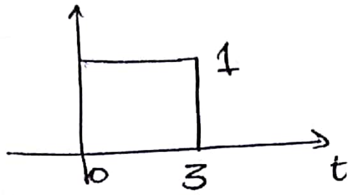
$-2y_1[n-2]$



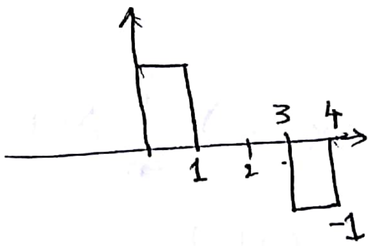
$y_2[n]$



b). $x_1(t)$



$x_2(t)$.

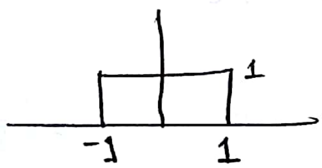


Notice:

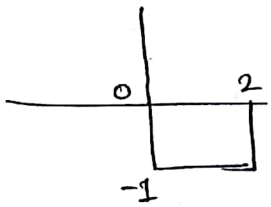
$x_2(t) = x_1(t) - x_1(t-1)$.

System is LTI $\Rightarrow$ $y_2(t) = y_1(t) - y_1(t-1)$.

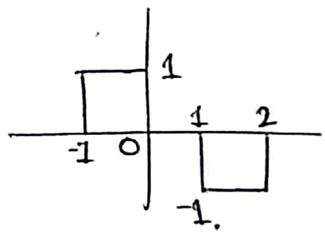
$y_1(t)$.



$-y_1(t-1)$



$y_2(t)$



4. $y(t) = x(t) * h(t)$.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

$$x(t+t_1) * h(t-t_2) = \int_{-\infty}^{\infty} x(\tau+t_1)\, h(t-t_2-\tau)\, d\tau$$

~~assume~~ (Let $\tau' = \tau + t_1 \Rightarrow \tau = \tau' - t_1$)

$$= \int_{-\infty}^{\infty} x(\tau')\, h(t-t_2-\tau'+t_1)\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau')\, h((t-(t_2-t_1))-\tau')\, d\tau.$$

$$\int_{-\infty}^{\infty} x(\tau')\, h((t-(t_2-t_1)) - \tau') = \underline{y(t-(t_2-t_1))}.$$

b) $$\int_{-\infty}^{\infty} x(t)\, \delta(t-t_0)\, dt = \int_{-\infty}^{\infty} x(t_0)\, \delta(t-t_0)\, dt$$

$$= x(t_0) \int_{-\infty}^{\infty} \delta(t-t_0)\, dt$$

$$= x(t_0)$$

c) $$\int_{-\infty}^{\infty} x(\tau)\, \delta(t+t_0-\tau)\, d\tau = \int_{-\infty}^{\infty} x(t+t_0)\, \delta(t+t_0-\tau)\, d\tau$$

$$= x(t+t_0) \int_{-\infty}^{\infty} \delta(t+t_0-\tau)\, d\tau$$

$$= x(t+t_0).$$

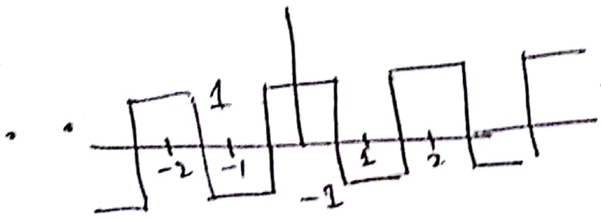
d). $$\int_{-\infty}^{\infty} x(\tau)\, \delta(t+\tau)\, d\tau = \int_{-\infty}^{\infty} x(-t)\, \delta(t+\tau)\, d\tau$$

$$= x(-t) \int_{-\infty}^{\infty} \delta(t+\tau)\, d\tau$$

$$= x(-t).$$

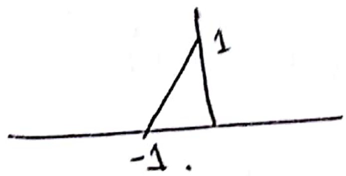
5.

a) x(t).



... Peridic T=2.

y(t)



$z(t) = x(t) * y(t).$

$x(t) = \sum_{k=-\infty}^{\infty} x_p(t-KT)$

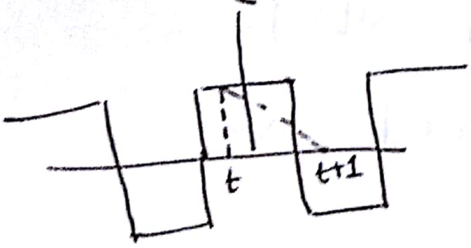
$z(t) = \left(\sum_{k=-\infty}^{\infty} x_p(t-KT)\right) * y(t).$

$\left(\sum_{k=-\infty}^{\infty} x_p(t-KT) * y(t)\right)$

$z(t) = \sum_{k=-\infty}^{\infty} (x_p(t-KT)$

∵ x(t) is periodic z(t) is also periodic with T=2. So it is enough if we find z(t) for one period. Let us find for $-0.5 \le t \le 1.5$.

(i) $-0.5 \le t \le 0.5$



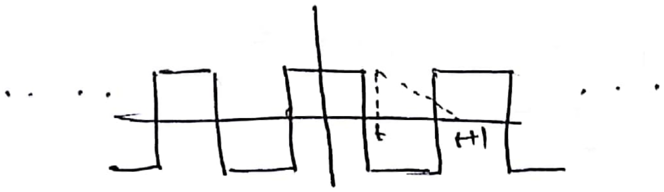
$y(t) = \frac{1}{2} - 2.\left(\frac{1}{2}(t+1-0.5)^2\right)$

$= \frac{1}{2} - (t+0.5)^2$

$= \frac{1}{2} - t^2 - t - \frac{1}{4}$

$y(t) = -t^2 - t + 1/4 \quad -0.5 \le t \le 0.5$

(ii) $0.5 \le t \le 1.5$



$$y(t) = -\frac{1}{2} + 2\left(\frac{1}{2}(t+1-1.5)^2\right)$$

$$= -1/2 + (t-0.5)^2$$

$$= -1/2 + t^2 - t + 0.25$$

$$= t^2 - t - 1/4$$

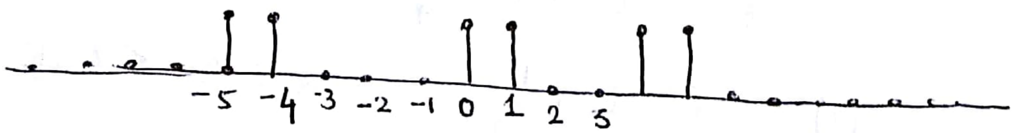
For one period $y(t) = \begin{cases} -t^2 - t + 1/4 & -0.5 \le t \le 0.5 \\ t^2 - t - 1/4 & 0.5 \le t \le 1.5 \end{cases}$

b). $x[n] = \delta[n+5] + \delta[n-4] + \delta[n]$.

$y[n] = u[n] - u[n-2]$. [Note: $x[n] * \delta[n-N] = x[n-N]$]

$$x[n] * y[n] = u[n+5] + u[n-4] + u[n] - u[n+3] - u[n-6] - u[n-2].$$

$x[n] * y[n]$.



c. $x_1[n] = \left(\frac{1}{3}\right)^n u[n]$.

$x_2[n] = u[n]$

$x_3[n] = \delta[n-2]$.

$x_1[n] * x_2[n] * x_3[n]$

$= x_1[n] * [x_2[n] * x_3[n]]$

$= \left(\left(\frac{1}{3}\right)^n u[n]\right) * [u[n] * \delta[n-2]]$

$= \left(\left(\frac{1}{3}\right)^n u[n]\right) * [u[n-2]]$.

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]\, y[n-k].$$

$x[n] = \left(\frac{1}{3}\right)^n u[n] \qquad y[n] = u[n-2]$.

$$\Rightarrow \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^k u[k] \cdot u[n-2-k]$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k u[n-2-k]. = 0 \quad \text{if} \quad n < 2$$

For $n \geq 2$. $\displaystyle\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k u[n-2-k] = \sum_{k=0}^{n-2} \left(\frac{1}{3}\right)^k$

$$= \frac{1 \cdot (1-(1/3)^{n-1})}{1-1/3}$$

$$= \frac{3}{2} \cdot (1-(1/3)^{n-1}).$$

$$x_1[n] * x_2[n] * x_3[n] = \begin{cases} 0 & n < 2 \\ \frac{3}{2} \cdot (1-(1/3)^{n-1}) & n \geq 2 \end{cases}$$