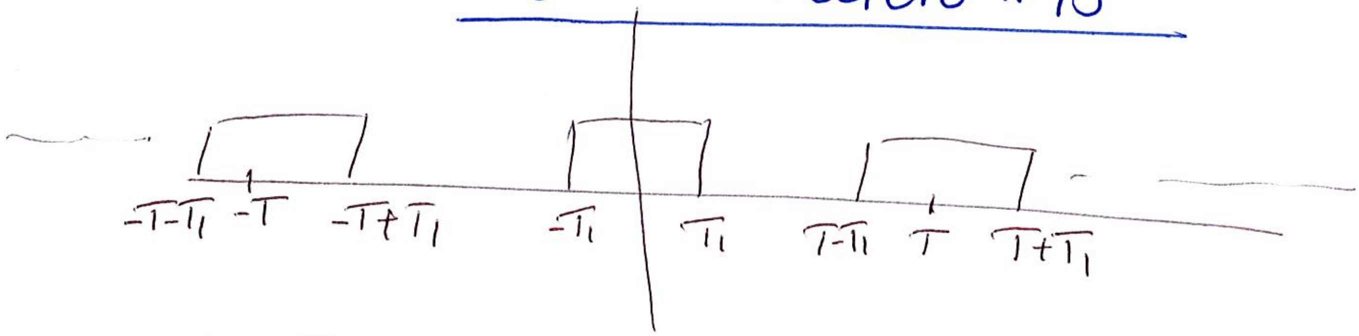
ECE 301 - Lecture #15



$T = 4T_1$

$$a_0 = \frac{2T_1}{T} = \frac{1}{2} \qquad a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}, \; k \neq 0$$

$$= \frac{\sin\left(k\frac{\pi}{2}\right)}{k\pi}$$

$$k = 1, -1 \Rightarrow a_1 = a_{-1} = \frac{\sin\left(\frac{\pi}{2}\right)}{\pi} = \frac{1}{\pi}$$

$$k = 2, -2, 4, -4, 6, -6, \ldots \Rightarrow a_k = 0$$

$$k = 3, -3 \Rightarrow a_3 = a_{-3} = \frac{\sin\left(\frac{3\pi}{2}\right)}{3\pi} = -\frac{1}{3\pi}$$

$$k = 5, -5 \Rightarrow a_5 = a_{-5} = \frac{\sin\left(\frac{5\pi}{2}\right)}{5\pi} = \frac{\sin\left(\frac{\pi}{2}\right)}{5\pi} = \frac{1}{5\pi}$$

$$k = 7, -7 \Rightarrow a_7 = a_{-7} = \frac{\sin(7\pi/2)}{7\pi} = \frac{\sin(3\pi/2)}{7\pi} = -\frac{1}{7\pi}$$

Two observations

For all non-zero a_k, $|a_k|$ diminishes as $|k|$ increases

They never die out

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Properties of Fourier Series Representation for Continuous-time Periodic Signals

$x(t) \xleftrightarrow{FS} a_k \qquad y(t) \xleftrightarrow{FS} b_k$

* Linearity $\longleftrightarrow$ $Ax(t) + By(t) \xleftrightarrow{FS} Aa_k + Bb_k$
* Time shifting
* Time Reversal

$x(t) \xleftrightarrow{FS} a_k \qquad x(t-t_0) \xleftrightarrow{FS} e^{-jk\omega_0 t_0} a_k$

$x(t) \xleftrightarrow{FS} a_k \qquad x(-t) \xleftrightarrow{FS} a_{-k}$

If $x(t)$ is even then $a_k = a_{-k}$, for all k

If $x(t)$ is odd then $a_k = -a_{-k}$, for all k

Multiplication

$x(t) \xleftrightarrow{FS} a_k$

$y(t) \xleftrightarrow{FS} b_k$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$$

$$z(t) = x(t)\, y(t) = \left(\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right)\left(\sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}\right)$$

For example

$$x(t) = a_1 e^{j\omega_0 t} + a_0 + a_{-1} e^{-j\omega_0 t}$$

$$y(t) = b_1 e^{j\omega_0 t} + b_0 + b_{-1} e^{-j\omega_0 t}$$

$$z(t) = a_1 b_1 e^{j2\omega_0 t} + (a_0 b_1 + a_1 b_0) e^{j\omega_0 t}$$
$$+ (a_0 b_0 + a_1 b_{-1} + a_{-1} b_1) + (a_{-1} b_0 + a_0 b_{-1}) e^{-j\omega_0 t}$$
$$+ a_{-1} b_{-1} e^{-j2\omega_0 t}$$
$$= \sum c_k e^{jk\omega_0 t}$$

$$\Rightarrow c_2 = a_1 b_1 \qquad c_1 = a_0 b_1 + a_1 b_0$$

$$c_0 = a_0 b_0 + a_1 b_{-1} + a_{-1} b_1 \qquad c_{-1} = a_{-1} b_0 + a_0 b_{-1}$$

$$c_{-2} = a_{-1} b_{-1}$$

$$z(t) = \sum c_k e^{jk\omega_0 t}$$

$$\Rightarrow c_k = \underbrace{\sum_{l=-\infty}^{\infty} a_l \, b_{k-l}}_{\text{Discrete-time Convolution}}$$

Multiplication of Periodic Continuous-Time Signals in time domain $\Longleftrightarrow$ Convolution of Discrete-~~time~~ sequences in Frequency domain

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Conjugation

$$x(t) \xleftrightarrow{FS} a_k$$

$$x^*(t) \xleftrightarrow{FS} a^*_{-k}$$

$$\boxed{\text{Real Signals } x(t) = x^*(t) \Longleftrightarrow a_k = a^*_{-k}}$$

Differentiation

$$x(t) \xleftrightarrow{FS} a_k$$

$$\frac{dx(t)}{dt} \xleftrightarrow{FS} ?\ a_k\, jk\omega_0$$

$$x(t) = \sum_k a_k e^{jk\omega_0 t}$$

$$\frac{dx(t)}{dt} = \sum_k \underbrace{a_k\, jk\omega_0}\, e^{jk\omega_0 t}$$

Integration

$$\int_{-\infty}^{t} x(t)\,dt \xleftrightarrow{FS} \frac{1}{jk\omega_0} a_k$$

Parserval's Relation

$$\frac{1}{T}\int_T |x(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

Proof of Parserval's Relation

$$\frac{1}{T}\int_T \left|\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right|^2 dt$$

$$\frac{1}{T}\int_T |a_k e^{jk\omega_0 t}|^2 dt$$

$$= \frac{1}{T}\int_T |a_k|^2 dt = |a_k|^2$$

$$\frac{1}{T}\int_T \sum_{k=-\infty}^{\infty} |a_k e^{jk\omega_0 t}|^2 dt$$

$$= \sum_{k=-\infty}^{\infty} \frac{1}{T}\int_T |a_k e^{jk\omega_0 t}|^2 dt$$

$$= \sum_{k=-\infty}^{\infty} |a_k|^2$$