ECE 301 - Lecture #23

Formal Derivation of the Convolution Property of the Fourier Transform

Bonus

$\mathcal{F}\Big\{$  $\Big\} =$ Due Friday Nov. 2

End of Lecture #20

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

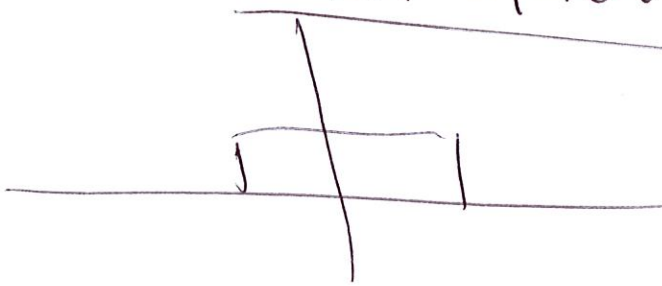
$$Y(j\omega) = \mathcal{F}\{y(t)\} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau\right] e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x(\tau) \underbrace{\left[\int_{-\infty}^{\infty} h(t-\tau)\, e^{-j\omega t}\, dt\right]}_{e^{-j\omega\tau} H(j\omega)} d\tau$$

$$Y(j\omega) = \int_{-\infty}^{\infty} x(\tau)\, e^{-j\omega\tau}\, H(j\omega)\, d\tau \qquad (2)$$

$$= H(j\omega) \underbrace{\int_{-\infty}^{\infty} x(\tau)\, e^{-j\omega\tau}\, d\tau}_{X(j\omega)}$$

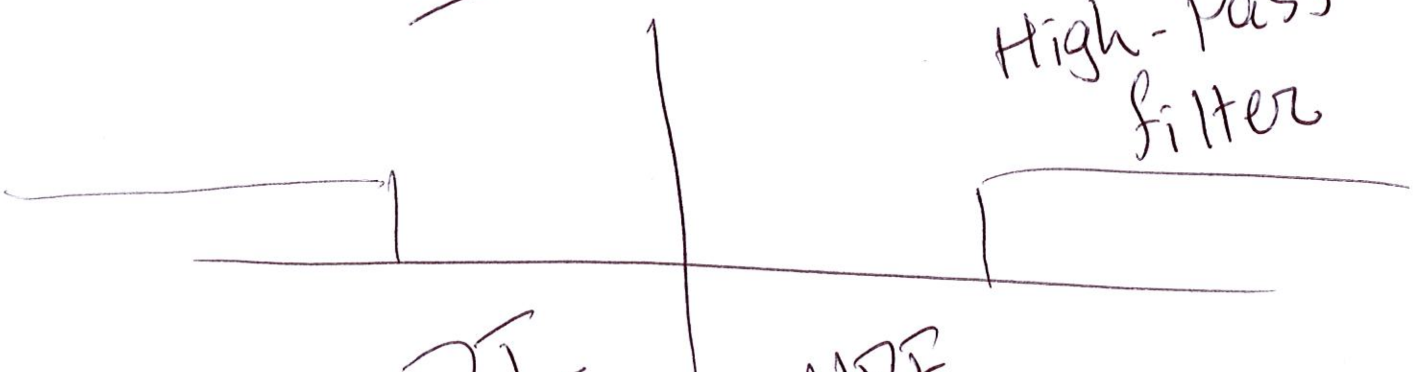
LTI System Freq. Response



Low-Pass filter

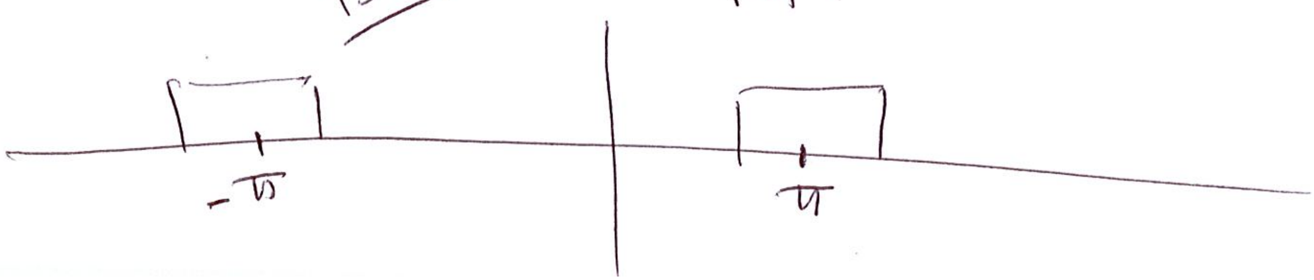
CT

High-Pass filter



DT

HPF



$$x(t) \longrightarrow \boxed{H(j\omega)} \longrightarrow y(t)$$

Dirichlet Condition 1

$$\int_{-\infty}^{\infty} |h(t)| < \infty \quad (1)$$

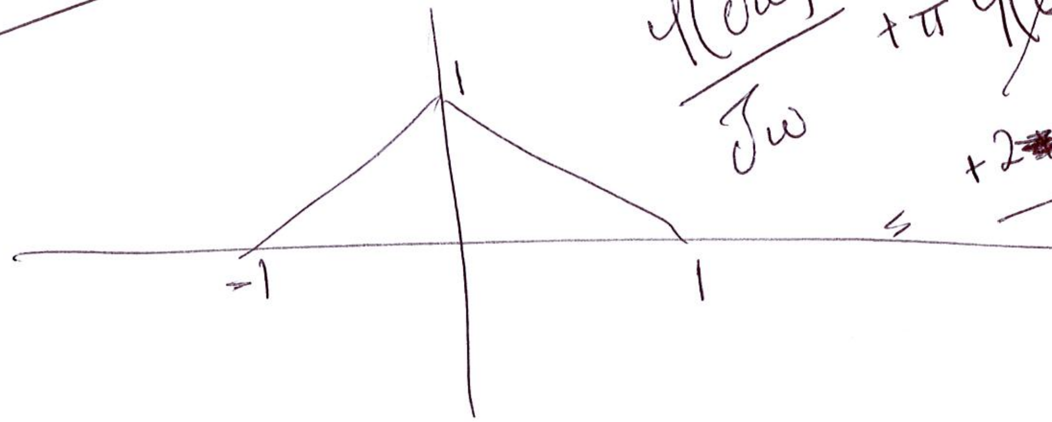
$$\hat{h}(t) = \frac{1}{2\pi} \int H(j\omega)\, e^{j\omega t}\, d\omega$$

If (1) is not true, it is not necessarily the case that $\hat{h}(t) = h(t)$

→ Condition for stability of the LTI System

→ FT is useful only for analyzing stable LTI Systems

HW 4 P.4



$$\frac{Y(j\omega)}{j\omega} + \pi \cancel{Y(0)\,\delta(\omega)}\,(4)$$

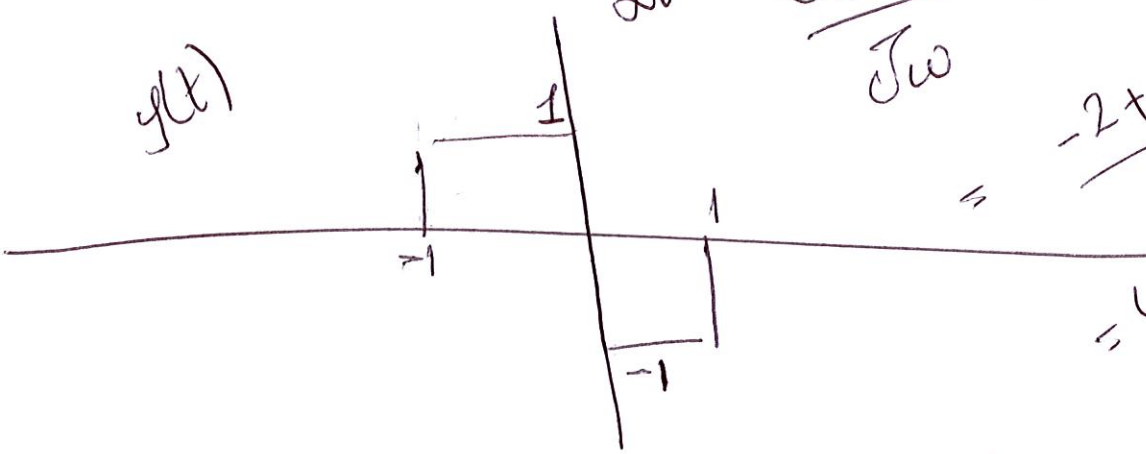
$$= \frac{+2 - e^{-j\omega} - e^{j\omega}}{\omega^2}$$

$\frac{d}{dt}$

$$\frac{G(j\omega)}{j\omega} + \pi \cancel{G(0)\,\delta(\omega)}$$

$$= \frac{-2+e^{-j\omega}+e^{j\omega}}{j\omega}$$

$$= Y(j\omega)$$



$\frac{d}{dt}$

$G(j\omega)$

$\downarrow$

$-2 + e^{-j\omega} + e^{j\omega}$