ECE 301 - Lecture #25

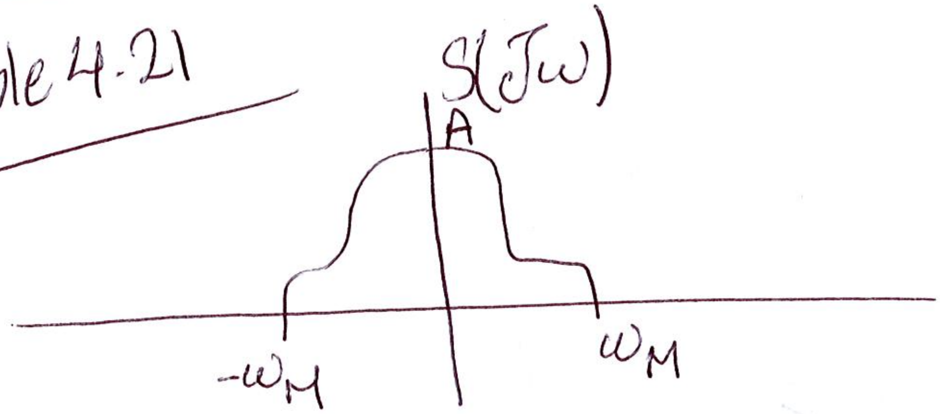
Multiplication Property

$$r(t) = s(t)p(t) \overset{\mathcal{F}}{\longleftrightarrow} R(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(j\theta)P(j(\omega-\theta))\,d\theta$$

Used in Amplitude modulation "Communication Systems"

Also called Modulation Property

Example 4.21



$$p(t) = \cos \omega_0 t$$

$$P(j\omega) = \pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)$$

$R(j\omega)$ $\boxed{\omega_0 > \omega_M}$

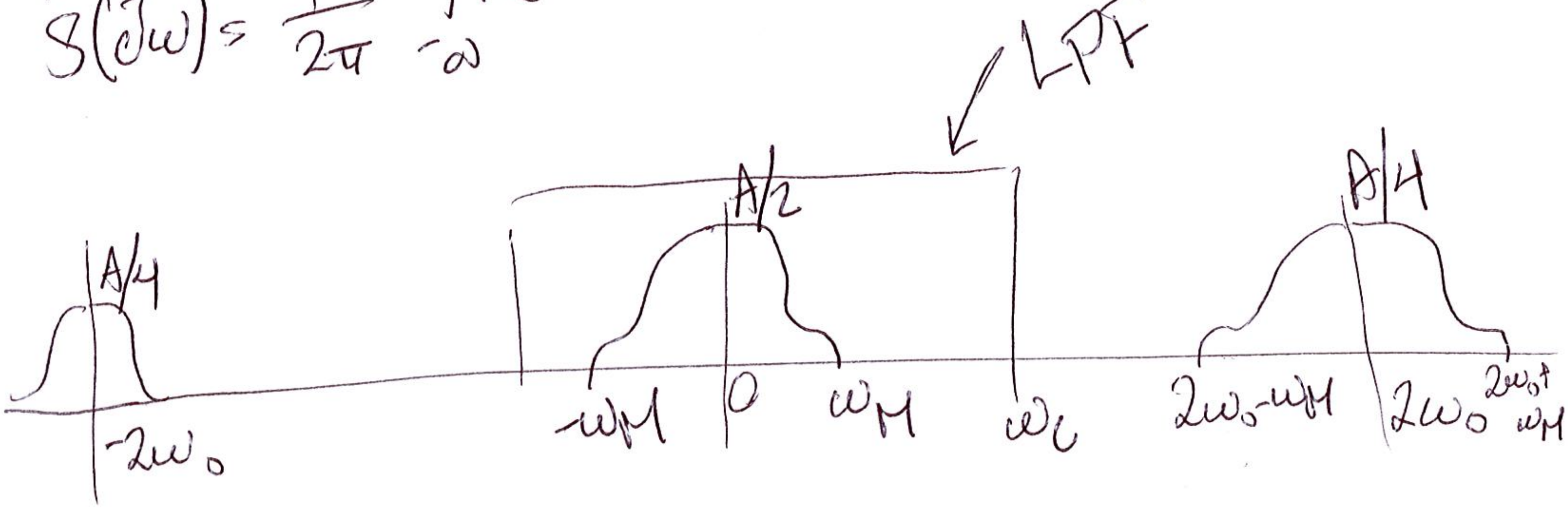
Information in original signal is presened

How to recover?

$\hat{s}(t) = r(t)\,p(t)$ ← $\cos \omega_0 t$

$P(j\omega)$

$$\hat{S}(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(j\theta)\,P(j(\omega-\theta))\,d\theta$$



then we pass $\hat{s}(t)$ by a LPF (3)

(with appropriate scaling factor)

and cut-off frequency $\omega_M < \omega_c < 2\omega_0 - \omega_M$

Note: Could also use LPF approximation

(for example, $h(t) = e^{-at} u(t)$, $a > 0$)

but make sure that energy leakage is very low at freq. $2\omega_0 - \omega_M$

Example 4.23

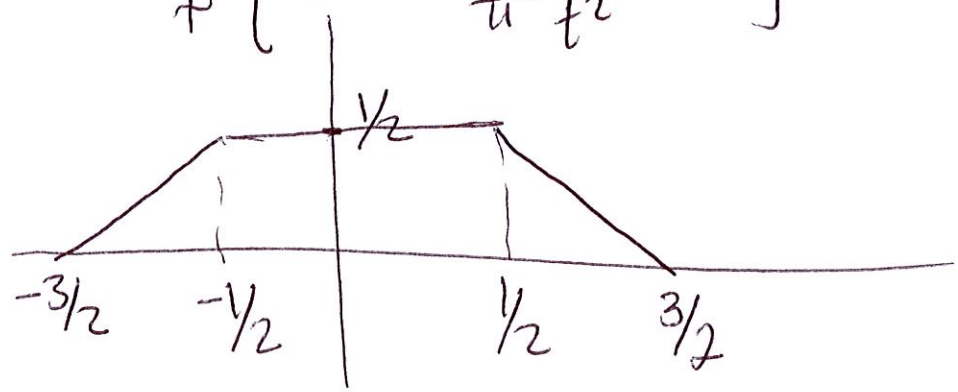
$$x(t) = \frac{\sin(t)\,\sin(t/2)}{\pi t^2}$$

$$= \pi \left(\frac{\sin t}{\pi t}\right)\left(\frac{\sin(t/2)}{\pi t}\right)$$

(4)

$$X(j\omega) = \frac{1}{2}\, \mathcal{F}\left\{\frac{\sin t}{\pi t}\right\} * \mathcal{F}\left\{\frac{\sin t/2}{\pi t}\right\}$$

$$\mathcal{F}\left\{\frac{\sin(t)\,\sin(t/2)}{\pi t^2}\right\}$$



Systems characterized by linear constant coefficient differential equations

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

N-th order differential equations

$$Y(j\omega) = X(j\omega)\, H(j\omega)$$

$$\mathcal{F}\left\{ \sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} \right\} = \mathcal{F}\left\{ \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \right\}$$

$$\sum_{k=0}^{N} a_k\, \mathcal{F}\left\{ \frac{d^k y(t)}{dt^k} \right\} = \sum_{k=0}^{M} b_k\, \mathcal{F}\left\{ \frac{d^k x(t)}{dt^k} \right\}$$

$$\sum_{k=0}^{N} a_k (j\omega)^k\, Y(j\omega) = \sum_{k=0}^{M} b_k (j\omega)^k\, X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k}$$

Example 4.24

$$\frac{dy(t)}{dt} + a\,y(t) = x(t)$$

LPF Approximation

$$H(j\omega) = \frac{1}{j\omega + a}$$

$$h(t) = e^{-at}\,u(t)$$

Example 4.25

$$\frac{d^2 y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

$$H(j\omega) = \frac{j\omega + 2}{(j\omega)^2 + 4j\omega + 3}$$

$$= \frac{2 + j\omega}{(j\omega+1)(j\omega+3)}$$

Partial Fraction Expansion

$$= \frac{1/2}{j\omega+1} + \frac{\frac{1}{2}}{j\omega+3}$$

$$h(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t)$$

$$x(t) = e^{-t}\,u(t)$$

$$Y(j\omega) = X(j\omega)\,H(j\omega)$$

$$= \left[\frac{2+j\omega}{(j\omega+1)(j\omega+3)}\right]\left[\frac{1}{1+j\omega}\right]$$

$$= \frac{2+j\omega}{(j\omega+1)^2\,(j\omega+3)}$$

$$Y(j\omega) = \frac{1/4}{1+j\omega} + \frac{1/2}{(1+j\omega)^2} - \frac{1/4}{3+j\omega}$$

$$y(t) = \left[\tfrac{1}{4}e^{-t} + \tfrac{1}{2}t\,e^{-t} - \tfrac{1}{4}e^{-3t}\right]u(t)$$