ECE 301 - Lecture #2

TA office hours : Tue - Thu 3-4 pm

EE 209 - Table 1 Fri 1.30 - 2.30 pm

My office hours : Mon-Wed 1-2 pm

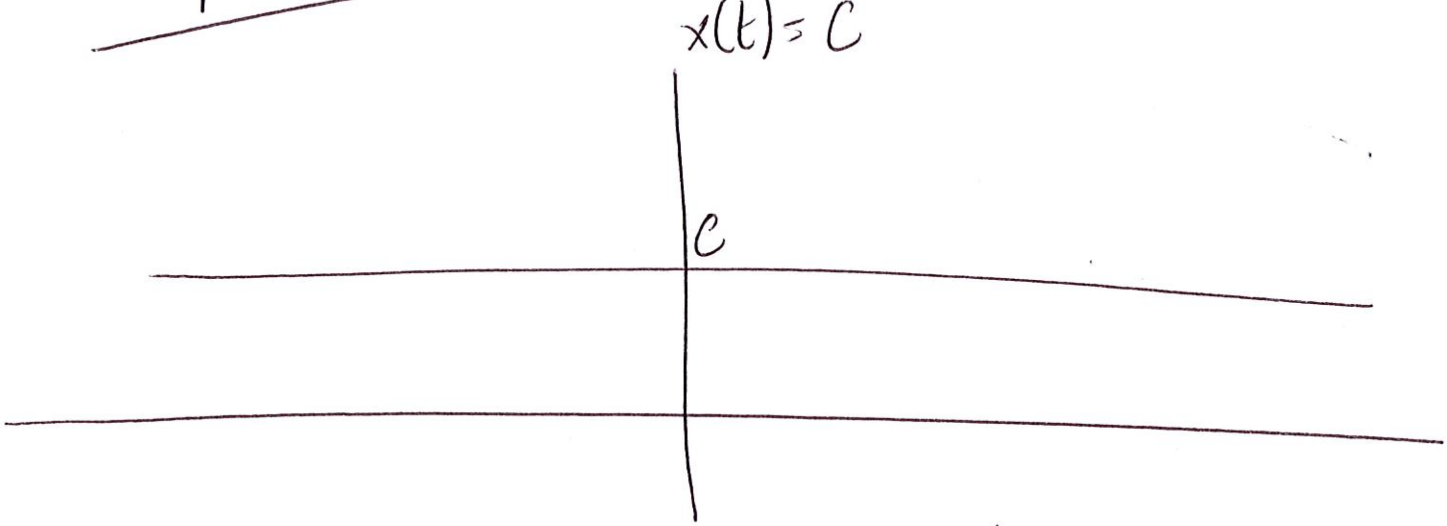
MSEE 350

Is there a signal with $E_\infty = \infty$ and $P_\infty = 0$?

In other words, Does $E_\infty = \infty$ imply that $P_\infty > 0$?

We will have example where $E_\infty = \infty$ and $P_\infty > 0$

Example #1



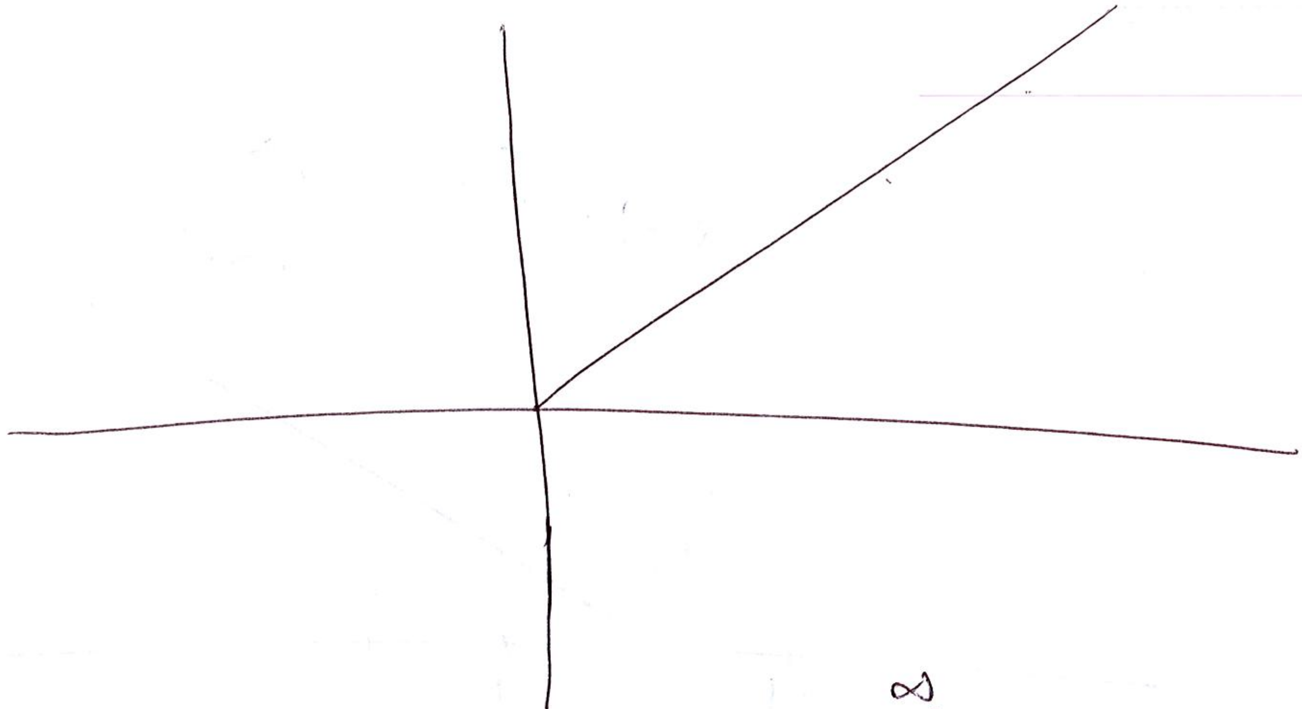
$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} C^2 \, dt = \infty$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} C^2 \, dt = C^2 > 0$$

$$x(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

(3)



$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} t^2 \, dt = \infty$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} t^2 \, dt$$

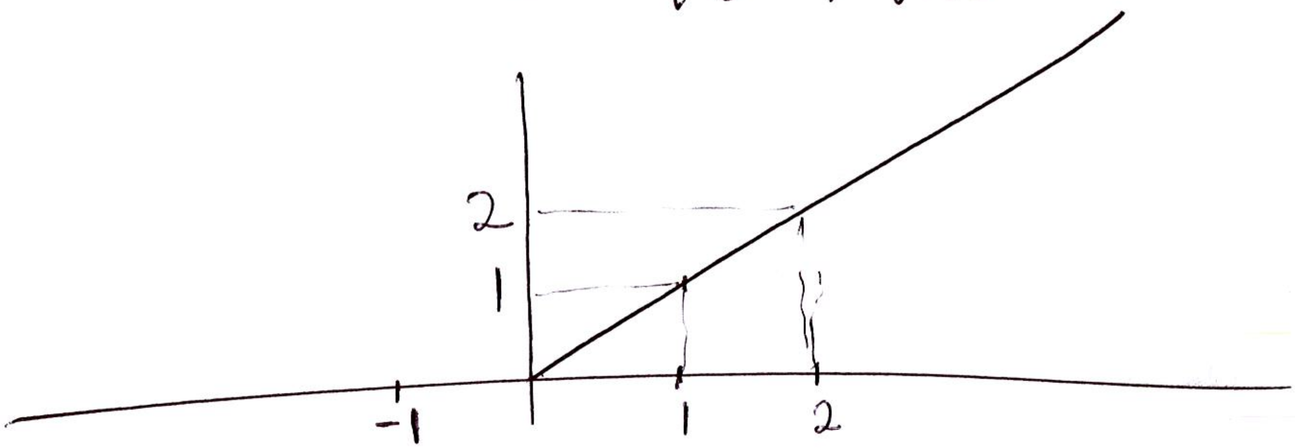
$$= \lim_{T \to \infty} \frac{1}{2T} \left. \frac{t^3}{3} \right|_{-T}^{T} = \lim_{T \to \infty} \frac{2T^3}{3 \cdot 2T}$$

$$= \lim_{T \to \infty} \frac{T^2}{3} = \infty$$

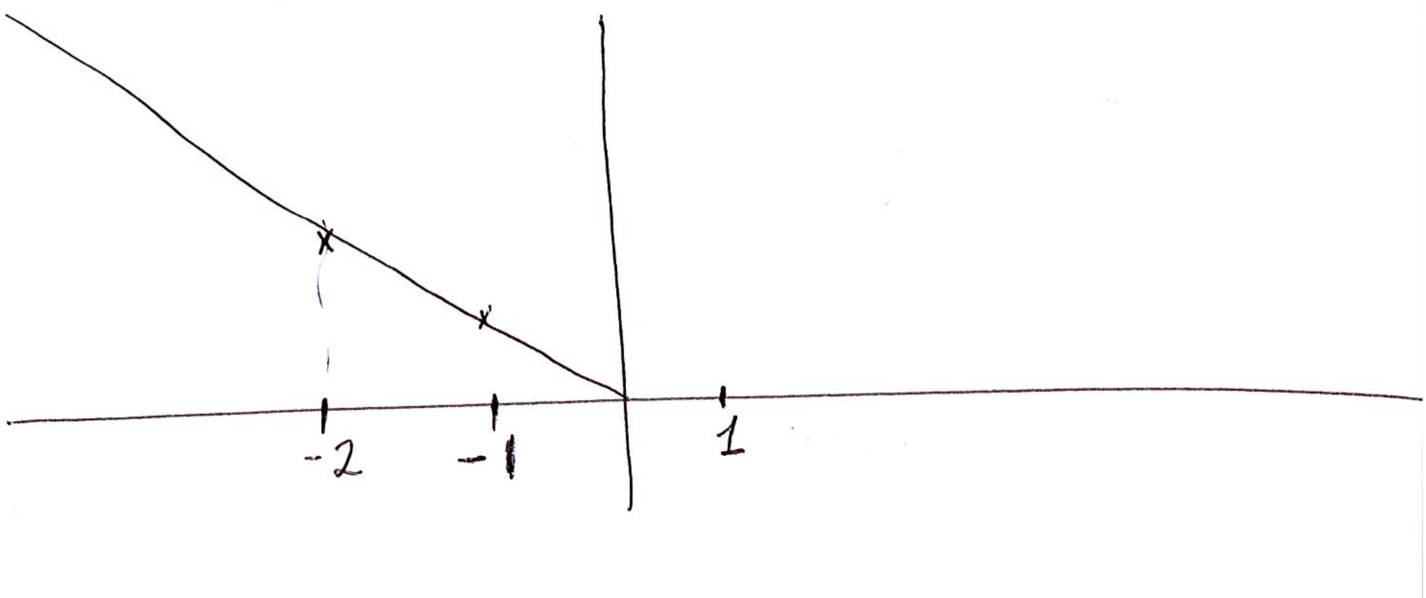
(4)

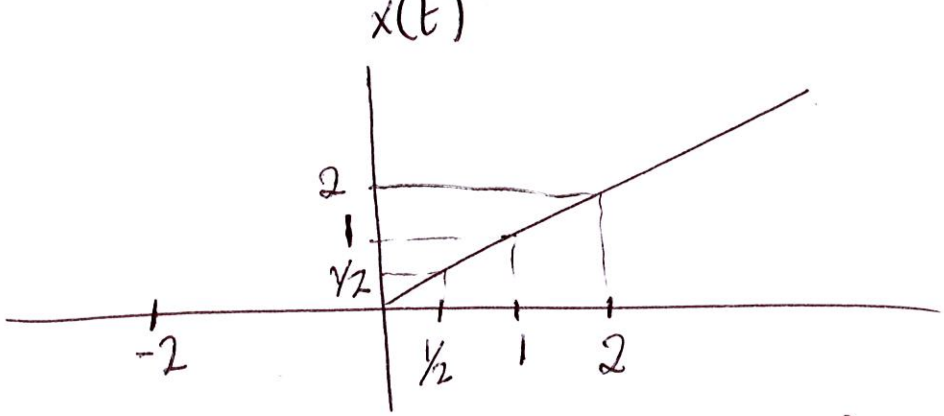
Transformations of the independent variable

$$x(t) = \begin{cases} t & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$



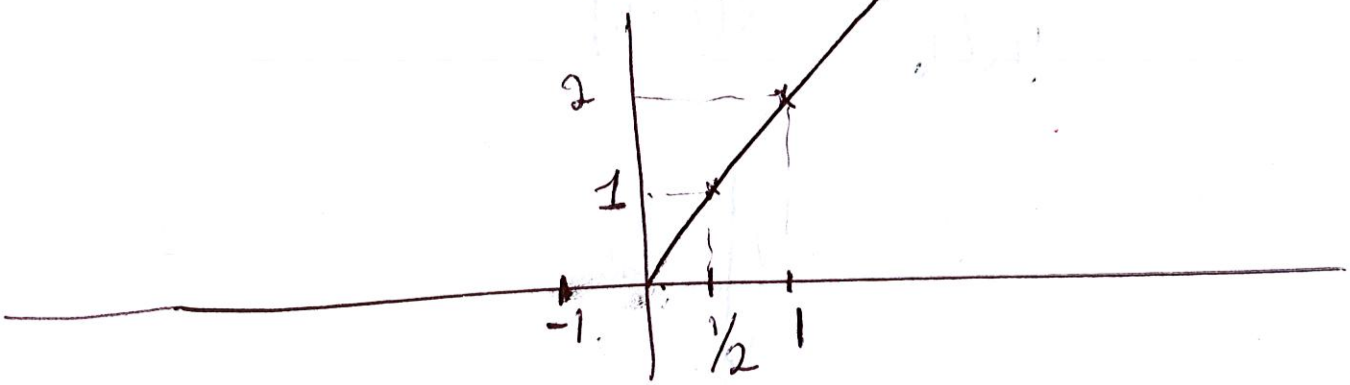
$y(t) = x(-t)$ ..





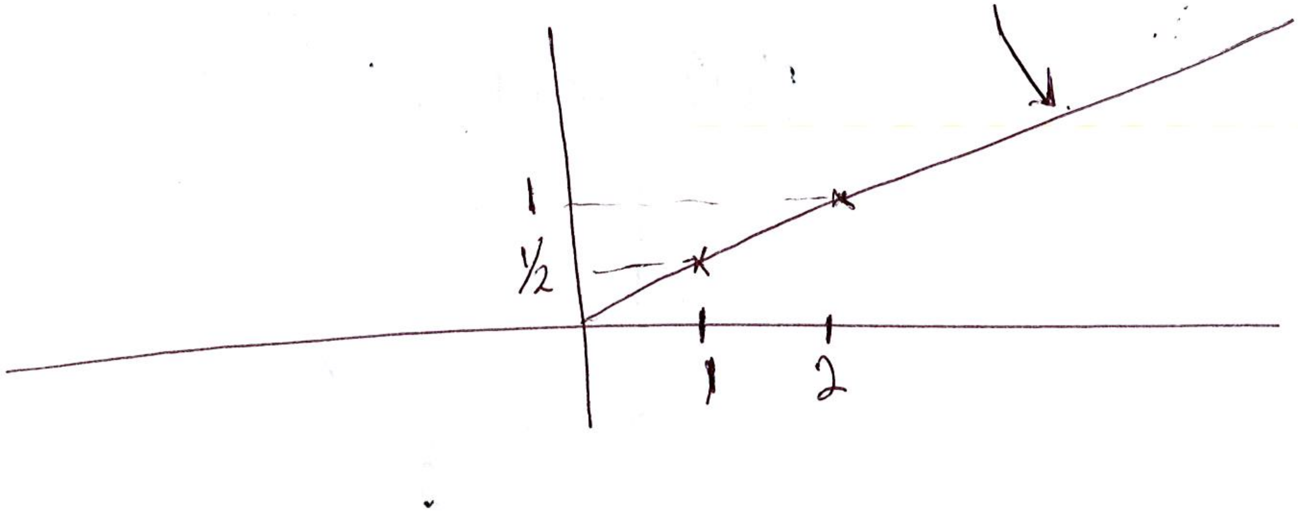
Grows faster than x(t)

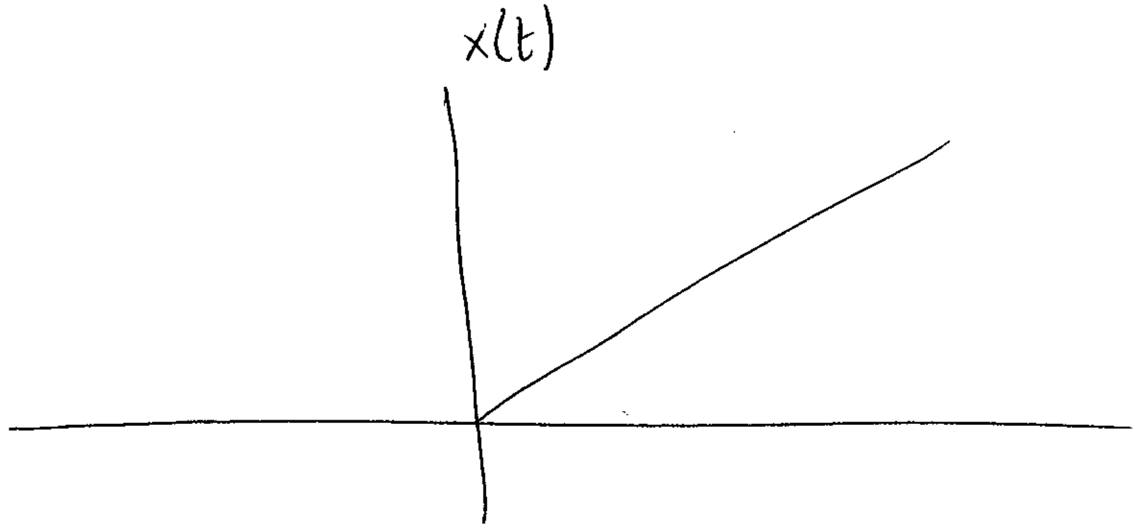
$y(t) = x(2t)$



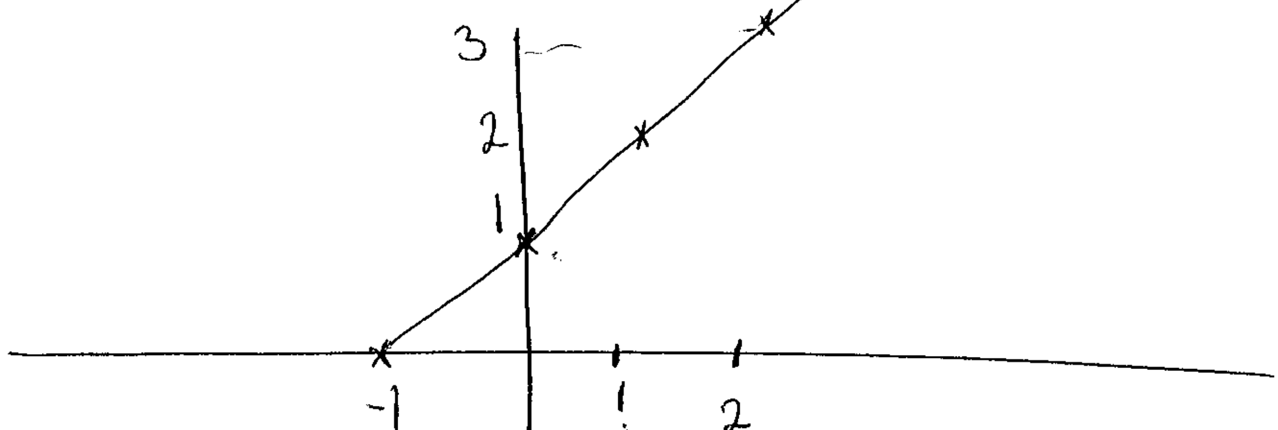
Grows slower than x(t)

$y(t) = x(t/2)$

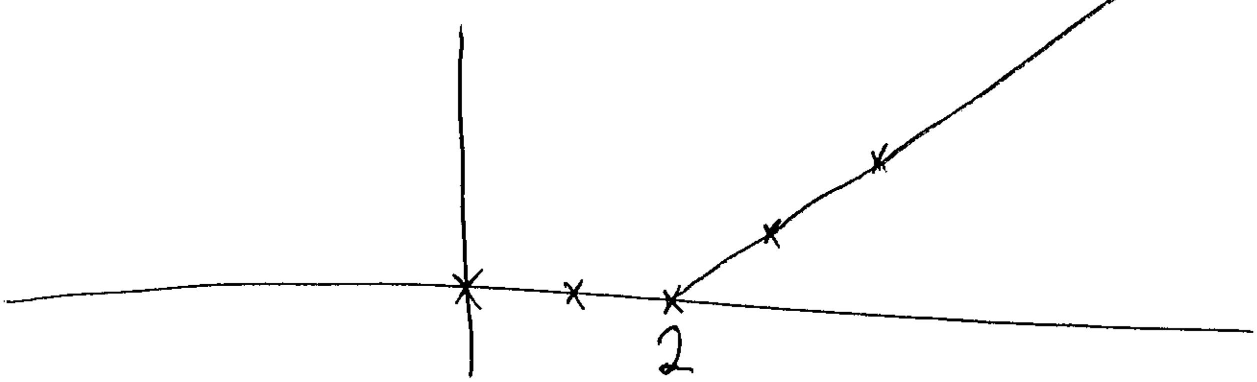


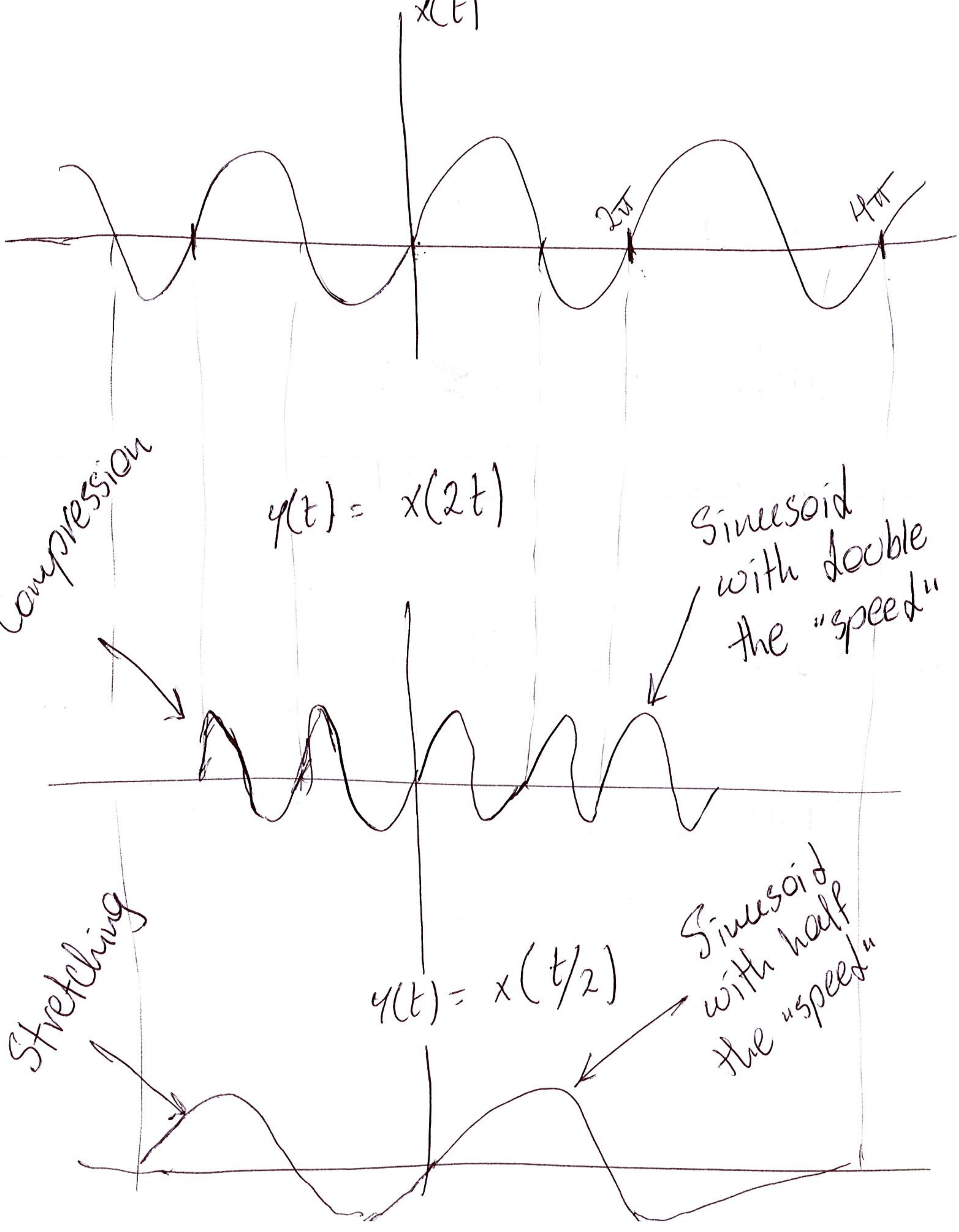


$y(t) = x(t+1)$



$y(t) = x(t-2)$




x(t)
2π
4π
Compression
y(t) = x(2t)
Sinusoid with double the "speed"
Stretching
y(t) = x(t/2)
Sinusoid with half the "speed"

real numbers

$$x(\alpha t + \beta)$$

1- Reverse if $\alpha < 0$

2- Stretch if $|\alpha| < 1$

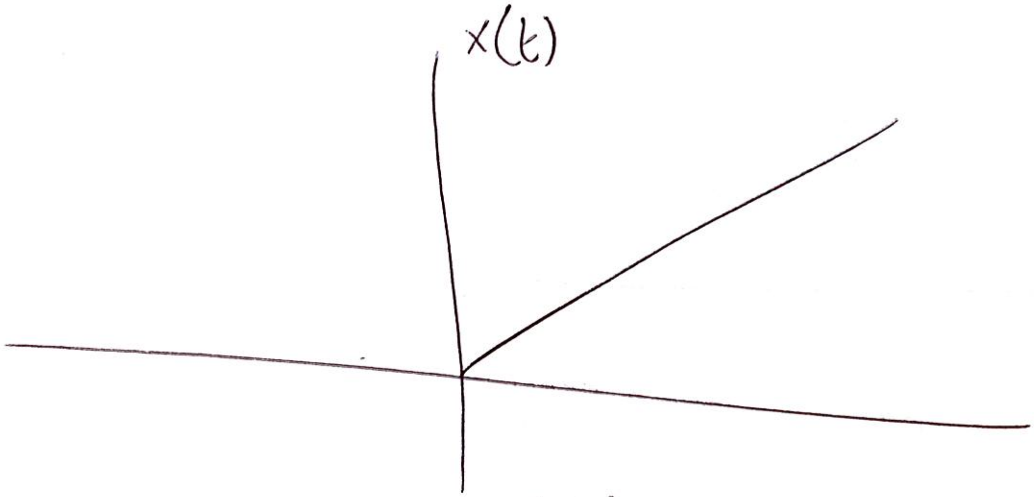
Compress if $|\alpha| > 1$

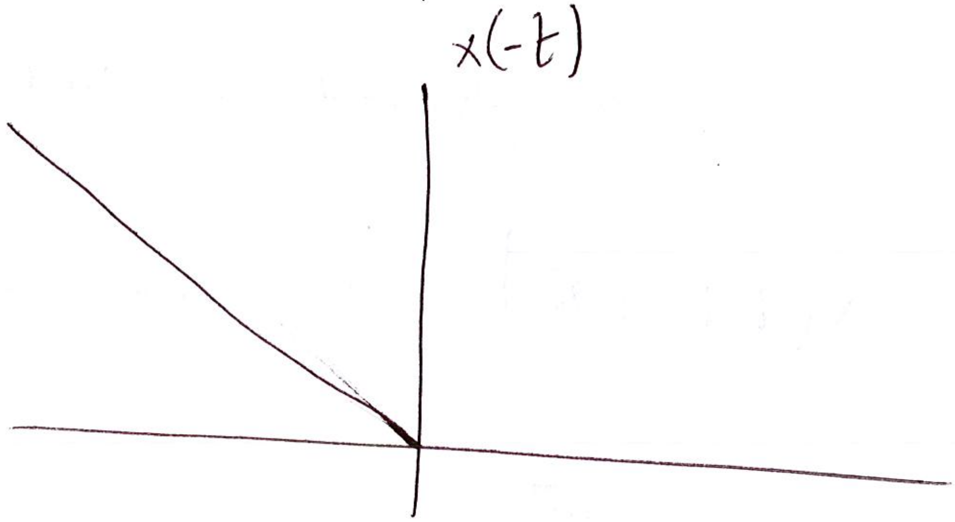
3- Shift to the left if $\beta > 0$ & $\alpha > 0$

OR $\beta < 0$ & $\alpha < 0$

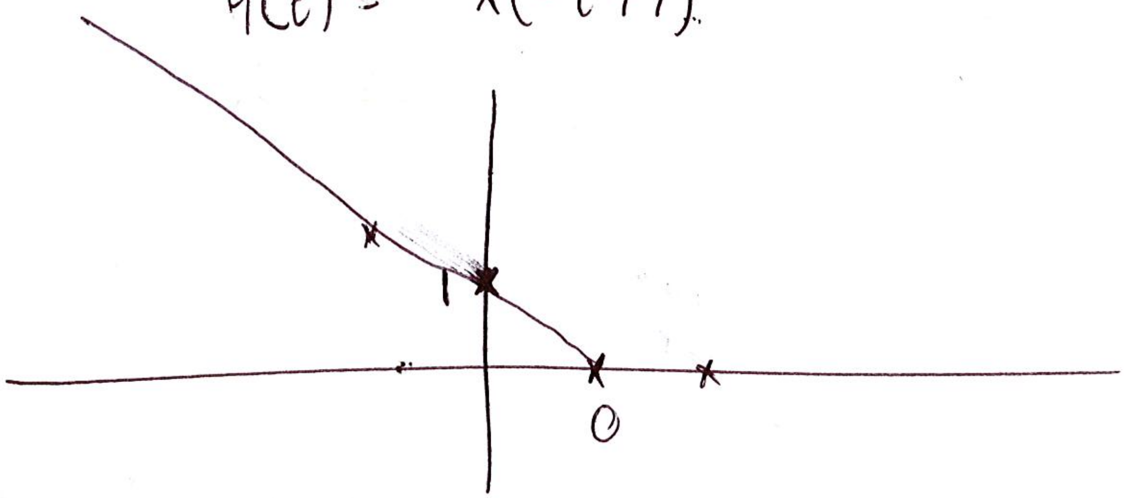
Shift to the right if $\beta < 0$ & $\alpha > 0$

OR $\beta > 0$ & $\alpha < 0$





$y(t) = x(-t+1)$



Periodic Signals

- We say that a signal $x(t)$ is periodic if there exists a period T such that

$$x(t) = x(t + mT), \text{ for any integer } m$$

- If a signal has period T, then it also has a period $2T$
$3T$
$4T$
$\vdots$

- Fundamental Period: Smallest Period of a signal (T_0)

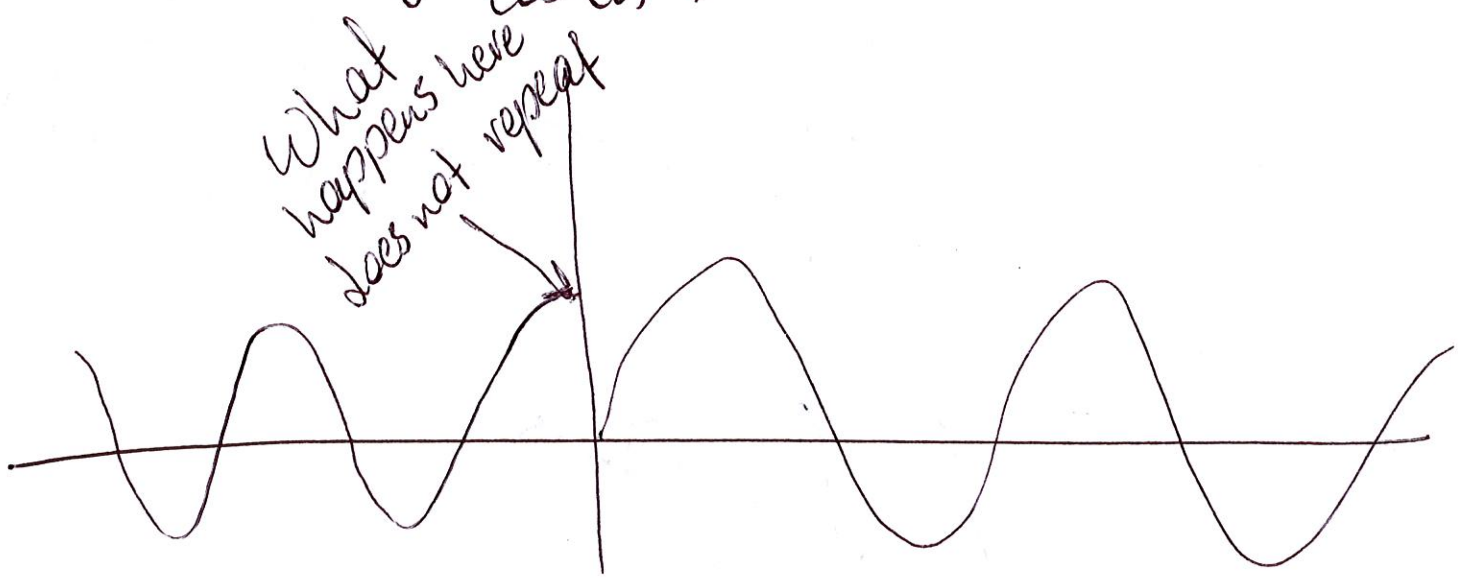
$x(t) = \cos t$

Periodic with Period 2π

Same for $x(t) = \sin(t)$

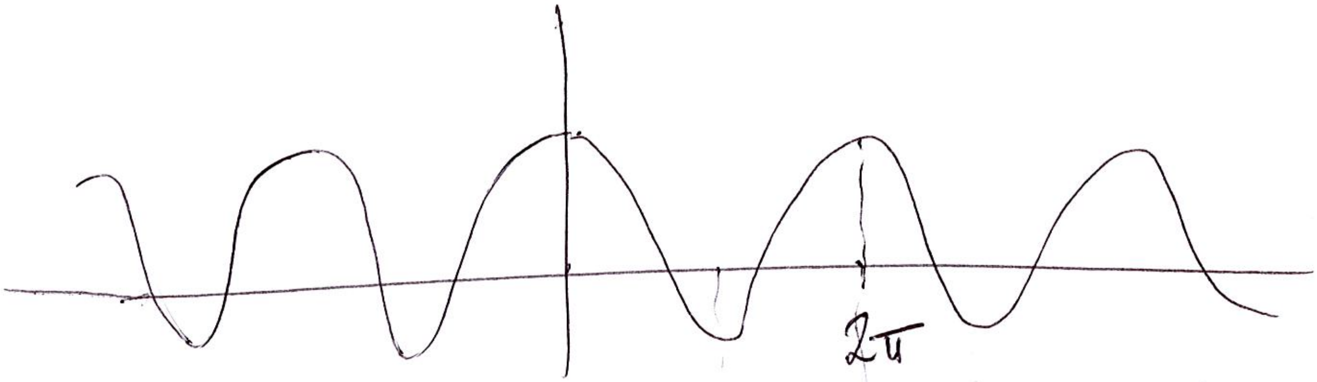
$$x(t) = \begin{cases} \sin(t), & t \geq 0 \\ \cos(t), & t < 0 \end{cases}$$

Not Periodic

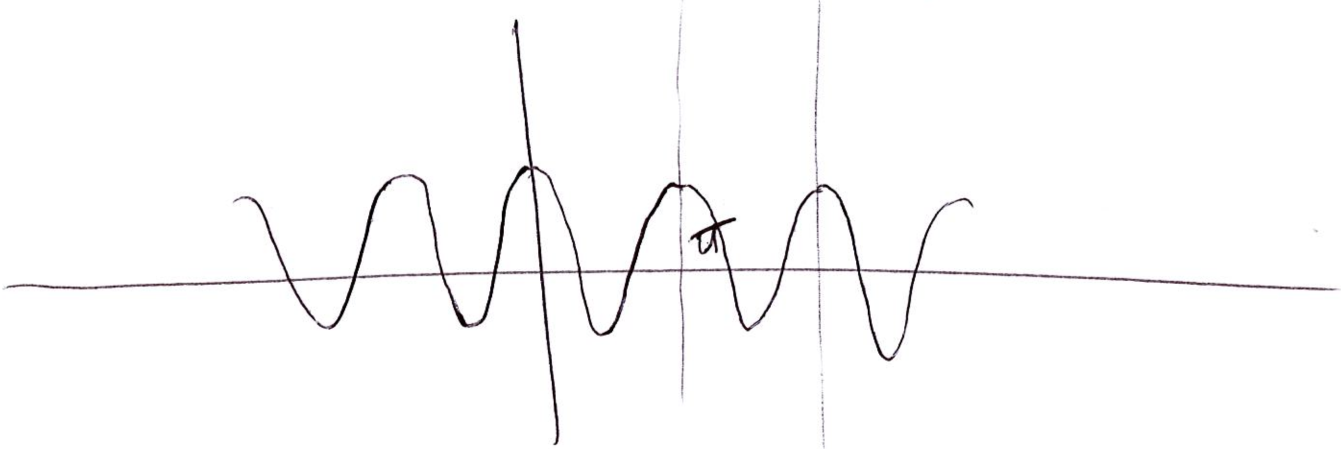


$x(t) = \cos t$

$y(t) = x(2t) = \cos(2t)$



$y(t)$ ← Periodic with period π



$y(t) = x(t/2) = \cos(t/2)$ Periodic with period 4π