ECE 301 - Lecture #3

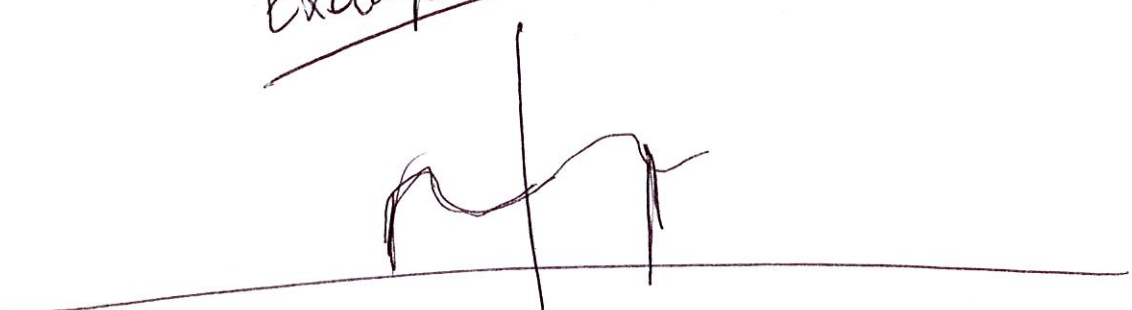
Even and Odd Signals

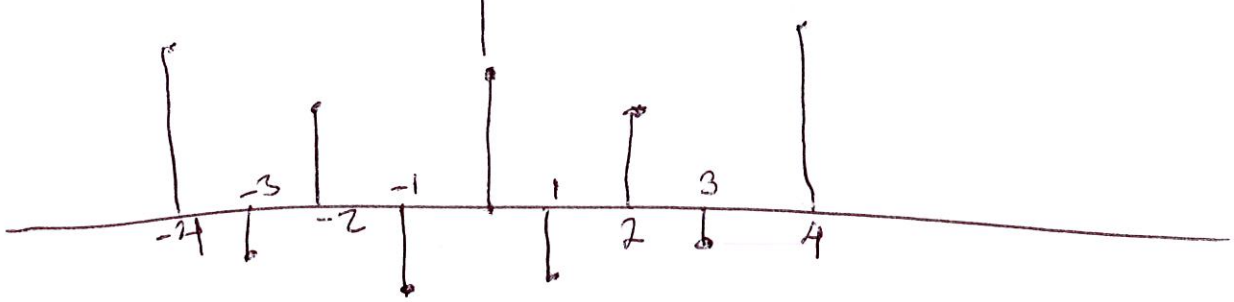
Even signal: - Identical to its time-reversed counterpart

- Positive-time part of the signal is identical to the negative-time part

$$x(t) = x(-t)$$

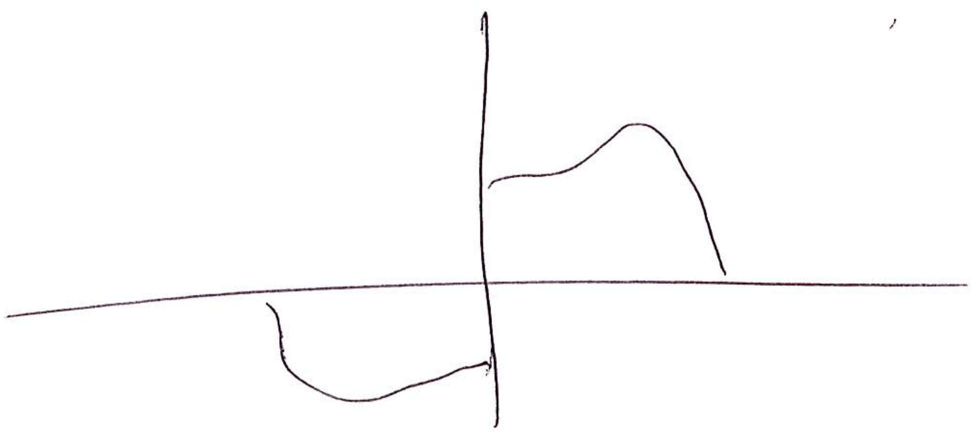
Example

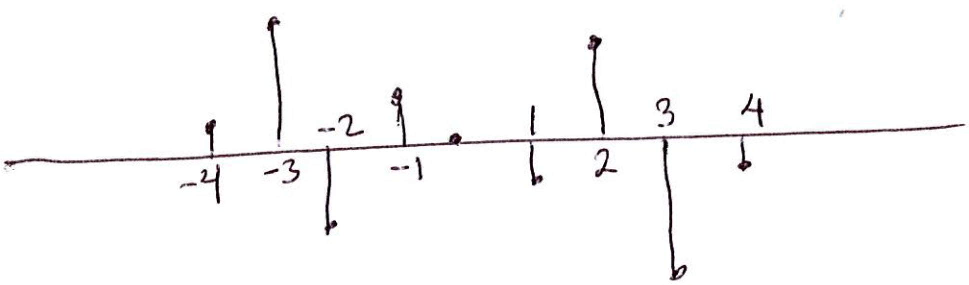




odd Signal:

$$x(t) = -x(-t)$$





The value (amplitude) of any odd signal at time $t=0$ is always 0

Even part of a signal

$$\mathcal{E}v\{x(t)\} = \frac{1}{2}\{x(t) + x(-t)\}$$

Odd part of a signal

$$\mathcal{O}d\{x(t)\} = \frac{1}{2}\{x(t) - x(-t)\}$$

Check that $\mathcal{E}v\{x(t)\}$ is always an even signal

and that $\mathcal{O}d\{x(t)\}$ is always an odd signal

Will be in Homework #1

Exponential and Sinusoidal Signals

Basic Building blocks

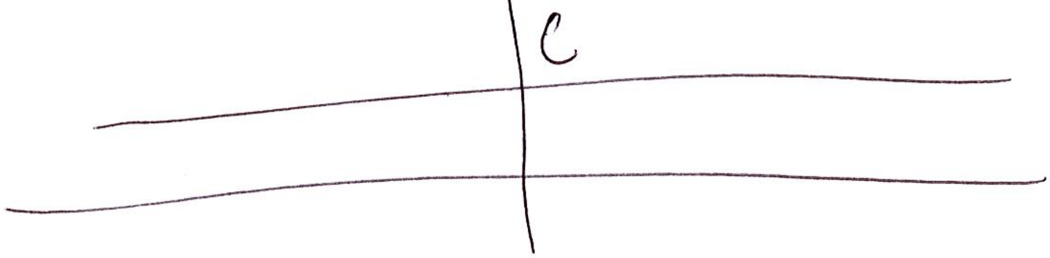
$$x(t) = C e^{at}$$

In general, C and a can be complex values with real and imaginary parts

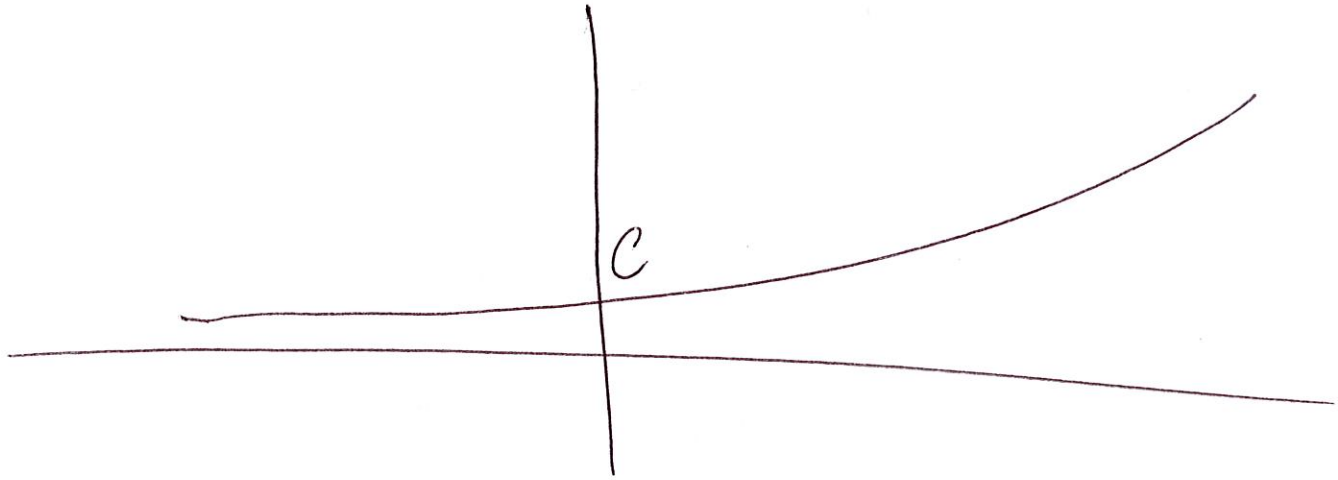
If C and a are both real, then we say that $x(t)$ is a real exponential

If $a=0$

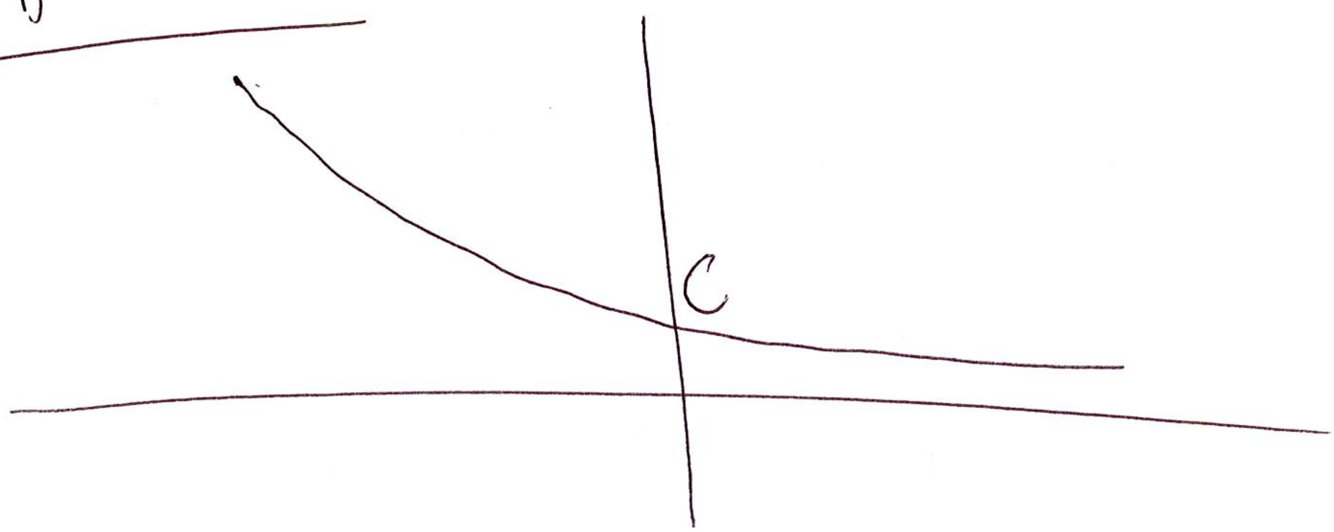
$$x(t) = C$$



If $a > 0$



If $a < 0$



If a is purely imaginary

$$x(t) = e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$$

$$e^{j\omega_0 t} = e^{j\omega_0 (t+T)}$$

$$= e^{j\omega_0 t} \, e^{j\omega_0 T}$$

If T is a period, then $e^{j\omega_0 T} = 1$

$$\Rightarrow \cos \omega_0 T + j \sin \omega_0 T = 1$$

$$\Rightarrow \cos \omega_0 T = 1$$

& $\sin \omega_0 T = 0$

$$\Rightarrow \omega_0 T = 2\pi$$

T_0 (fundamental period) $= \dfrac{2\pi}{|\omega_0|}$

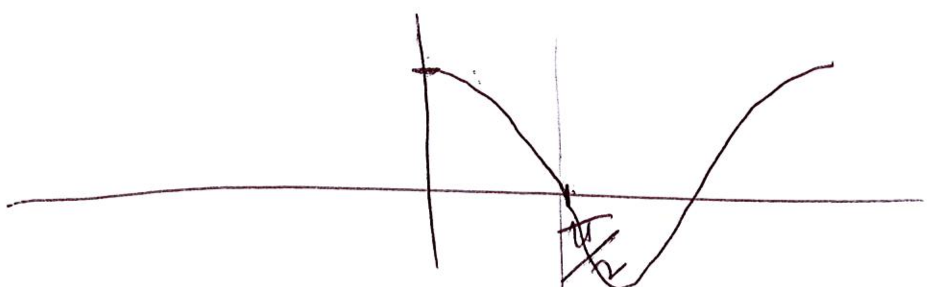
$$x(t) = e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} \left|\cos\omega_0 t + j\sin\omega_0 t\right|^2 dt$$

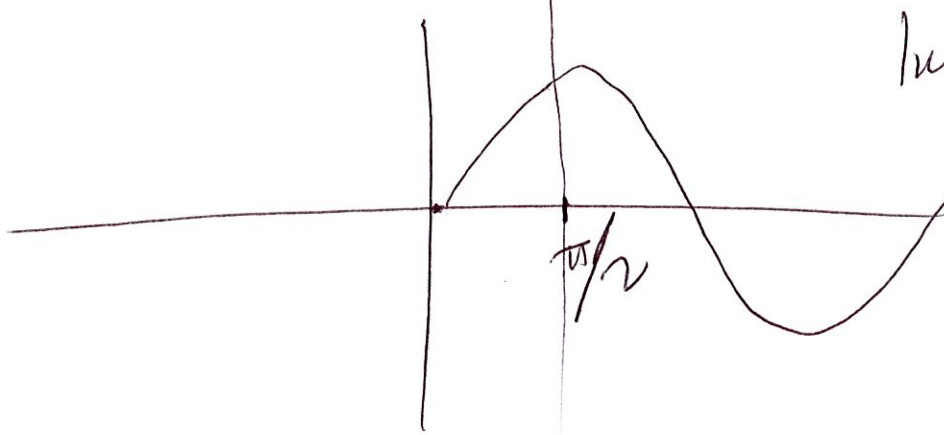
$$= \int_{-\infty}^{\infty} (\cos^2\omega_0 t + \sin^2\omega_0 t)\, dt$$

$$= \int_{-\infty}^{\infty} 1 \, dt = \infty$$

Real Part



$P_\infty =$



Imaginary Part

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |\underbrace{x(t)}_{e^{j\omega_0 t}}|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1 \, dt$$

$$= \lim_{T\to\infty} \frac{2T}{2T} = 1$$

$$x(t) = C\, e^{at}$$

General Complex Exponential Signals

$C = |C|\, e^{j\theta}$ polar form

$a = r + j\omega_0$ rectangular form

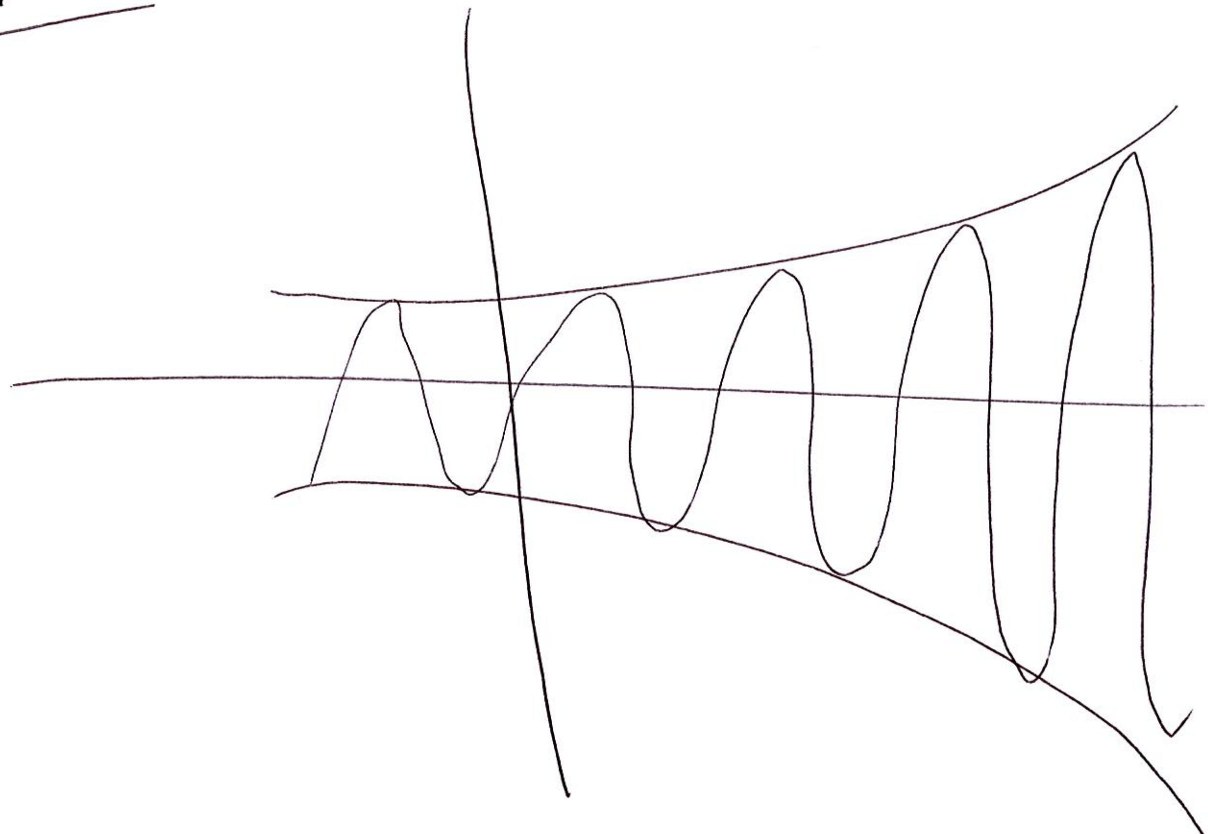
$$x(t) = |C|\, e^{j\theta}\, e^{(r + j\omega_0)t}$$

$$= |C|\, e^{rt}\, e^{j(\omega_0 t + \theta)}$$

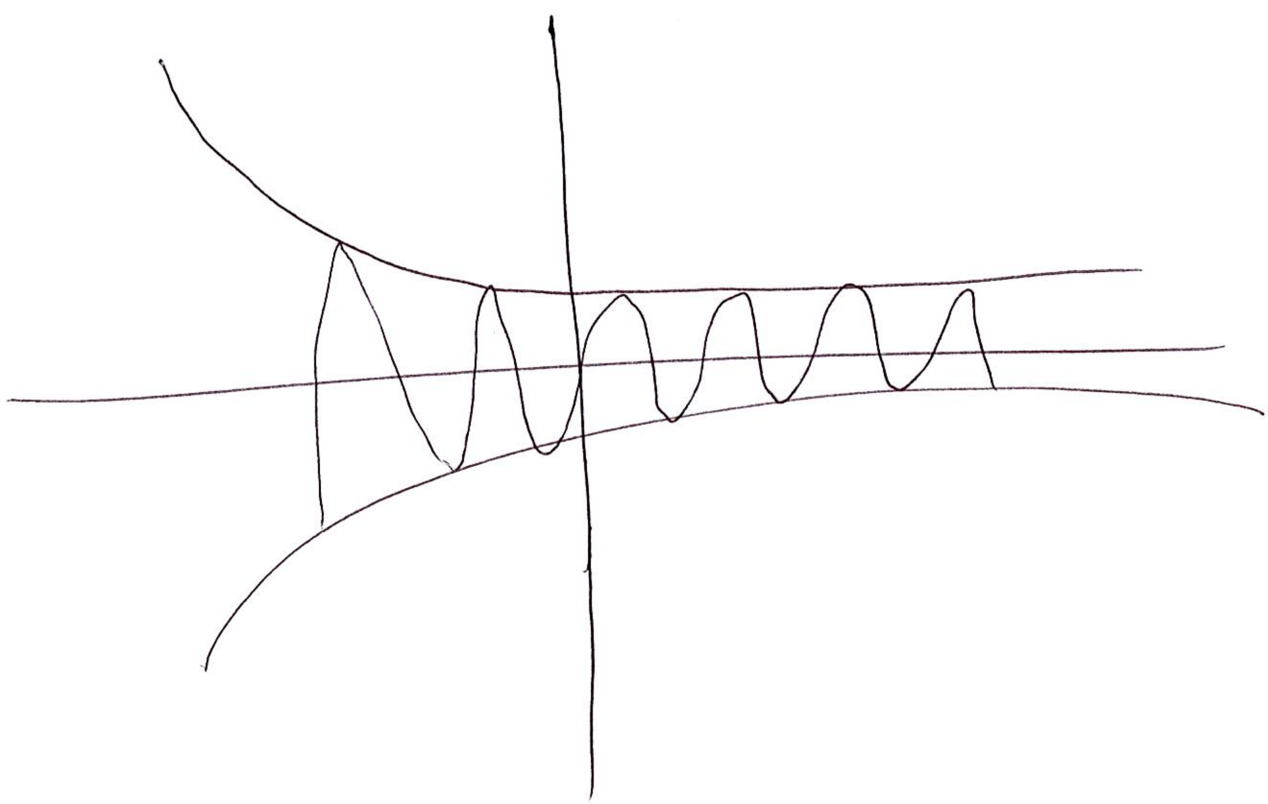
$$= |C|\, e^{rt} \cos(\omega_0 t + \theta)$$

$$+ j\, |C|\, e^{rt} \sin(\omega_0 t + \theta)$$

$r > 0$



$r < 0$



Harmonically related set of complex exponentials is a set of periodic signals whose frequencies are all multiples of a single fundamental frequency

positive

$$\omega_0 = \frac{2\pi}{T_0}$$

$$\omega_k = k\,\omega_0 \qquad k = 0, \pm 1, \pm 2, \pm 3, \ldots$$

$$\phi_k(t) = e^{j\omega_k t}$$

Fundamental frequency

$$|\omega_k| = |k|\,\omega_0$$

Fundamental Period $= \dfrac{2\pi}{|k|\,\omega_0} = \dfrac{T_0}{|k|}$