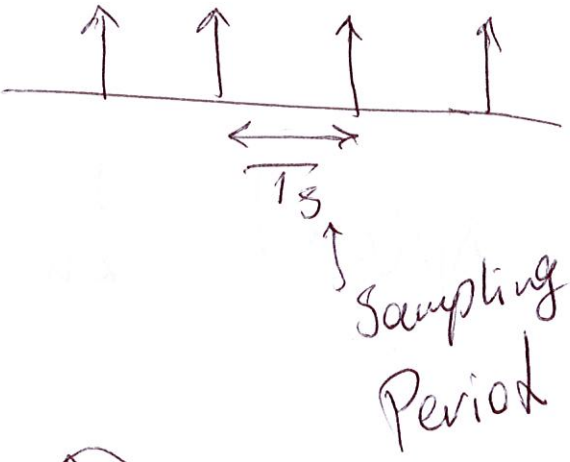
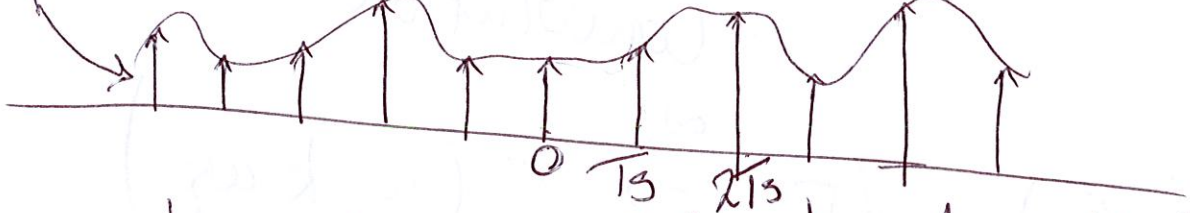
Impulse Train Sampling



$x_p(t) = x(t)\ p(t)$



When can we reconstruct $x_p(t)$ from $x(t)$?

Interpolation

If there is any way to interpolate the samples to get the original signal?

What happens in Frequency domain?

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s)$$

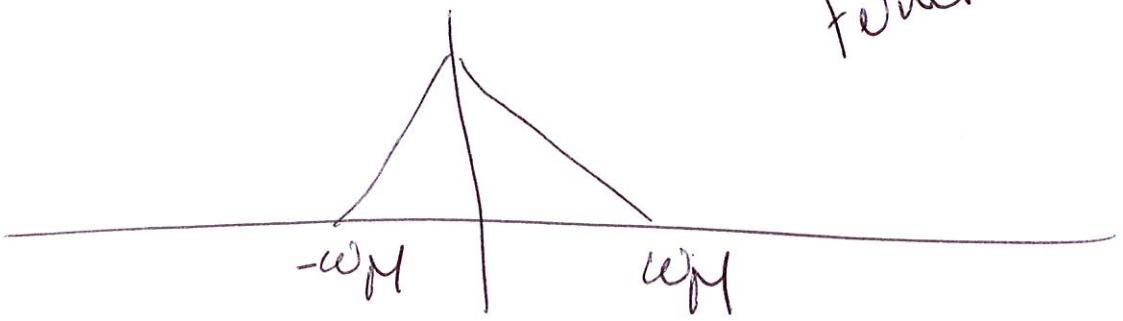
$$X_p(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta)\,P(j(\omega-\theta))\,d\theta$$

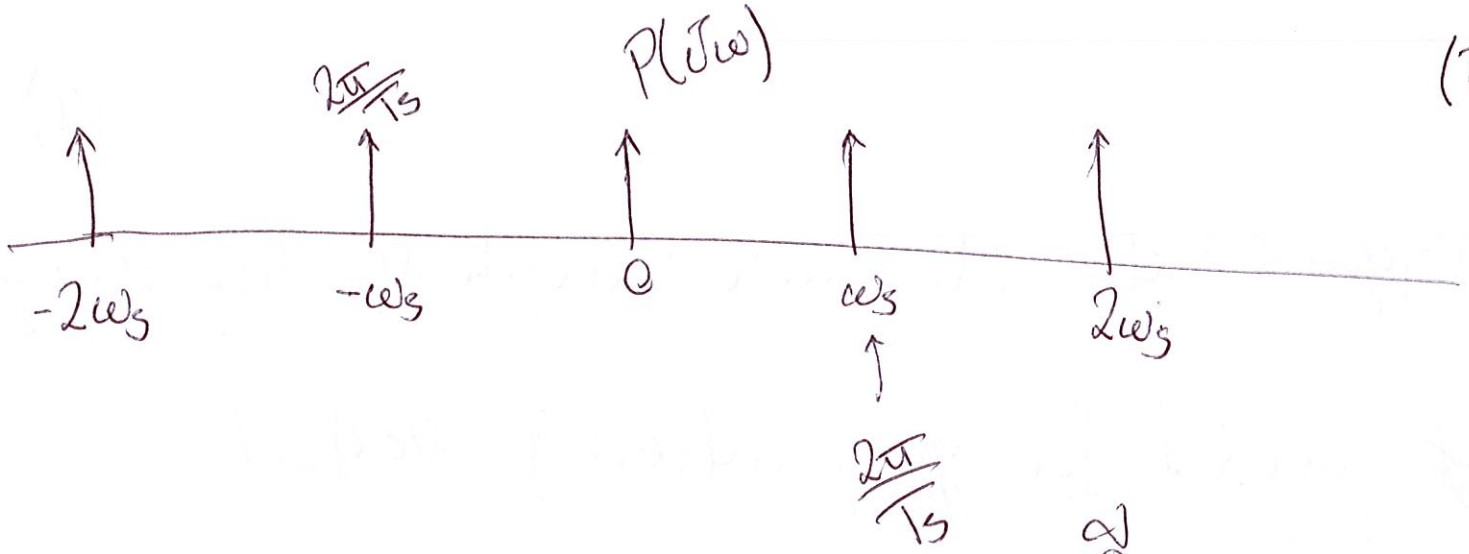
Convolution

$$P(j\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

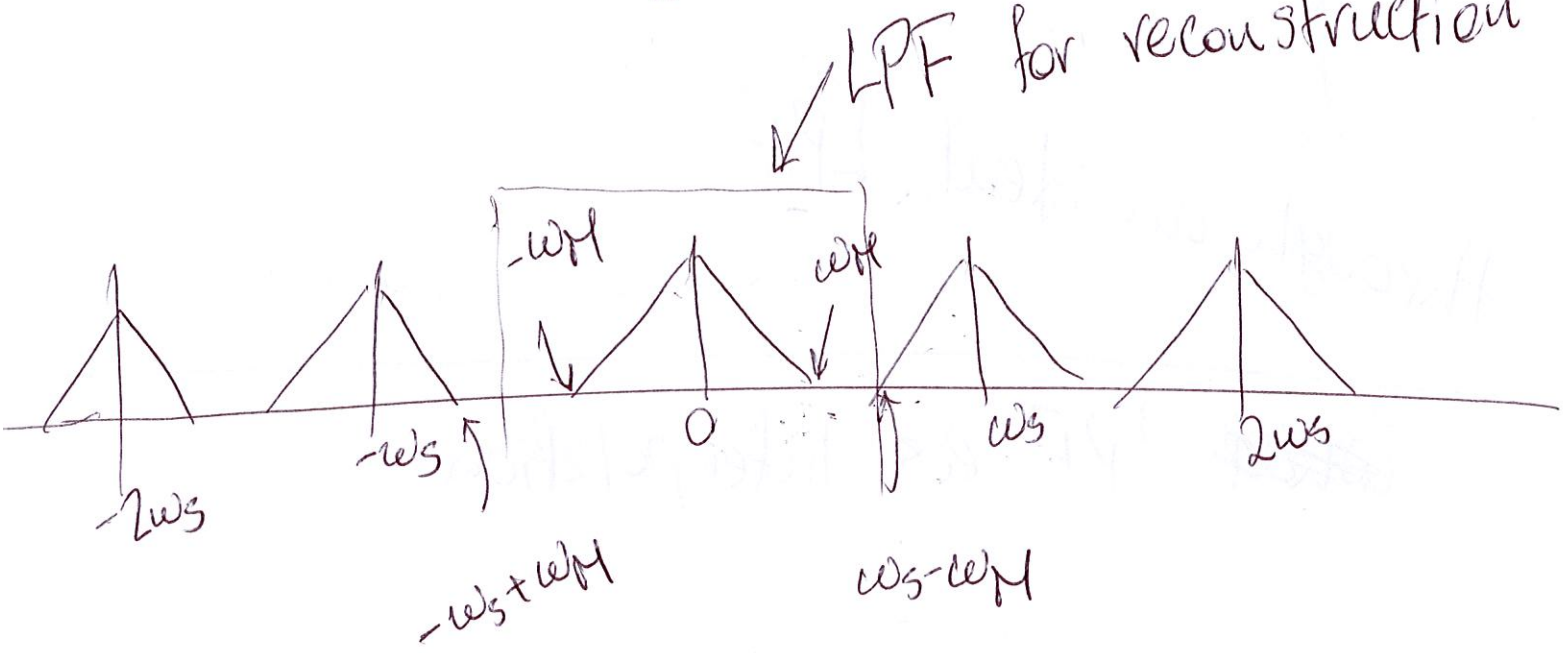
$\omega_s = \frac{2\pi}{T_s}$

Assume $X(j\omega)$ Band-limited function





$$\Downarrow \quad X_p(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\,P(j(\omega-\theta))\,d\theta$$



That will happen only if

$$\omega_s - \omega_M > \omega_M$$

$$\omega_s > 2\omega_M$$

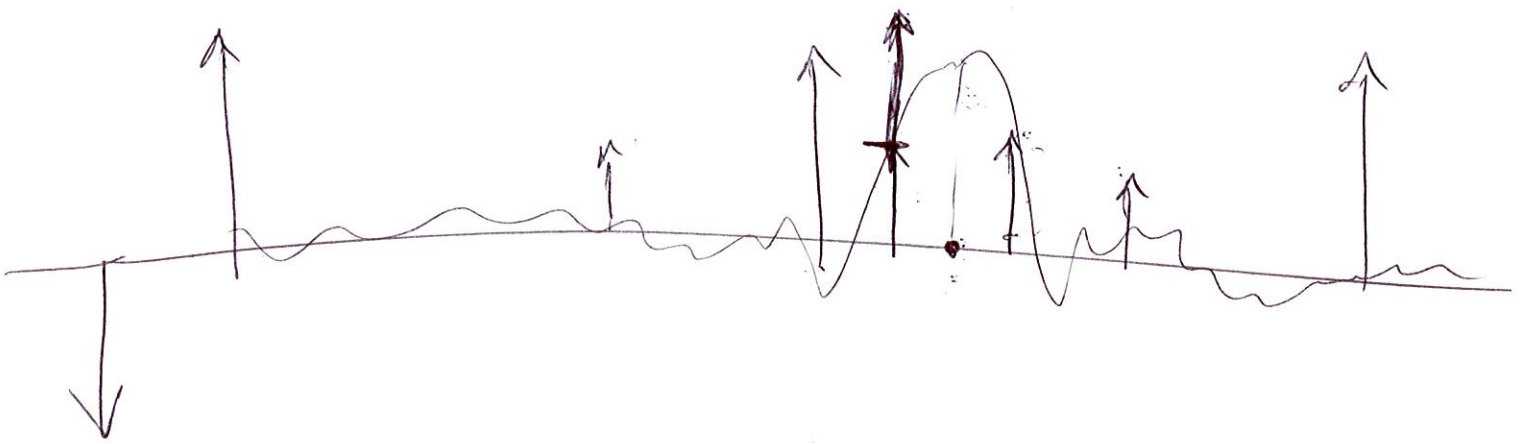
Nyquist Rate

Nyquist Rate: Minimum sampling frequency needed for guaranteeing perfect reconstruction of the original signal

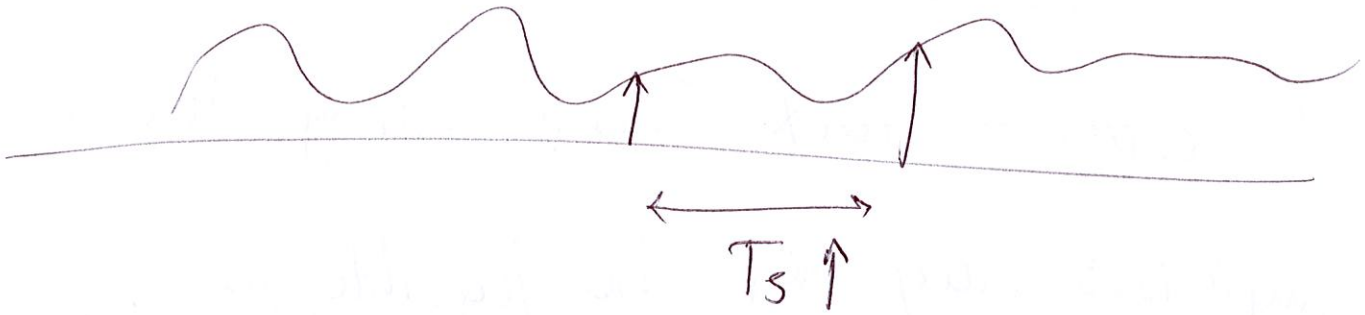
↑

through an ideal LPF

LPF as Interpolation



What if we sample too slowly?



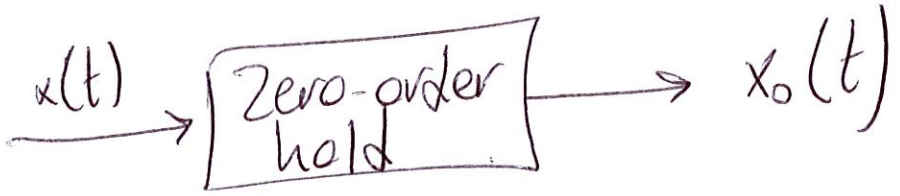
Impossible to reconstuct perfectly

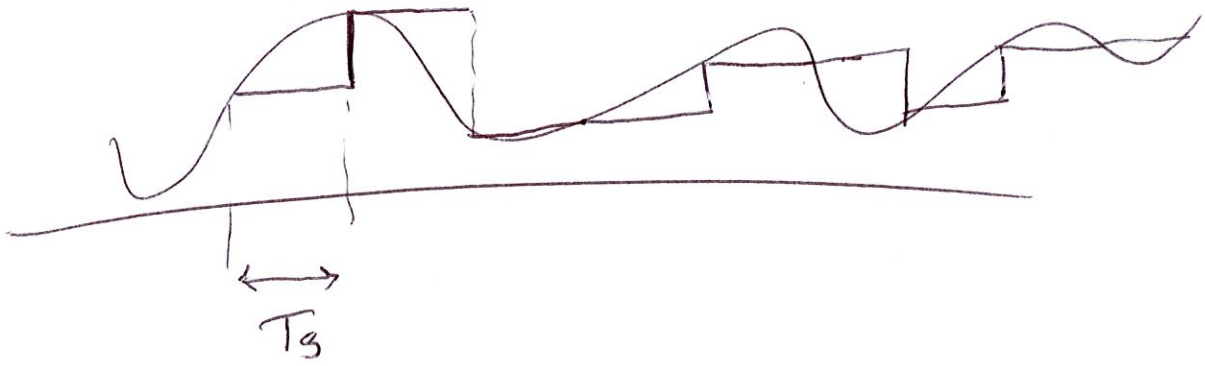
Aliasing

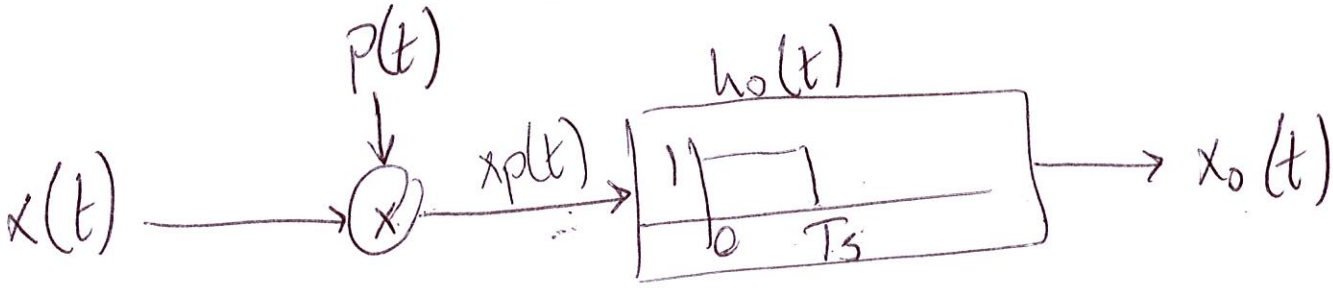
That happens if $\omega_s < 2\omega_M$

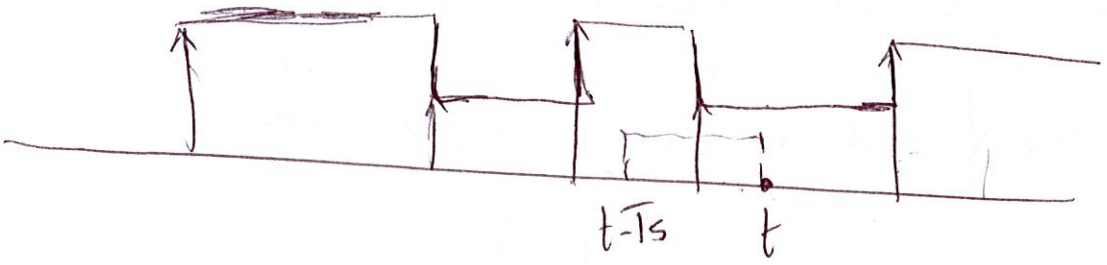
Sampling with Zero-Order Hold

Generating and Transmitting these impulses may not be feasible in practice

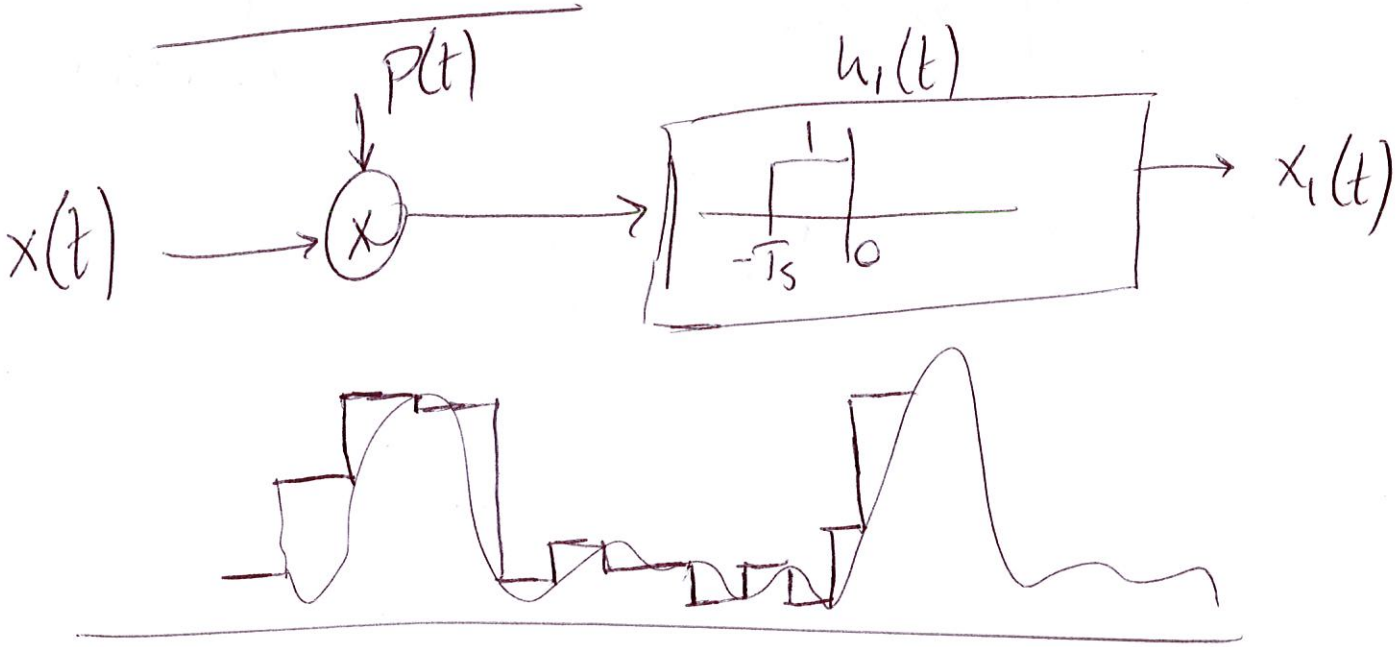








Let's say



How do we reconstruct the original signal from zero-order hold?