

Fall 2018 - Problem Set 6

ECE 301: Signals and Systems

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Due Date : November 30, 2018

Instructions

1. Please write clearly and legibly.
2. Your solutions must include detailed steps and/or explanations. Do not simply state the answer.
3. Write your full name (first,last), PUID on your homework submission.
4. All problems carry equal weight.

Problem 1

Consider an LTI system described by the following differential equation

$$\frac{dy(t)}{dt} + ay(t) = x(t).$$

- a Find the frequency response $H(j\omega)$.
- b Find impulse response $h(t)$.
- c Comment on the functionality of the system.
- d What impact does changing the value of a have on the system?

Problem 2

- a Write the synthesis and analysis equations for Continuous Time Fourier Series(CTFS) and Discrete Time Fourier Transform(DTFT).
- b Consider the equations for DTFS. Let a_k be the DTFS coefficients for the discrete periodic signal $x[n]$. We know that the sequence a_k is periodic. So, find the Fourier Series coefficients for a_k .
- c In the Continuous Time Fourier Series equations, if we switch the time and frequency variables, what is the condition under which there is duality?
- d What are the Fourier Series coefficients for the periodic signal $X(e^{j\omega})$ (which is the DTFT of a discrete time signal $x[n]$) if we switch time and frequency.

Problem 3

For the system defined by the differential equation

$$\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 8y(t) = 2x(t)$$

- a Find impulse response $h(t)$ of the system
- b Find the output of the system $y(t)$ for the input signal $x(t) = te^{-4t}u(t)$

Problem 4

a Given the following details about a discrete signal $x[n]$ with DTFT $X(e^{j\omega})$, find $x[n]$.

- i $x[n] = 0$ for $n < 0$
- ii $x[0] > 0$
- iii $\text{Im}(X(e^{j\omega})) = \sin(3\omega) - \sin(6\omega)$
- iv $\frac{1}{2\pi} \int_{-\infty}^{\infty} |X(e^{j\omega})|^2 d\omega = 9$

b Given the DTFT $Y(e^{j\omega})$ of a signal $y[n]$, answer the following questions

$$Y(e^{j\omega}) = \cos(\omega)$$

- i Is $y[n]$ real?
- ii Is $y[n]$ even?
- iii Evaluate $\sum_{n=-\infty}^{\infty} y[n]$
- iv Evaluate $\sum_{n=-\infty}^{\infty} (-1)^n y[n]$

Problem 5

- a Find the DTFT of the signal $x[n] = \frac{1}{4} |n|^{-1}$
- b Find the DTFT of the signal in Figure 1.
- c Find the DTFT of the signal $x[n] = \delta[-n] + 5\delta[n - 2] + \delta[n + 4]$

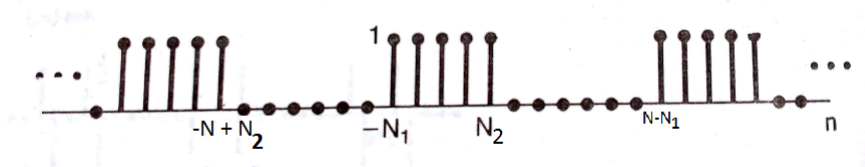


Figure 1: $x[n]$