ECE 301 - Lecture #1

Web page:

web.ics.purdue.edu/~elgamala/ECE301-F18

Signals and Systems

Homeworks: 35% 7 Homeworks
each 5%

Team Quizzes: 15% 5 Quizzes
3% each

Extra Bonus Quiz (3%)

Exams (50%): 2 Midterms (12.5% each)
Final 30%

5% Bonus

What the course is about

1st Concept

Duality

* The intercheangeable relationships between time and frequency domains

What's a signal

A function that has an index and a value at every point of the index

In this course, we will always assume that the index is time

n: time index

$x[n]$: signal

$x[0] = 5$ $x[1] = 3$ $x[2] = 4$

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2nd Concept

- why Linearity and Time Invariance are important properties for analyzing systems

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Continuous-time and Discrete-time

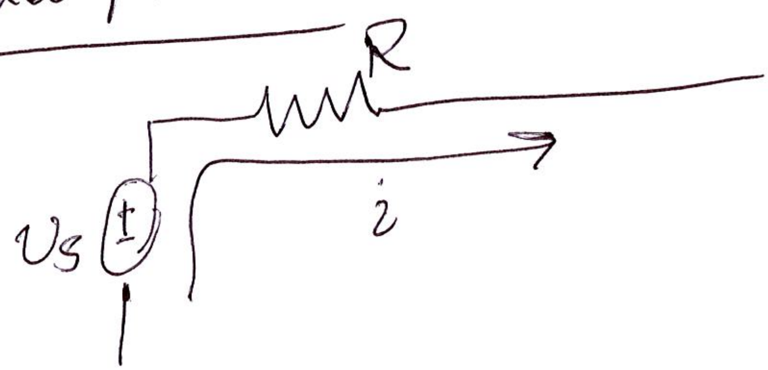
DT-signals: n as the time index

$x[n]$, $n = -5, -4, -3, \cdots, 0, 1, 2, 3, \cdots$

CT-signals: t as the time index

$x(t)$, $t = -0.245,\ 2.6,\ 4.30101$

Example



instantaneous power: $p(t) = i(t)\, v(t)$

$= \frac{1}{R} v^2(t)$

Total energy over $t_1 \le t \le t_2$

$$\int_{t_1}^{t_2} p(t)\, dt = \frac{1}{R} \int_{t_1}^{t_2} v^2(t)\, dt$$

Average power over $t_1 \le t \le t_2$

$$\frac{1}{t_2 - t_1} \frac{1}{R} \int_{t_1}^{t_2} v^2(t)\, dt$$

We will frequently consider signals that take complex values

For a signal $x(t)$, we define the total energy over $t_1 \le t \le t_2$ as

$$\int_{t_1}^{t_2} |x(t)|^2\, dt$$

Average power over $t_1 \leq t \leq t_2$

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

DT- signal $x[n]$

Total energy over $n_1 \leq n \leq n_2$

$$\sum_{n=n_1}^{n_2} |x[n]|^2$$

Average power over $n_1 \leq n \leq n_2$

$$\frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

CT

$$E_\infty \triangleq \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

DT

$$E_\infty \triangleq \lim_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$\boxed{E_\infty < \infty ?}$$

CT

$$P_\infty \triangleq \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

DT

$$P_\infty \triangleq \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Example

$x[n]$: $x[-1] = x[0] = x[1] = 2$

$x[n] = 0$, for all $n \neq -1, 0, 1$

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-1}^{1} |x[n]|^2$$

$$= |x[-1]|^2 + |x[0]|^2 + |x[1]|^2$$

$$= 2^2 + 2^2 + 2^2 = 12$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$N = 0 \Rightarrow \frac{1}{1} |x[0]|^2 = 4$$

$$N = 1 \Rightarrow \frac{1}{3} \left[|x[-1]|^2 + |x[0]|^2 + |x[1]|^2 \right]$$

$$= 4$$

$$N=2 \Rightarrow \frac{1}{5}\sum_{n=-2}^{2}|x[n]|^2$$

$$= \frac{1}{5}\cdot 12 = \frac{12}{5} = 2.4$$

$$N=3 \Rightarrow \frac{1}{7}\sum_{n=-3}^{3}|x[n]|^2$$

$$= \frac{12}{7} < 2$$

$P_\infty = 0$ $\quad\quad E_\infty = 12$

Can we infer a general rule?

If $E_\infty < \infty$ then $P_\infty = 0$

$\boxed{\text{If } E_\infty = \infty \text{ then } P_\infty > 0\text{ ?}}$

Bonus Exercise (Due Friday Aug. 31st)

(0.2 %)

If yes, Justify

If no, give example

(DT and CT)