Fall 2018 - Problem Set 7
ECE 301: Signals and Systems

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Due Date : December 7, 2018

Instructions

1. Please write clearly and legibly.
2. Your solutions must include detailed steps and/or explanations. Do not simply state the answer.
3. Write your full name(first,last), PUID on your homework submission.
4. All problems carry almost equal weight.

Problem 1

Find the output signals $y_1(t)$, $y_2(t)$ $y_3(t)$, and $y_4(t)$, obtained when input x(t) is passed through LTI systems with impulse responses $h_1(t)$, $h_2(t)$, $h_3(t)$, and $h_4(t)$ (as shown in figures 1 and 2), respectively.

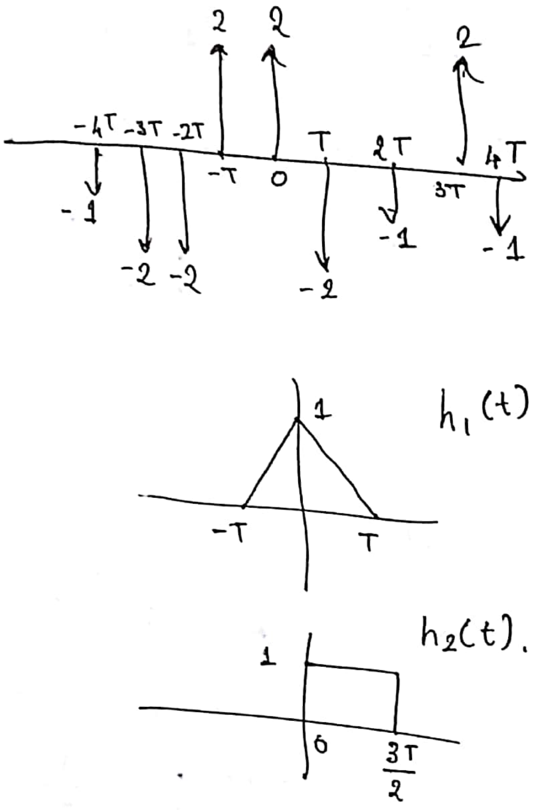

Figure 1

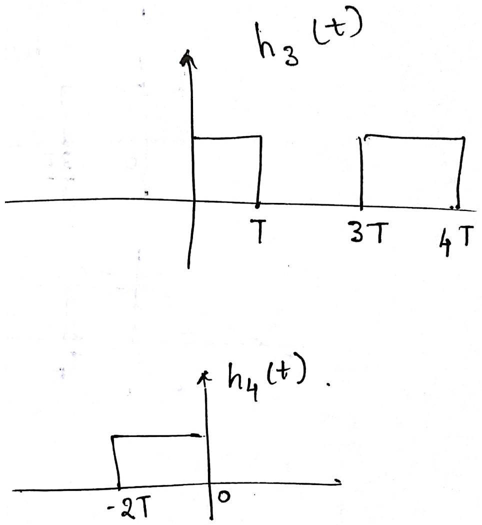

Figure 2

Problem 2

For each of the two signals depicted in Figure 3, can we sample at a frequency $\omega_s = 2\omega_M$, and guarantee perfect reconstruction? Justify your answer.

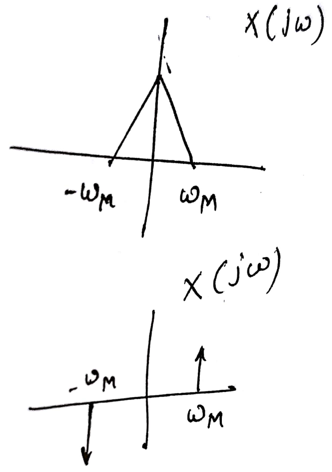

Figure 3

Problem 3

A signal $x(t)$ with continuous time Fourier transform $X(j\omega)$ (as shown in Figure 5) is sampled using an impulse train $p(t)$ with sampling period T. The sampled signal is then converted to a discrete time signal $x_d[n]$ with discrete time fourier transform $X_d(e^j\omega)$. The sequence of operations is as shown in Figure 4. Note that $\omega_s >> 2\omega_m$.

a Sketch $x_p(t)$ where

$$x_p(t) = x(t)p(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t - KT)$$

b Derive and sketch $X_p(e^{j\omega})$

c Derive the relation between $X_p(e^{j\omega})$ and $X_d(e^{j\omega})$.

d Sketch $X_d(e^{j\omega})$.

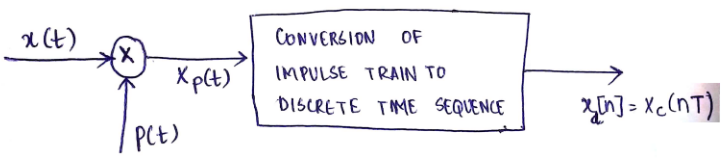

Figure 4

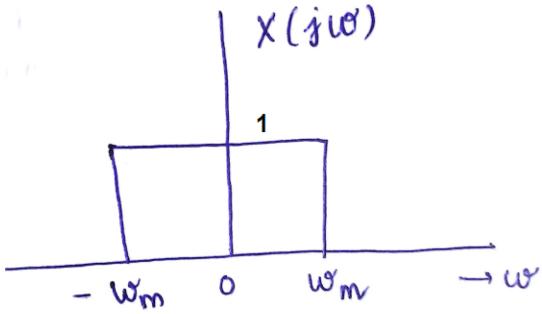

Figure 5

Problem 4

Consider the discrete-time sampling process as given below

$$x_p[n] = \begin{cases} x[n], \text{if n is an integer multiple of the sampling period N} \\ 0, otherwise \end{cases}$$

For a given $X(e^{j\omega})$ as shown in Figure 6, Derive and sketch $X_p(e^{j\omega})$.

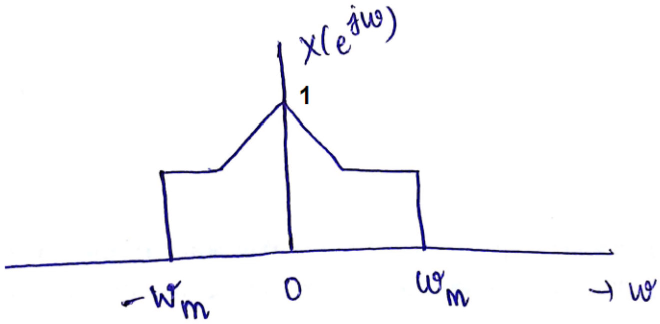

Figure 6

Problem 5

Find the most efficient downsampling and/or upsampling scheme for the signal $x[n]$ whose Fourier Transform $X(e^j\omega)$ is as shown in Figure 7.

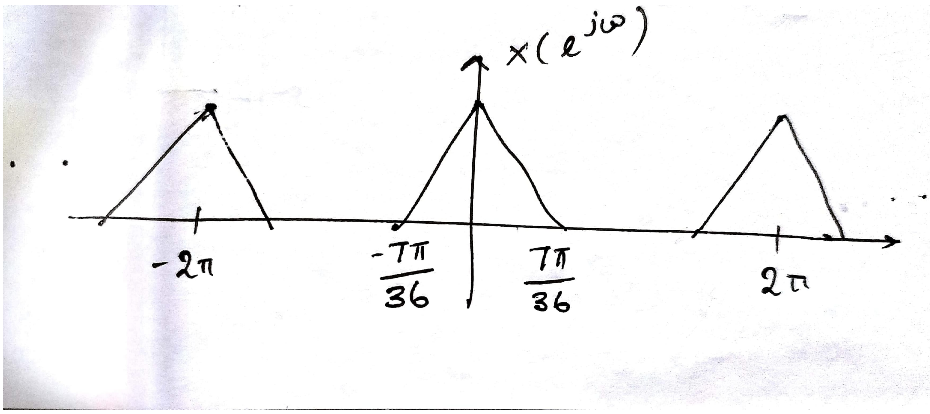

Figure 7