ECE 301 - Lecture #10

Invertibility of LTI Systems

An LTI System with impulse response $h(t)$ is invertible if there is a signal $h_1(t)$ such that $h(t) * h_1(t) = \delta(t)$

Examples

$$h(t) = \delta(t - t_0)$$

$$y(t) = x(t) * h(t)$$

$$= \int x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{\tau=-\infty}^{\infty} x(\tau)\, \delta(t - t_0 - \tau)\, d\tau$$

$$= x(t - t_0)$$

We know that the inverse system

$y_1(t) = x_1(t+t_0)$

$h_1(t) = \delta(t+t_0)$

Let's verify that $h(t) * h_1(t) = \delta(t)$

$$h(t) * h_1(t) = \delta(t-t_0) * \delta(t+t_0)$$

$$= \int_{\tau} \delta(\tau - t_0)\, \delta(t+t_0-\tau)\, d\tau$$

$$= \delta(t+t_0-t_0) = \delta(t)$$

Example

$$h[n] = u[n]$$

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, u[n-k]$$

$$= \sum_{k=-\infty}^{n} x[k]$$ Accumulator

$$y[n] - y[n-1] = \sum_{k=-\infty}^{n} x[k] - \sum_{k=-\infty}^{n-1} x[k]$$

$$= x[n]$$

The inverse system to the accumulator (4)

is the differentiator

$$y_1[n] = x_1[n] - x_1[n-1]$$

$$h_1[n] = \delta[n] - \delta[n-1]$$

<u>verification</u>

$$h[n] * h_1[n] = u[n] * [\delta[n] - \delta[n-1]]$$

$$= u[n] * \delta[n] - u[n] * \delta[n-1]$$

$$= u[n] - u[n-1]$$

$$= \delta[n] \checkmark$$

The unit-step response

$$s[n] = u[n] * h[n]$$

$$= h[n] * u[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$$

$$= \sum_{k=-\infty}^{n} h[k]$$

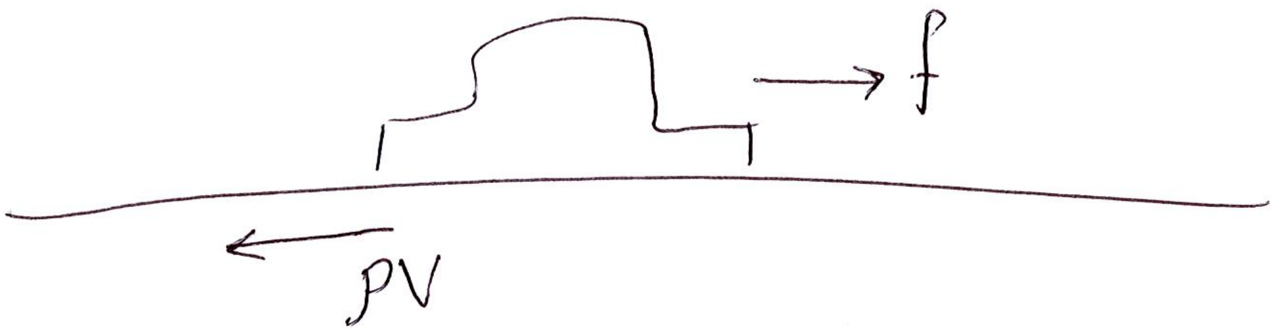
$$h[n] = s[n] - s[n-1]$$

CT

$$s(t) = \int_{\tau=-\infty}^{t} h(\tau)\, d\tau$$

$$h(t) = \frac{d\, s(t)}{dt}$$

Systems described by linear constant coefficient differential equations



$$\frac{dv(t)}{dt} = \frac{1}{m}\left[f(t) - \rho v(t)\right]$$

$$\frac{dv(t)}{dt} + \frac{\rho}{m} v(t) = \frac{1}{m} f(t)$$

Constants

$$\frac{dy(t)}{dt} + a\, y(t) = b\, x(t)$$

First-order diff. equ.

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \quad - (*)$$

$$x(t) = k\, e^{3t}\, u(t)$$

$$y(t) = y_p(t) + y_h(t)$$

$y_p(t)$ is a solution to $(*)$

$y_h(t)$ is a solution to

$$\frac{d\, y_h(t)}{dt} + 2y_h(t) = 0$$

<u>Finding $y_p(t)$</u>

Assume $y_p(t) = \underset{\text{constant}}{\Psi}\, e^{3t}$

$$3\Psi e^{3t} + 2\Psi e^{3t} = k\, e^{3t} \quad , \quad t \geq 0$$

$$y = \frac{k}{5}$$

$$y_p(t) = \frac{k}{5} e^{3t} \cancel{u(t)} , \quad t > 0$$

Finding $y_h(t)$

$$\frac{d y_h(t)}{dt} + 2 y_h(t) = 0$$

Constant

Assume $y_h(t) = A e^{st}$

$$A s e^{st} + 2 A e^{st} = 0$$

$$A e^{st} (s+2) = 0$$

$$\Rightarrow s = -2$$

$$y_h(t) = A e^{-2t}$$

$$y(t) = y_p(t) + y_h(t)$$

$$= \frac{k}{5} e^{3t} + A e^{-2t} \quad , \quad t > 0$$