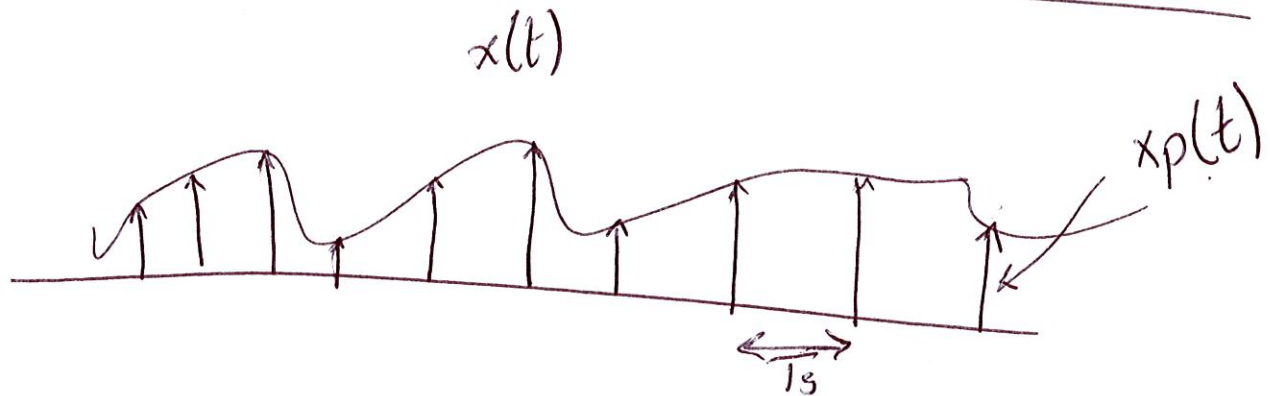
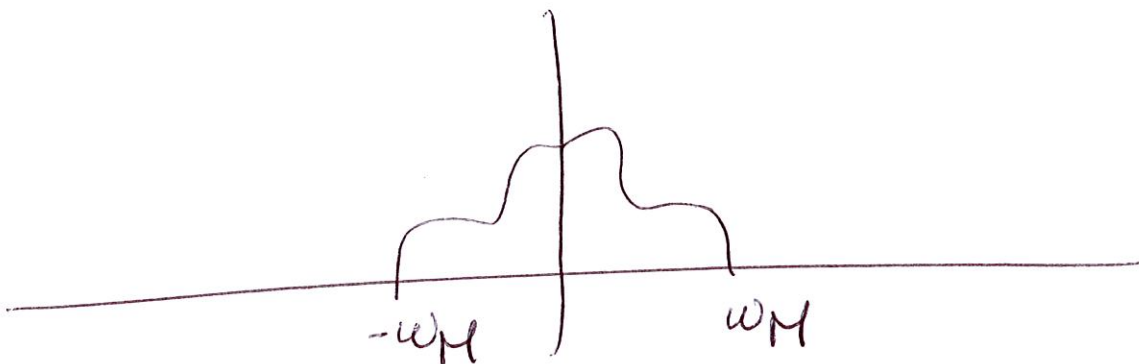


Sampling & Reconstruction



$X(j\omega)$ Band limited

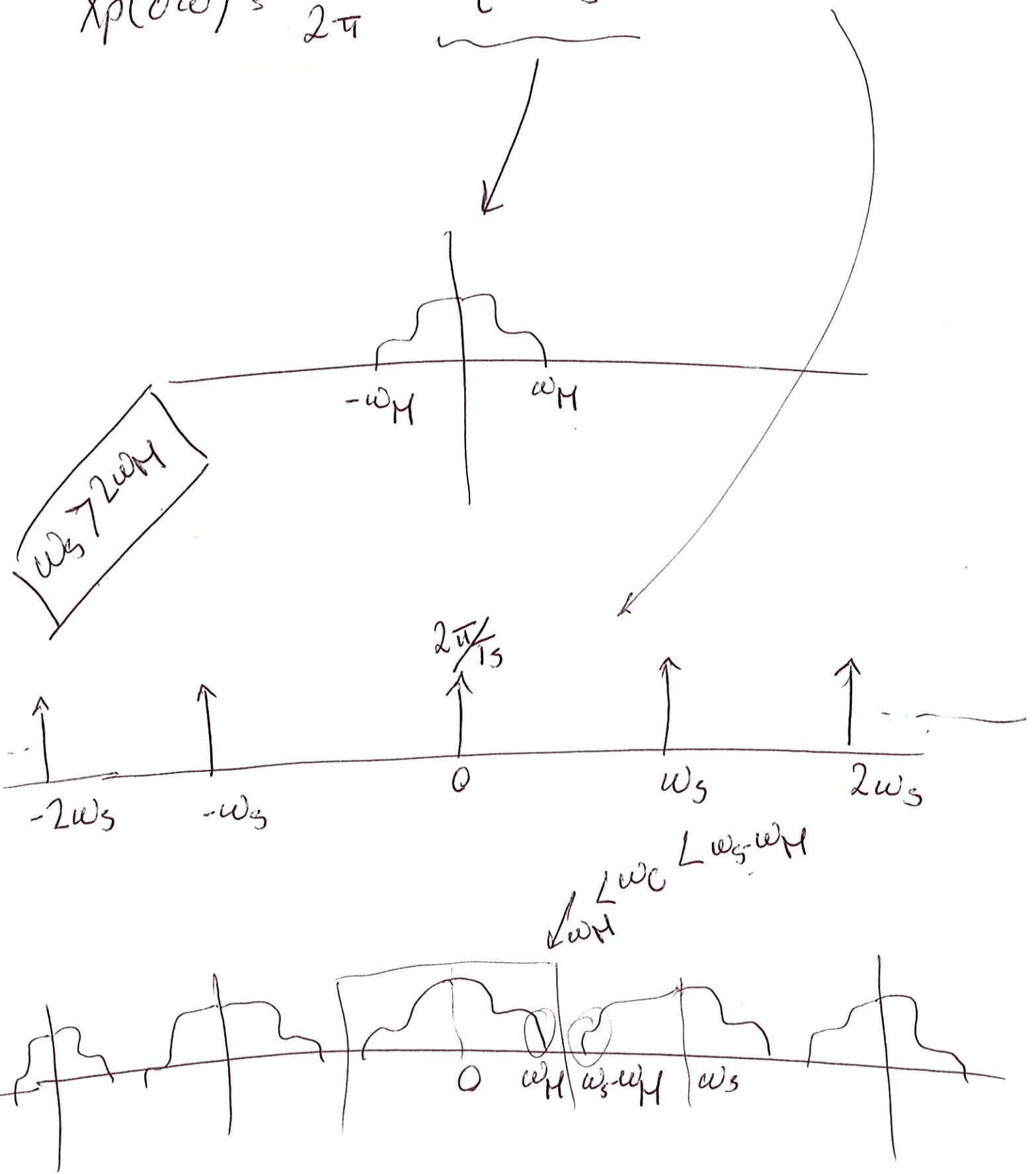


$$x_p(t) = x(t) p(t) \leftarrow \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

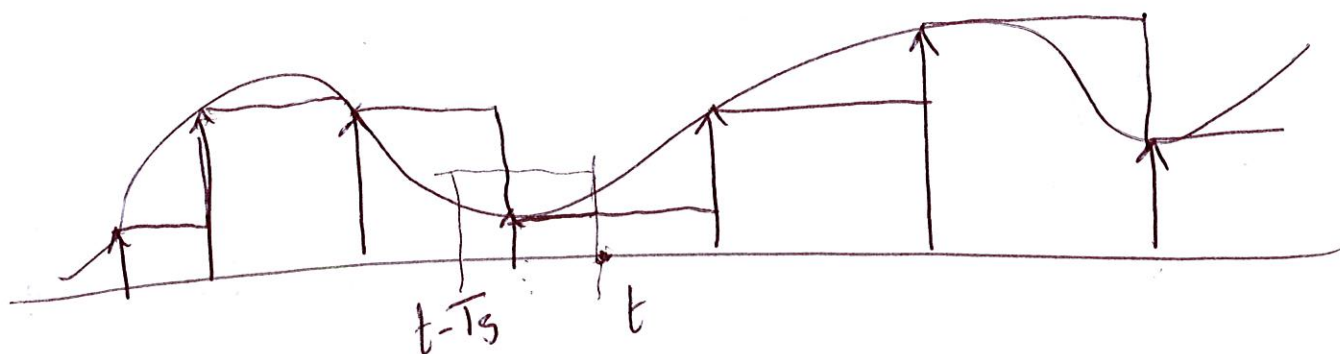
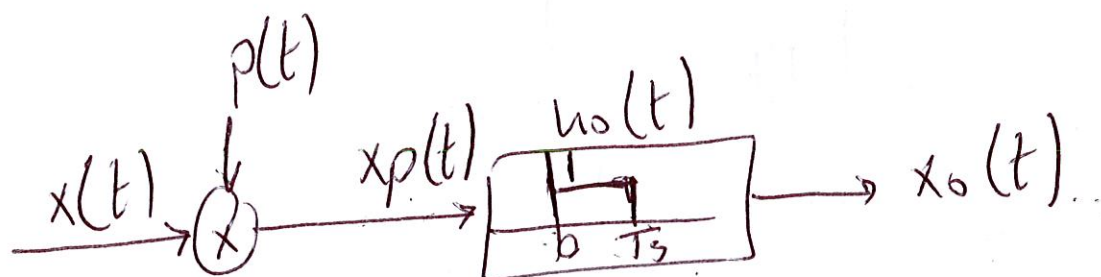
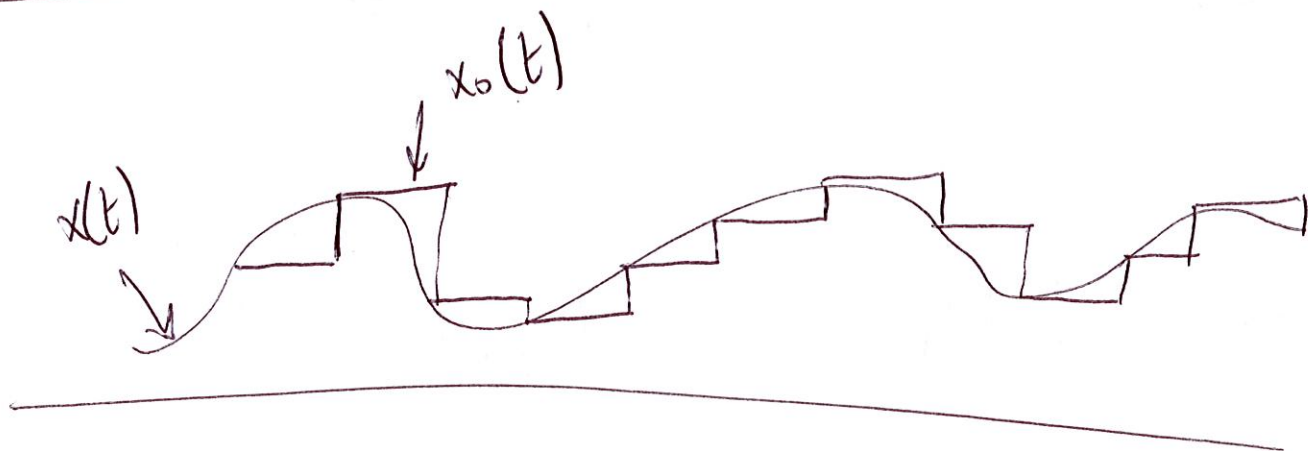
$$P(j\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s) \quad \frac{2\pi}{T_s}$$

(2)

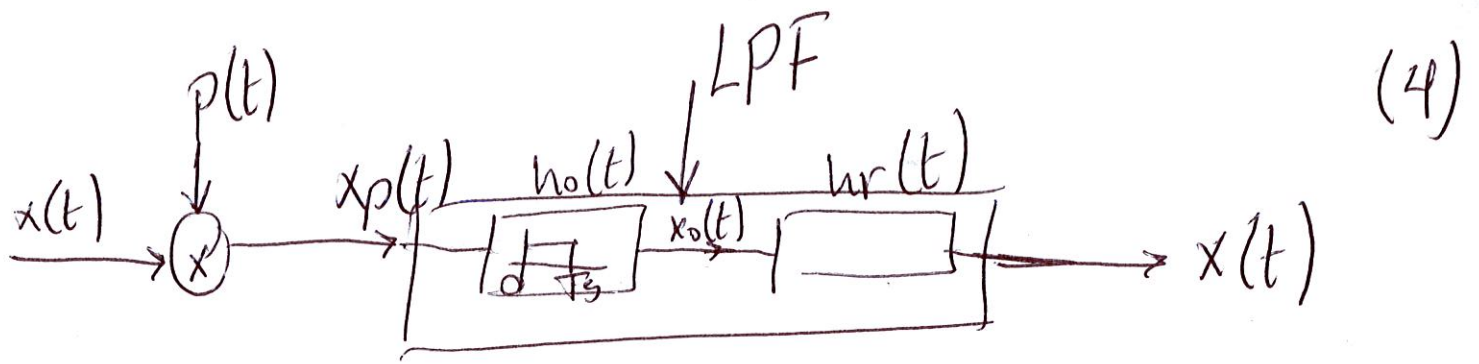
$$X_p(j\omega) = \frac{1}{2\pi} \underbrace{\mathcal{F}\{x(t)\}} * \mathcal{F}\{p(t)\}$$



Sampling with a Zero-order Hold (3)



How to reconstruct?



$$H_o(j\omega) H_r(j\omega) = H_{LPF}(j\omega)$$

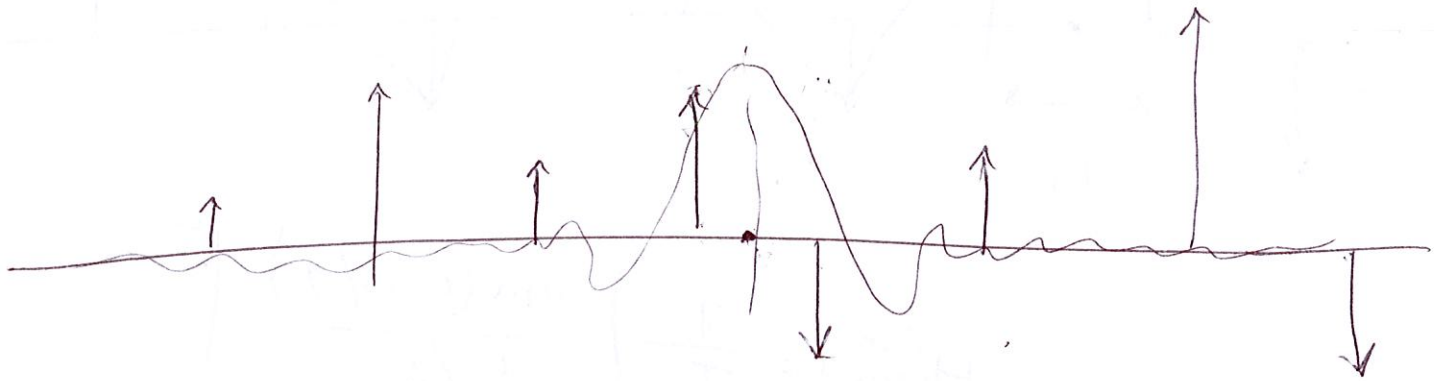
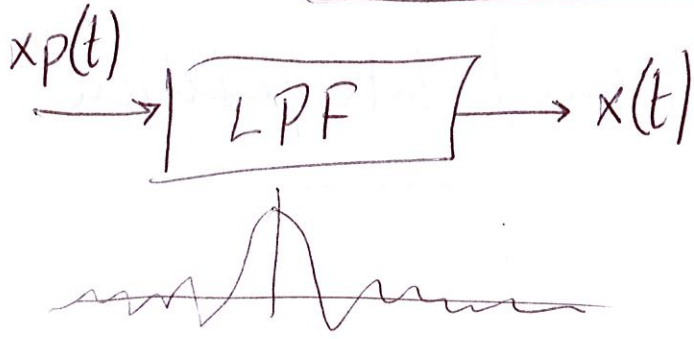
$$H_r(j\omega) = \frac{H_{LPF}(j\omega)}{H_o(j\omega)}$$

$$H_o(j\omega) = e^{-j\omega \frac{T_s}{2}} \left[\frac{2 \sin(\omega T_s/2)}{\omega} \right]$$

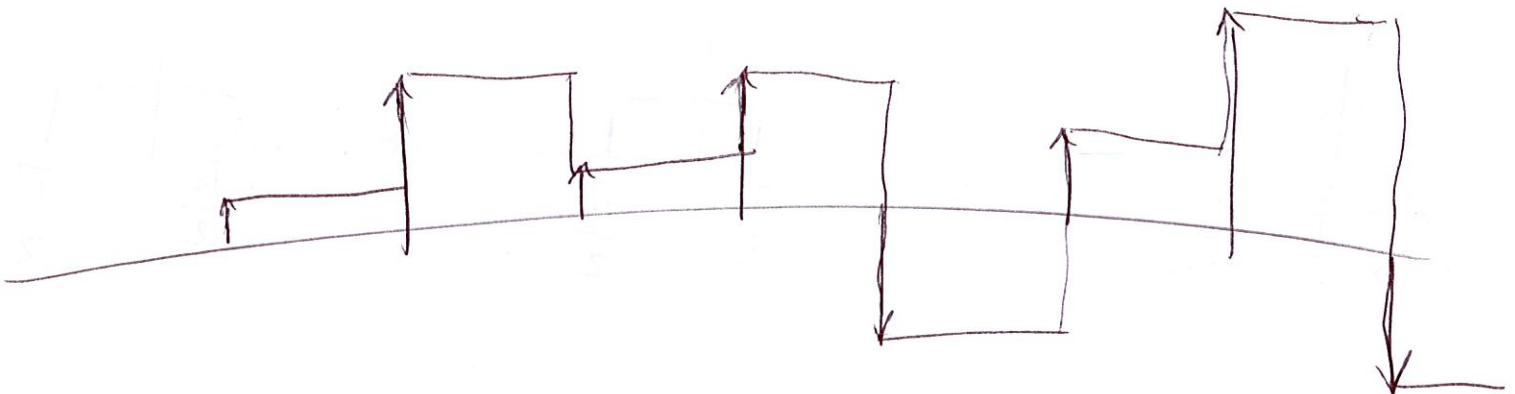
$$H_r(j\omega) = \frac{\omega e^{j\omega T_s/2} H_{LPF}(j\omega)}{2 \sin(\omega T_s/2)}$$

$\omega_c = 2\omega_M$

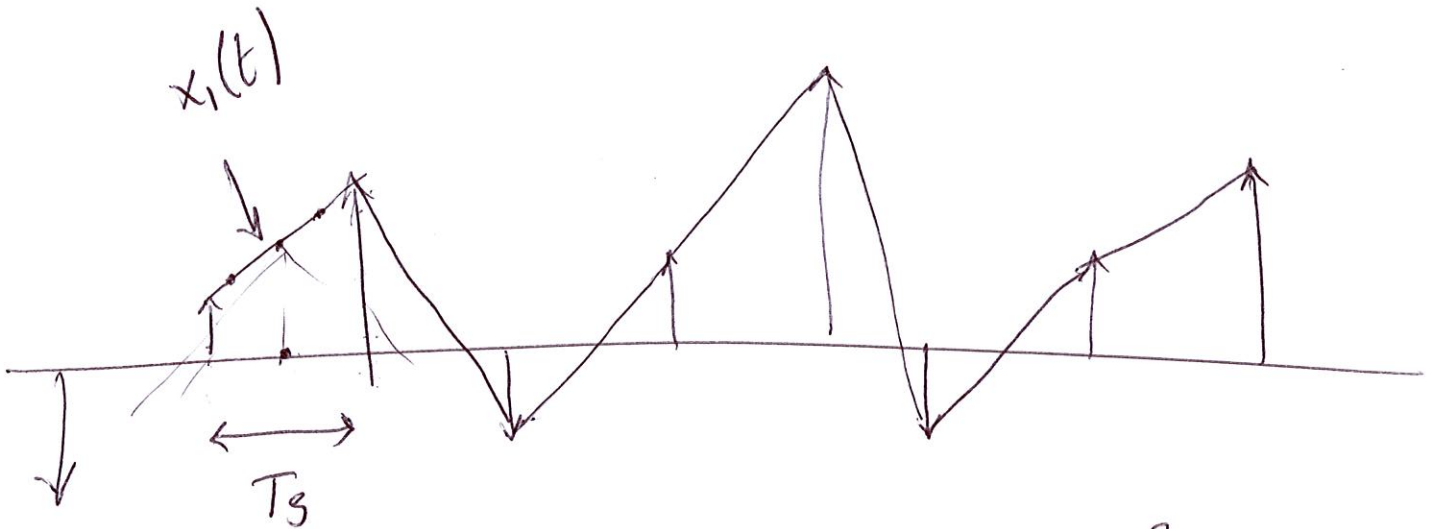
Reconstruction as Interpolation (5)



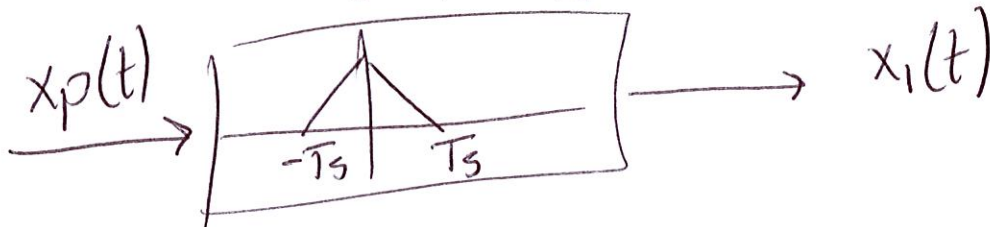
Zero-order Hold for reconstruction



First-order (Linear) Interpolation

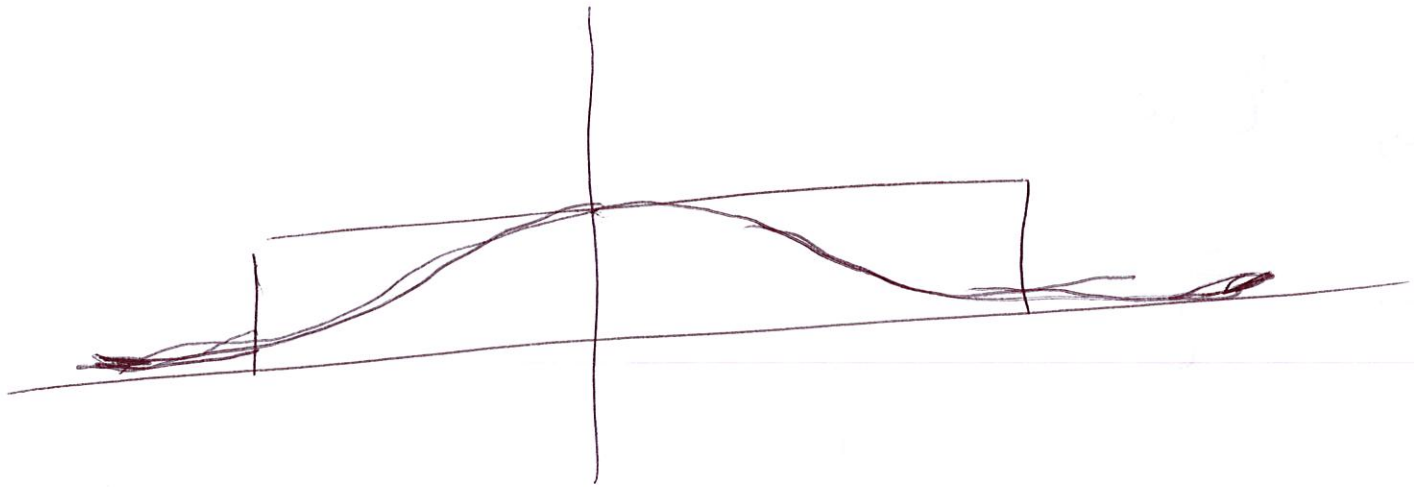


$$H(j\omega) = \frac{1}{T_s} \left[\frac{\sin(\omega T_s/2)}{\omega/2} \right]^2$$



A diagram illustrating the convolution process. On the left is a triangular pulse centered at zero with base from $-T_s$ to T_s . This is followed by an equals sign, then a rectangular pulse centered at zero with base from $-\frac{T_s}{2}$ to $\frac{T_s}{2}$. This is followed by an asterisk, then another rectangular pulse centered at zero with base from $-\frac{T_s}{2}$ to $\frac{T_s}{2}$.

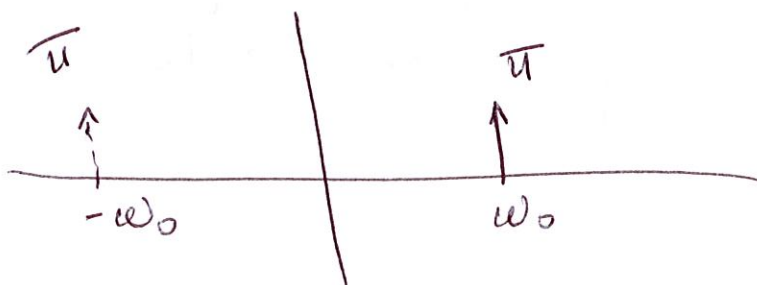
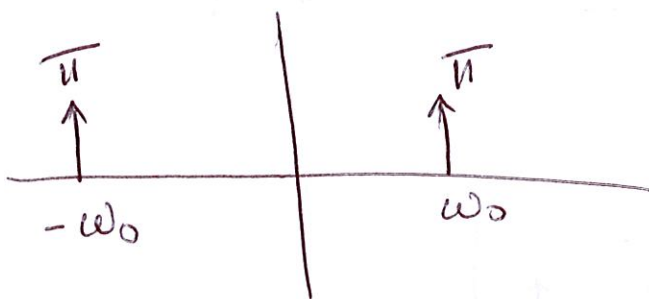
(7)



What would happen if we under-sample?

Sub-Nyquist
Sampling

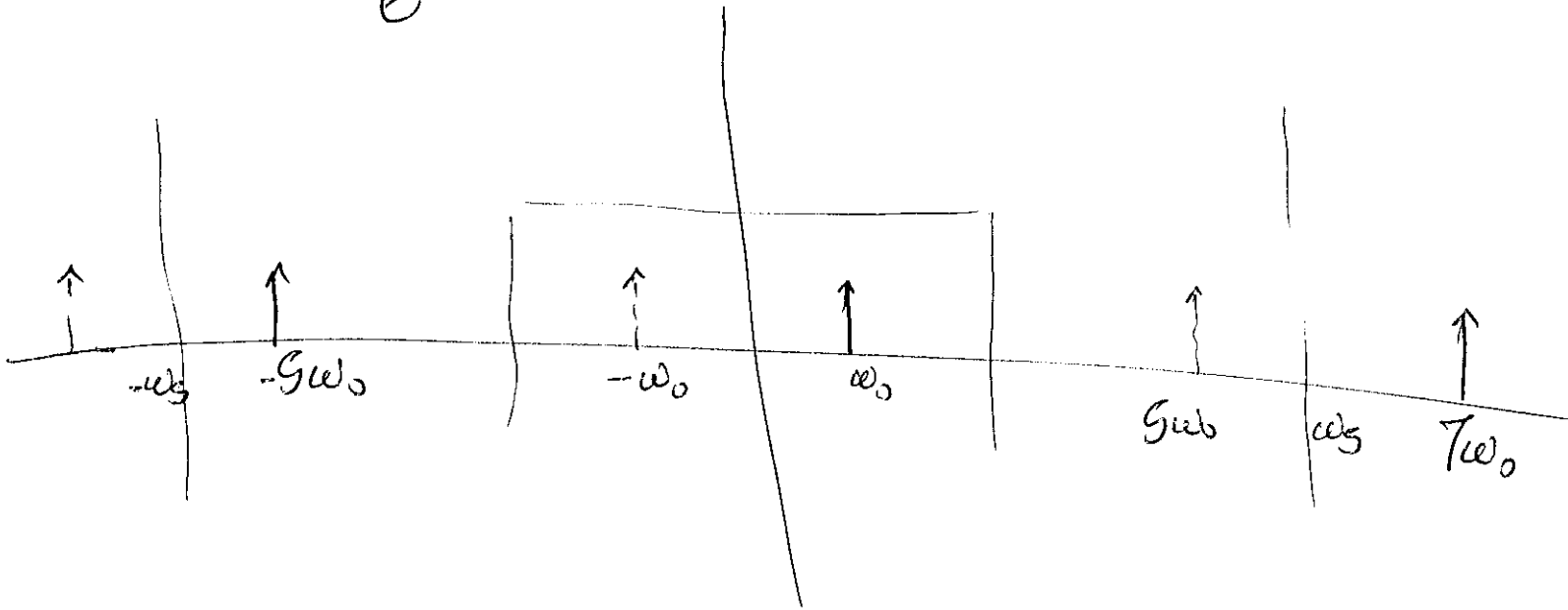
$$x(t) = \cos \omega_0 t$$



(8)

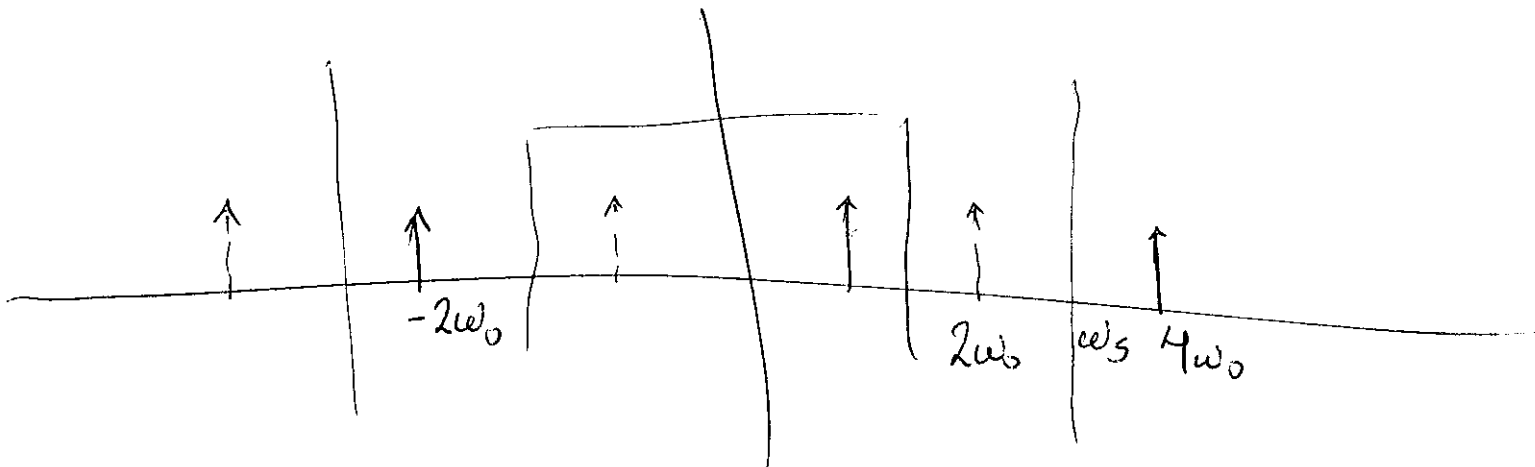
$$\omega_s = 6 \omega_0$$

$$\omega_0 = \frac{\omega_s}{6}$$



$$\omega_s = 3\omega_0$$

$$\omega_0 = \frac{\omega_s}{3}$$

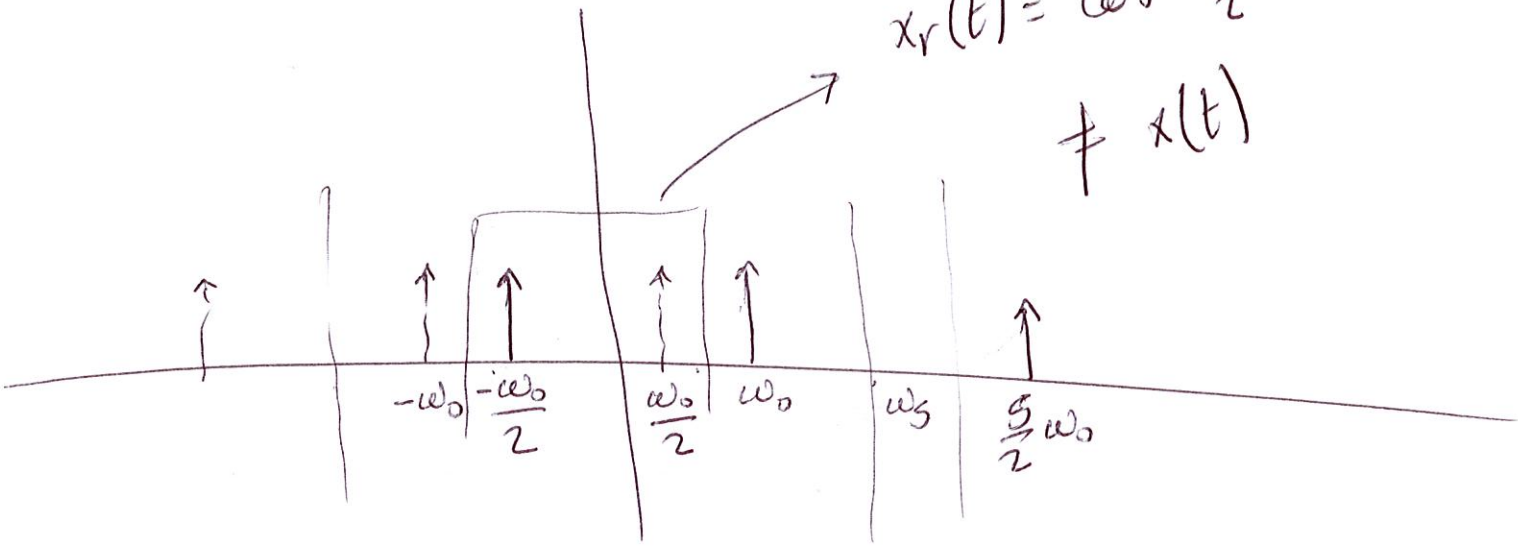


(9)

$$\omega_0 = \frac{2\omega_s}{3}$$

$$\omega_s = \frac{3}{2}\omega_0$$

$$x_r(t) = \cos\frac{\omega_0}{2}t \neq x(t)$$



Aliasing

