

Fall 2018 - Problem Set 4
ECE 301: Signals and Systems

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Due Date : October 26, 2018

Instructions

1. Please write clearly and legibly.
2. Your solutions must include detailed steps and/or explanations. Do not simply state the answer.
3. Write your full name (first, last), PUID on your homework submission.
4. All problems carry almost equal weight.

Problem 1

For each of the Fourier Transforms shown in figure 1 and 2, evaluate the following

a $E = \int_{-\infty}^{\infty} |x(t)|^2$

b $D = \frac{d}{dt}x(t)|_{t=0}$

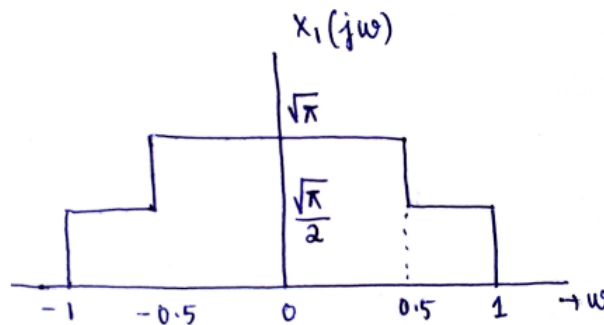


Figure 1: $X_1(j\omega)$

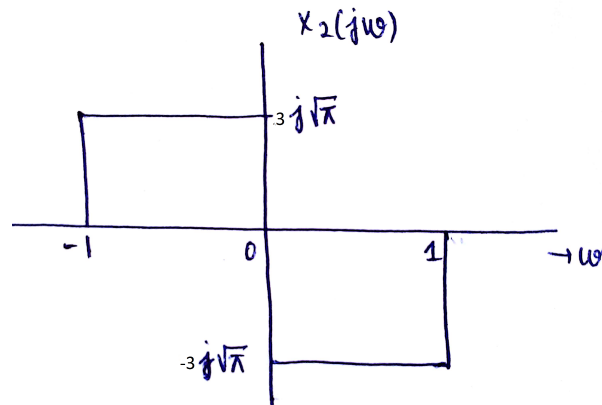


Figure 2: $X_2(j\omega)$

Problem 2

What are the practical considerations while designing a Low Pass Filter. Given an example of the approximation of the Fourier Transform of the Low Pass Filter.

Problem 3

Consider the modulation system as shown below in figure 3. The input signal is $x(t)$ and has Fourier Transform $X(j\omega)$ that is zero for $|\omega| > \omega_M$ as shown in figure 4. The output of the system is $y(t)$.

- Sketch $Y(j\omega)$ which is the Fourier transform of $y(t)$. Show your work in detail and give explanations wherever necessary.
- Is $x(t)$ real or purely imaginary? Is $x(t)$ even or odd signal?
- Is $y(t)$ real or purely imaginary? Is $y(t)$ even or odd signal?
- For the demodulation system shown in figure 5, find the value of the constant 'c' which is required to reconstruct $x(t)$ accurately. Show your work in detail and give explanations wherever necessary

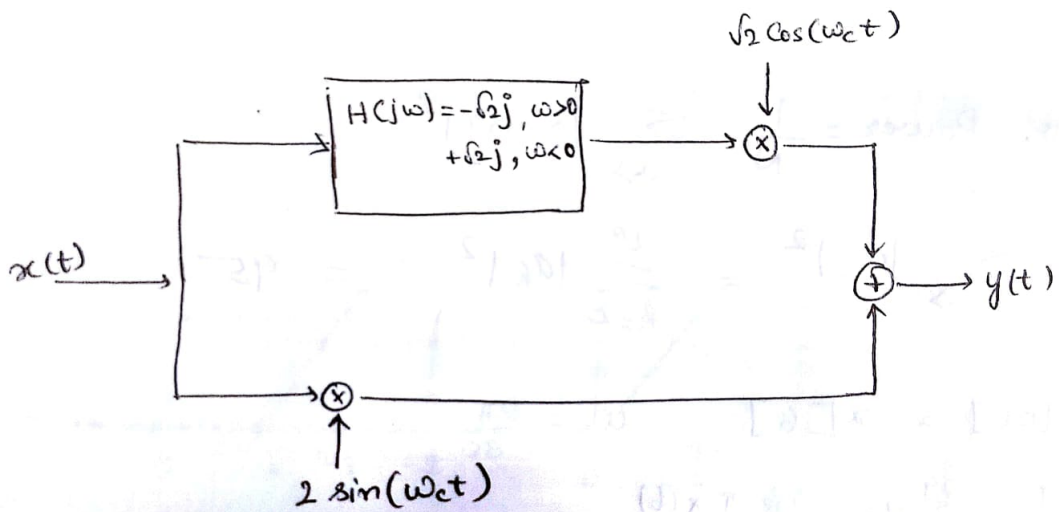


Figure 3: Modulation System

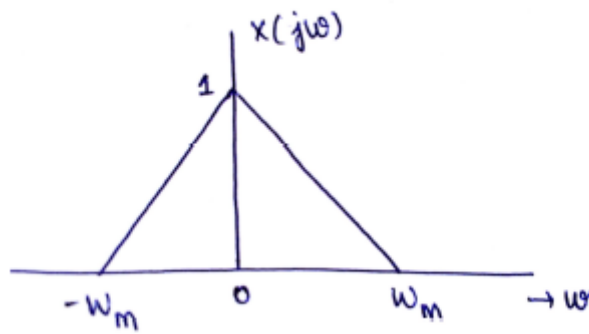


Figure 4: $X_2(j\omega)$

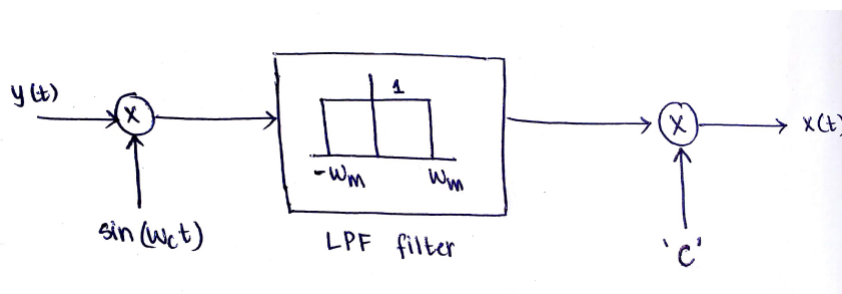


Figure 5: Demodulation System

Problem 4

- a Given $X(j\omega) = \frac{5}{2} \frac{\sin(4\omega)}{\omega}$, find $x(t)$.
- b Let $Y(j\omega)$ be obtained by multiplying $X(j\omega)$ with an impulse train as shown in figure 6. Note that $X(j\omega)$ is as given in part (a). Find $y(t)$.

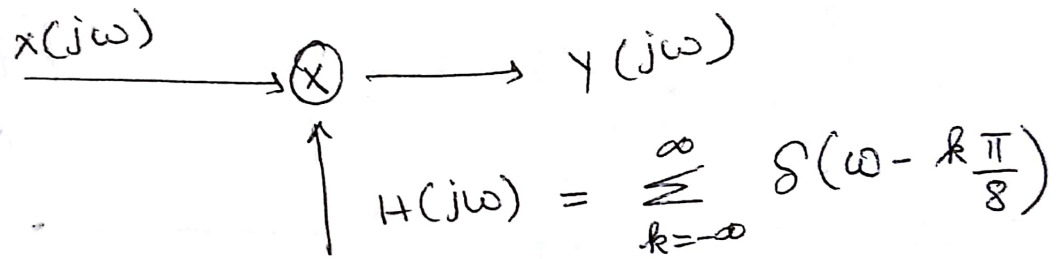


Figure 6: $x_2(t)$

Problem 5

- Find the Fourier Transform of the signal $\frac{\sin(2t)\sin(3t)}{t^2}$
- Find the Inverse Fourier Transform of $\frac{\sin^2(2\omega)}{\omega^2}$
- Find the Fourier Transform of the signal in figure 7.

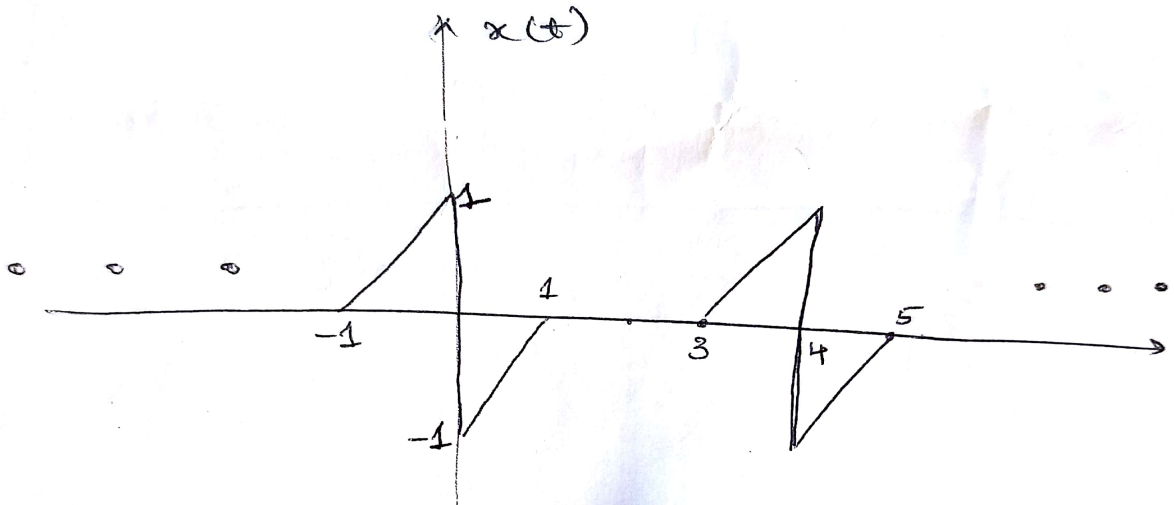


Figure 7: Caption