Comment on HW 4 P. 2

$$x[n] = \sum a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

and $a_k = 0$ for all odd values of k

then we could replace N by $\tilde{N} = \frac{N}{2}$

$$x_1[n] = \sum a_k e^{jk\left(\frac{2\pi}{4}\right)n}$$

$$= a_0 + a_1 e^{j\left(\frac{2\pi}{4}\right)n} + a_2 e^{j\left(\frac{4\pi}{4}\right)n} + a_3 e^{j\left(\frac{6\pi}{4}\right)n}$$

$$= a_0 + a_1 e^{j\left(\frac{6\pi}{12}\right)n} + a_2 e^{j\left(\frac{12\pi}{12}\right)n} + a_3 e^{j\left(\frac{18\pi}{12}\right)n}$$

$$= \sum b_k e^{jk\left(\frac{2\pi}{12}\right)n}$$

$b_0 = a_0$ $\quad b_3 = a_1$ $\quad b_6 = a_2$ $\quad b_9 = a_3$

Preparing for Midterm 2

- Spring '18 Exam 2
- Fall '17 Exam 2
- Homework 4 & Homework 3
- Quizzes 3 & 4
- Lectures between Exam 1 & 2
- Practice Exam 2

2 Crib sheets Front & back

Parserval's Relation (Fourier Transform) (3)

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(e^{j\omega})|^2 \, d\omega$$

Proof:

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} x(t) \, x^*(t) \, dt$$

$$= \int_{-\infty}^{\infty} x(t) \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(e^{j\omega}) \, e^{-j\omega t} \, d\omega \right] dt$$

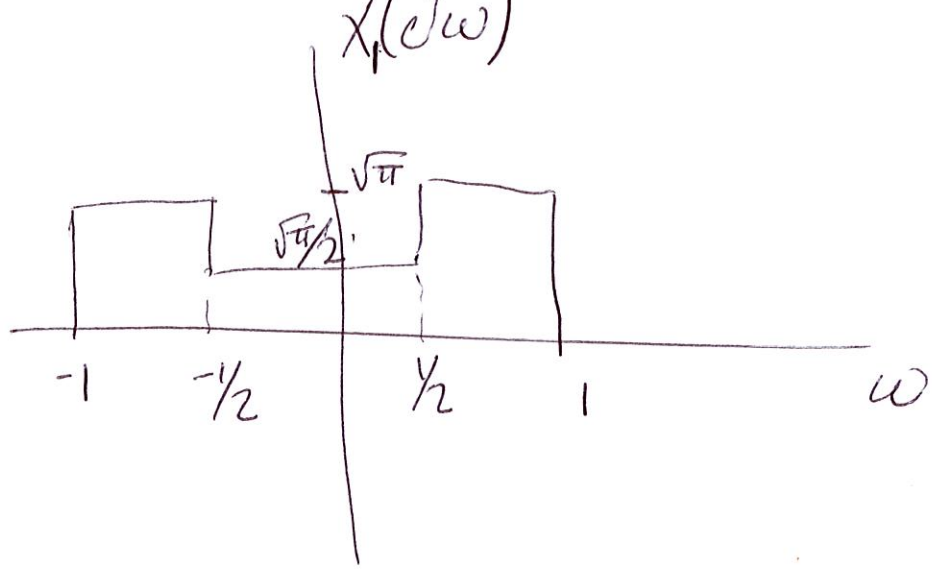
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(e^{j\omega}) \underbrace{\left[\int_{-\infty}^{\infty} x(t) \, e^{-j\omega t} \, dt \right]}_{X(e^{j\omega})} d\omega$$

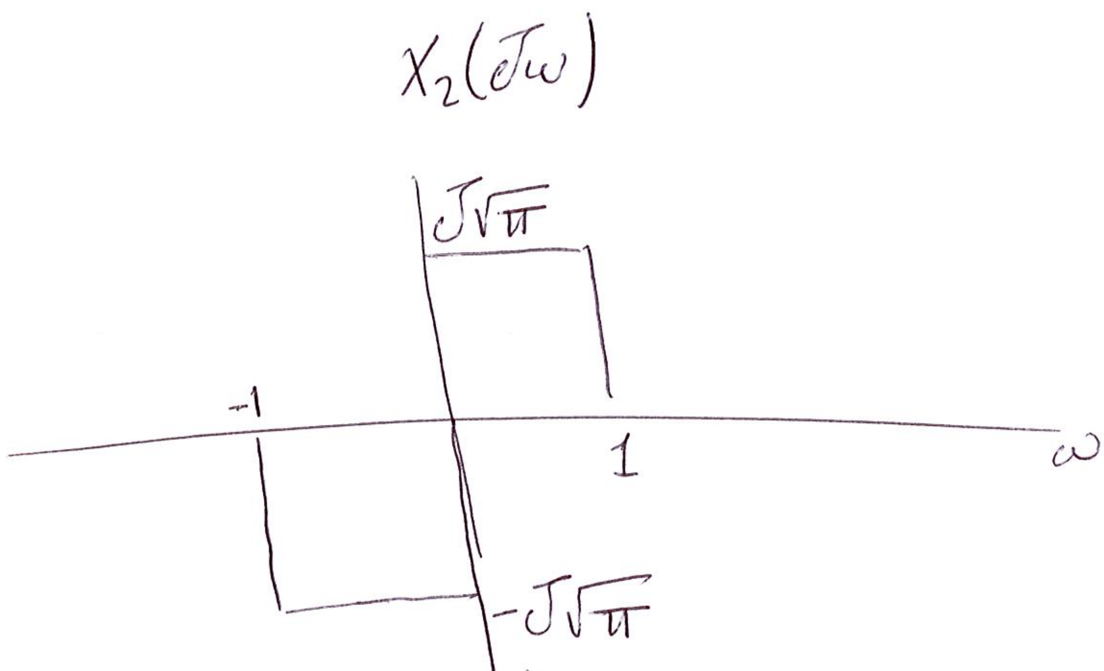
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(e^{j\omega})|^2 \, d\omega$$

$$\frac{|X(e^{j\omega})|^2}{2\pi}$$

Energy density Spectrum

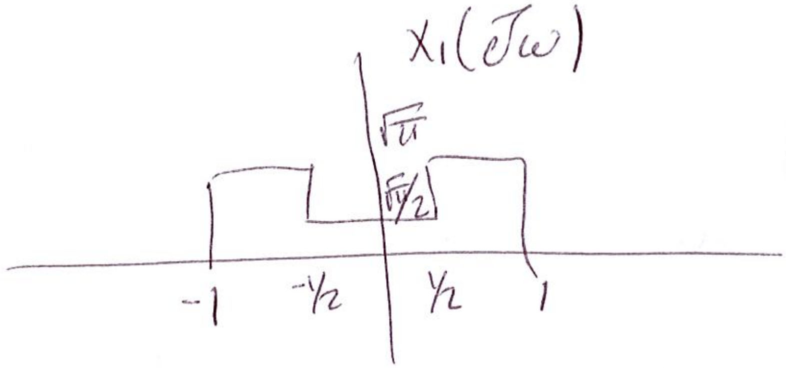
Example 4.14





$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

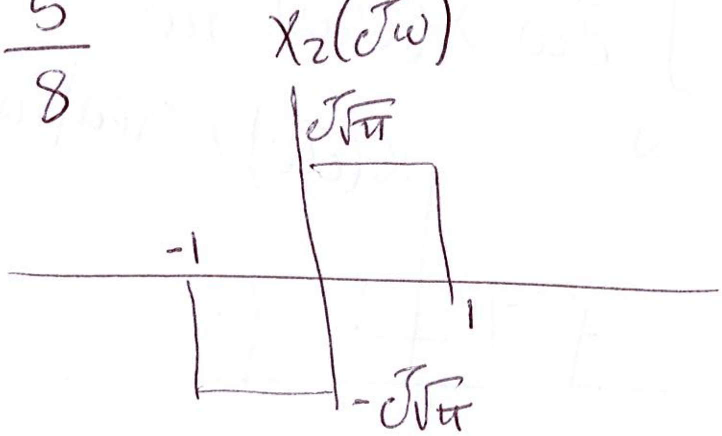
$$D = \frac{d}{dt} x(t) \Big|_{t=0}$$



$$E_1 = \frac{1}{2\pi} \int |X_1(j\omega)|^2 \, d\omega$$

$$= \frac{1}{2\pi} \left[\overset{[-1,-1/2]}{\frac{\pi}{2}} + \overset{[\frac{1}{2},1]}{\frac{\pi}{2}} + \overset{[-\frac{1}{2},\frac{1}{2}]}{\frac{\pi}{4}} \right]$$

$$= \frac{5}{8}$$



$$E_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X_2(j\omega)|^2 \, d\omega$$

$$= \frac{1}{2\pi} \left[\overset{[-1,0]}{\pi} + \overset{[0,1]}{\pi} \right] = 1$$

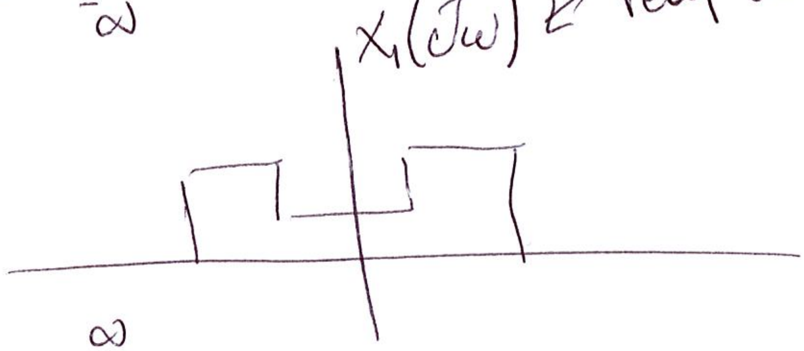
$$D = \frac{d}{dt} x(t) \Big|_{t=0}$$

Let $g(t) = \frac{d}{dt} x(t)$, then $D = g(0)$ #

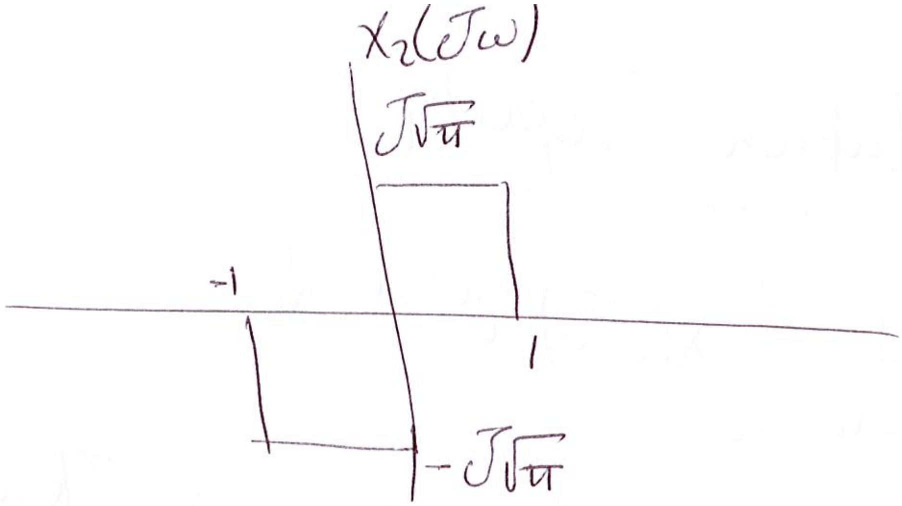
$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(j\omega)\, e^{j\omega t}\, d\omega$$

$$g(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(j\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega\, X(j\omega)\, d\omega$$



$$D_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega\, X_1(j\omega)\, d\omega = 0$$



$$D_2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} j\omega\, X_2(j\omega)\, d\omega = -\frac{1}{2\sqrt{\pi}}$$

$$= \underbrace{\frac{1}{2\pi}\int_{-1}^{0} j\omega\, X_2(j\omega)\, d\omega}_{} + \underbrace{\frac{1}{2\pi}\int_{0}^{1} j\omega\, j\sqrt{\pi}\, d\omega}_{}$$

$$\underbrace{\frac{1}{2\pi}\int_{-1}^{0} j\omega \cdot -j\sqrt{\pi}\, d\omega}_{} \qquad \underbrace{\frac{1}{2\pi}\int_{0}^{1} -\sqrt{\pi}\,\omega\, d\omega}_{}$$

$$\underbrace{\frac{1}{2\pi}\int_{-1}^{0} \sqrt{\pi}\,\omega\, d\omega}_{} \qquad \underbrace{-\frac{1}{2\sqrt{\pi}}\,\frac{\omega^2}{2}\Big|_{0}^{1}}_{-\frac{1}{4\sqrt{\pi}}}$$

$$\underbrace{\frac{1}{2\sqrt{\pi}}\,\frac{\omega^2}{2}\Big|_{-1}^{0}}_{-\frac{1}{4\sqrt{\pi}}}$$

The convolution Property

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \lim_{\omega_0 \to 0} \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(jk\omega_0)\, e^{jk\omega_0 t}\, \omega_0$$

If we process $x(t)$ through an LTI system we know that the response to $e^{jk\omega_0 t}$ is $H(jk\omega_0)\, e^{jk\omega_0 t}$

$$H(jk\omega_0) = \int_{-\infty}^{\infty} \underbrace{h(\tau)}_{\text{Impulse response}}\, e^{-jk\omega_0 \tau}\, d\tau$$

(9)

$$x(t) \xrightarrow[h(t)]{LTI} \lim_{\omega_0 \to 0} \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(jk\omega_0)\, H(jk\omega_0)\, e^{jk\omega_0 t}\, \omega_0$$

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{X(j\omega) H(j\omega)}_{Y(j\omega)} e^{j\omega t}\, d\omega$$

$$y(t) = x(t) * h(t)$$

$$Y(j\omega) = X(j\omega)\, H(j\omega)$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= FT\{h(t)\}$$