ECE 301 - Lecture #29

Example 5.15

$$x[n] = x_1[n]\, x_2[n]$$

$$x_1[n] = \frac{\sin\left(\frac{3\pi n}{4}\right)}{\pi n}$$

$$x_2[n] = \frac{\sin(\pi n/2)}{\pi n}$$

$$X(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} X_1(e^{j\theta})\, X_2\left(e^{j(\omega-\theta)}\right) d\theta$$

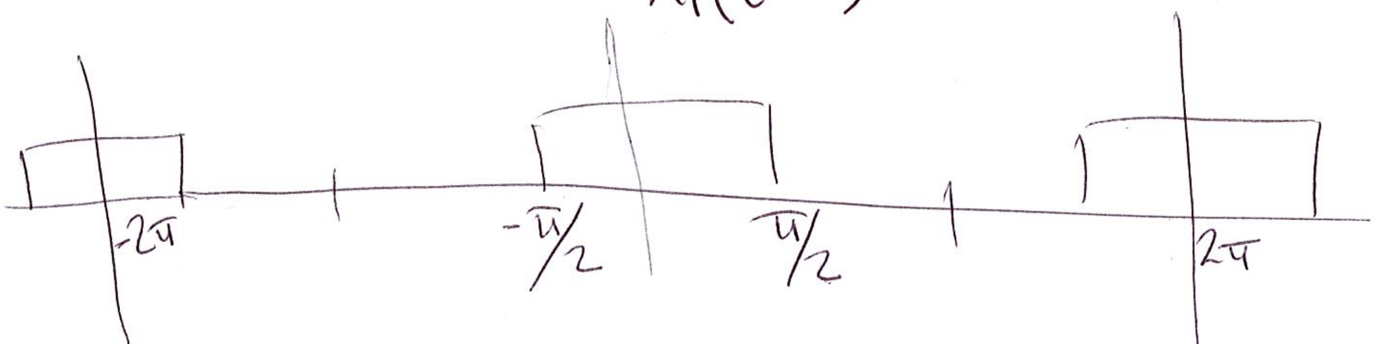
$-\pi \leq \theta \leq \pi$

$$\hat{X}_1(e^{j\omega}) = \begin{cases} X_1(e^{j\omega}), & -\pi \leq \omega \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

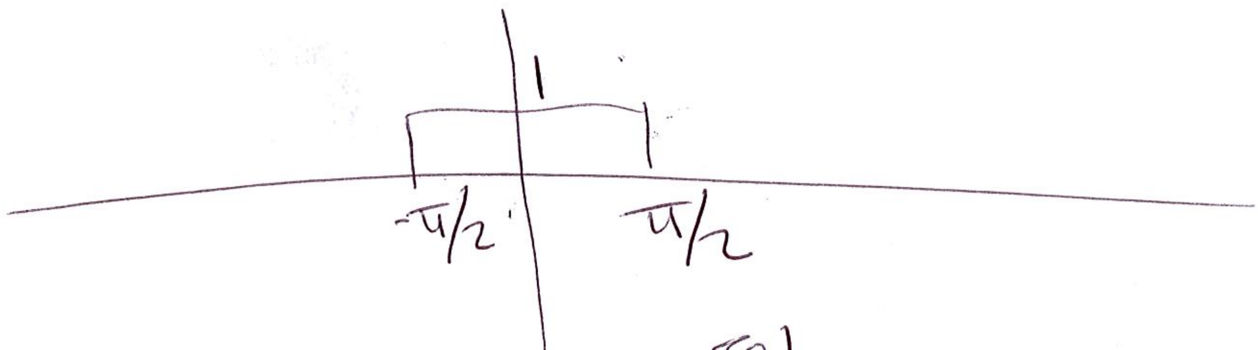
$$X(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{X}_1(e^{j\theta})\, X_2(e^{j(\omega-\theta)})\, d\theta$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{X}_1(e^{j\theta})\, X_2(e^{j(\omega-\theta)})\, d\theta$$

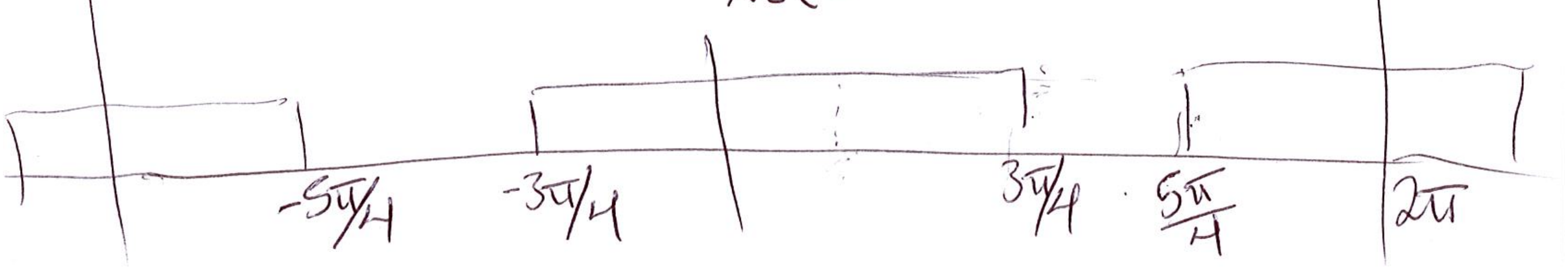
$X_1(e^{j\theta})$

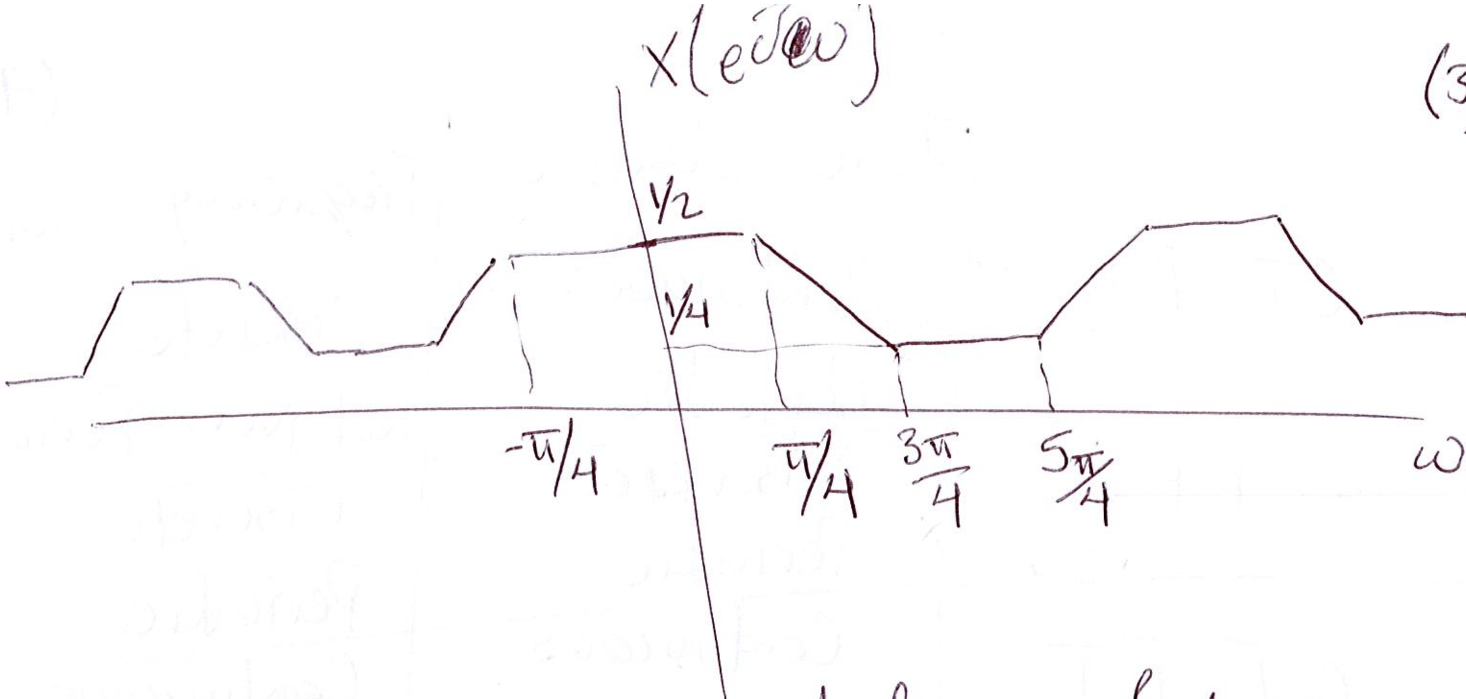


$\hat{X}_1(e^{j\theta})$



$X_2(e^{j\theta})$





Freq. domain representation of two Ideal LPFs ($\omega_{c_1} = \frac{\pi}{2}$, $\omega_{c_2} = \frac{3\pi}{4}$) multiplied in time domain

Recap (FS + FT)

CT FS — Continuous and Periodic in time

DT FS — Discrete and Periodic in time

CT FT — Continuous and not necessarily periodic in time

DTFT — Discrete and not nec. per. time

	Time - Domain	Frequency Domain
CT FS	Continuous Periodic	Discrete Not Nec. Periodic
DTFS	Discrete Periodic	Discrete Periodic
CT FT	Continuous Not Nec. Periodic	Continuous Not Nec. Periodic
DT FT	Discrete Not Nec. Periodic	Continuous Periodic with Period 2π

Duality for DTFS

Duality for CTFT

Duality between CTFS and DTFT only if the continuous time period is 2π

Periodic in time domain $\longrightarrow$ Discrete in Freq. Domain

Not Nec. Periodic in TD $\longrightarrow$ Continuous in Freq. Domain

Continuous in TD $\longrightarrow$ Not Nec. Periodic in Freq. Domain

Discrete in TD $\longrightarrow$ Periodic in Freq. Domain

Discrete in one domain $\longleftrightarrow$ Periodic in the other domain

Duality in the Discrete-time Fourier Series

$x[n]$ DT signal
with period N

FS coeff. a_k Discrete
with period N

What if we have a discrete-time periodic signal $f[n] = a_n$
with period N

$$f[n] \xrightarrow{FS} ? \quad \frac{1}{N} x[-k]$$

Proof

$$f[m] = \frac{1}{N} \sum_{r=\langle N \rangle} g[r] \, e^{-jr\left(\frac{2\pi}{N}\right)m}$$

Let $m=k$, $r=n$

$$f[k] = \frac{1}{N} \sum_{n=\langle N \rangle} g[n] \, e^{-jn\left(\frac{2\pi}{N}\right)k}$$

$$\Longrightarrow \quad g[n] \xrightarrow{FS} f[k]$$

Let $m=n$, $r=-k$

$$f[n] = \sum_{k=\langle N \rangle} \frac{1}{N} g[-k] \, e^{jk\left(\frac{2\pi}{N}\right)n}$$

$$f[n] \xrightarrow{FS} \frac{1}{N} g[-k]$$