ECE 301 - Lecture #11

Midterm 1 - Monday Sep. 24 In-Class

5 Problems (30 pts - 3% each)

1 ~~[illegible]~~ sheet (Back and Front) allowed

Revise: Notes - HW 1 (& 2) - Q1 & 2 - F'17 & S'18 Exam 1

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$$x(t) = k e^{3t} u(t)$$

$$y(t) = A e^{-2t} + \frac{k}{5} e^{3t}, \quad t > 0$$

The condition of initial rest

$$y(0) = 0$$

$$A e^{-2t} + \frac{k}{5} e^{3t} \Big|_{t=0} = 0$$

$$A + \frac{k}{5} = 0$$

$$A = -\frac{k}{5}$$

$$\Rightarrow \quad y(t) = \frac{k}{5}\left[e^{3t} - e^{-2t}\right] u(t)$$

Systems described by linear constant-coefficient difference equations

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x[n-k] \quad (*)$$

N-th order

With the condition of initial rest, the system described by (*) is LTI and Causal

$$y[n] = \frac{1}{a_0}\left[\sum_{k=0}^{M} b_k\, x[n-k] - \sum_{k=1}^{N} a_k\, y[n-k]\right]$$

Example

$$y[n] - \frac{1}{2} y[n-1] = x[n]$$

$$y[n] = x[n] + \frac{1}{2} y[n-1]$$

Assume initial rest and

$$x[n] = k\delta[n]$$

$$y[n] = \left(\frac{1}{2}\right)^n k \; u[n]$$

$$y[n] = 0 \quad \text{for } n < 0$$

$$y[0] = x[0] + \frac{1}{2} y[-1]$$

$$= k$$

$$y[1] = x[1] + \frac{1}{2} y[0]$$

$$= \frac{k}{2}$$

$$y[2] = x[2] + \frac{1}{2} y[1] = \frac{k}{4}$$

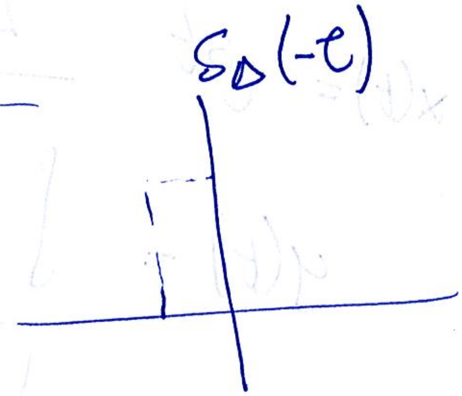
$$y[n] - \frac{1}{2} y[n-1] = x[n]$$

with initial rest

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

$\boxed{\text{IIR System}}$

Singularity Functions



$$\delta_\Delta(t) * \delta_\Delta(t) = \int \delta_\Delta(\tau)\, \delta_\Delta(t-\tau)\, d\tau$$

Area under the curve = 1

$$\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t) \qquad \lim_{\Delta \to 0} r_\Delta(t) = \delta(t)$$

$$\lim_{\Delta \to 0} \delta_\Delta(t) * r_\Delta(t) = \delta(t)$$

Fourier Series Representation of Periodic Signals

$$x(t) = e^{st} \xrightarrow{LTI} ?$$

$$y(t) = \int h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int h(\tau)\, \underbrace{e^{s(t-\tau)}}_{e^{st}\, e^{-s\tau}}\, d\tau$$

$$= e^{st} \underbrace{\int h(\tau)\, e^{-s\tau}\, d\tau}_{H(s)}$$

Eigen function

Eigen value

(7)

CT

$$e^{st} \xrightarrow{LTI} e^{st} H(s)$$

DT

$$x[n] = z^n \xrightarrow{LTI} \;?$$

$$y[n] = x[n] * h[n]$$

$$= h[n] * x[n]$$

$$= \sum_k h[k] \, x[n-k]$$

$$= \sum_k h[k] \, z^{n-k}$$

$$= z^n \underbrace{\sum_k h[k] \, z^{-k}}_{H(z)}$$

Example

$$y(t) = x(t-3)$$

$$x(t) = e^{j2t}$$

$$y(t) = e^{j(2t-6)} = \underbrace{e^{-j6}}_{H(2j)} e^{j2t}$$

$$H(s) = \int h(\tau) e^{-s\tau} \, d\tau$$

$$= \int \delta(\tau - 3) e^{-s\tau} \, d\tau$$

$$= e^{-3s}$$

$$\Rightarrow H(2j) = e^{-6j}$$

$$\Rightarrow y(t) = e^{j2t} e^{-6j}$$

$$x_2(t) = e^{\cancel{3t}3jt} \Longrightarrow y(t) = e^{j3t} e^{-9j}$$