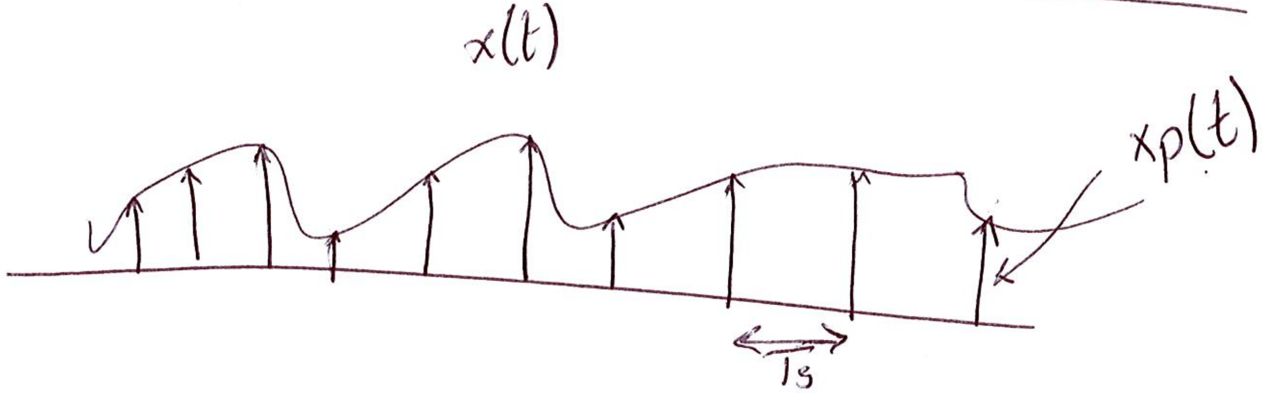
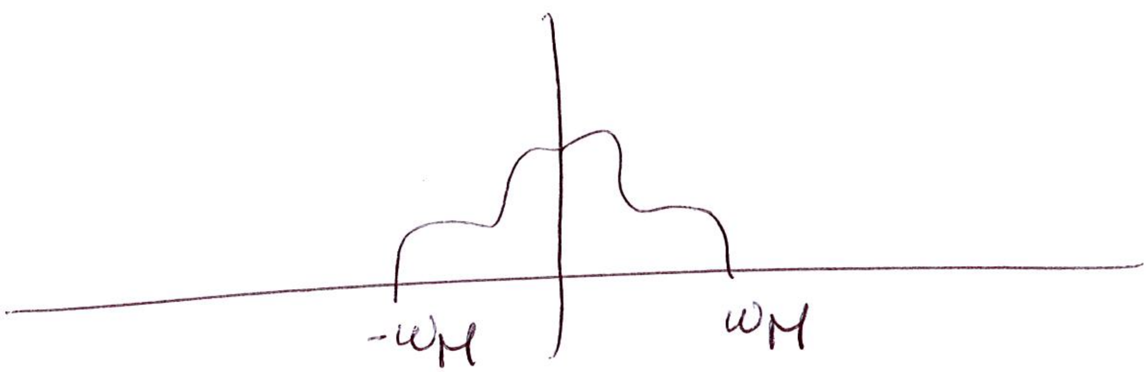
ECE 301 - Lecture # 32

Sampling & Reconstruction



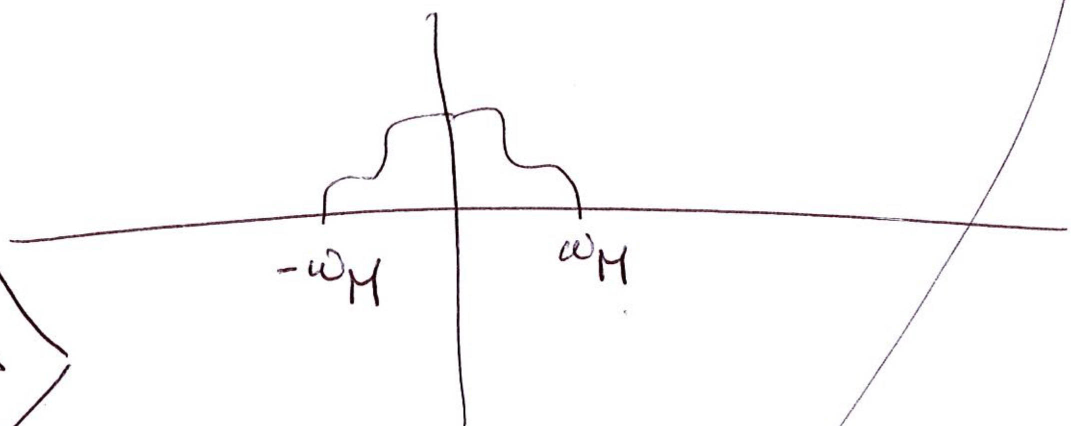
$X(j\omega)$ Band limited



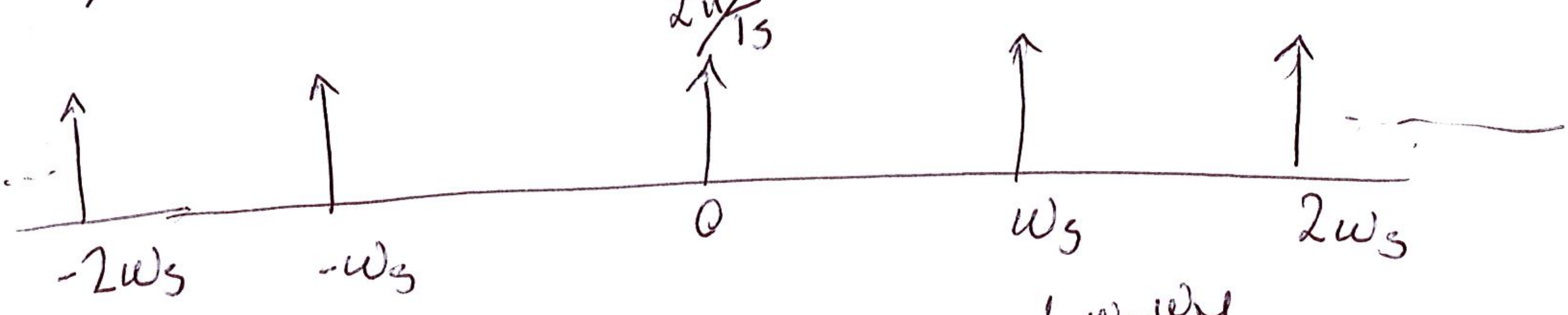
$$x_p(t) = x(t)\,p(t) \qquad p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$P(j\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s) \qquad \omega_s = \frac{2\pi}{T_s}$$

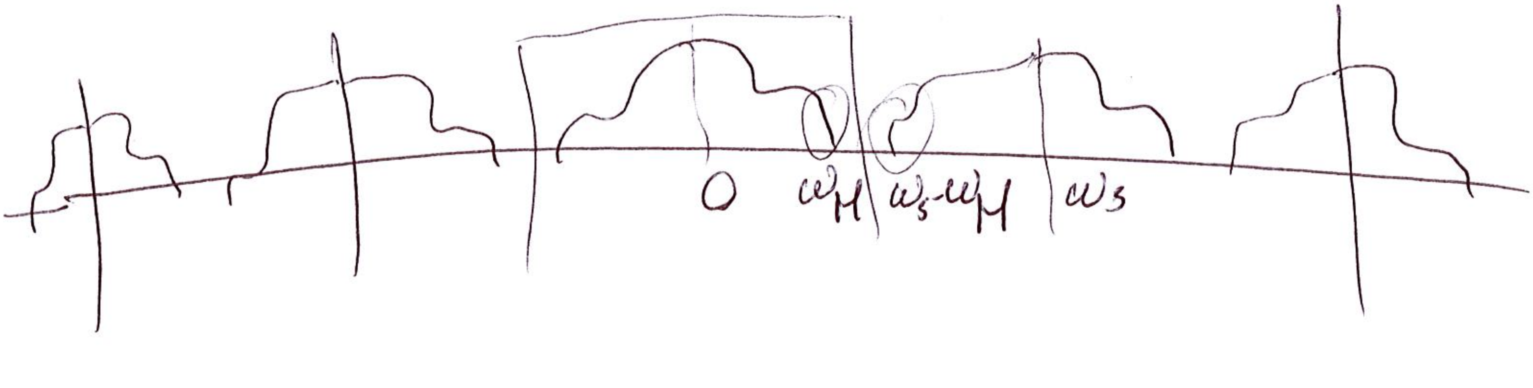
$$X_p(j\omega) = \frac{1}{2\pi} \mathcal{F}\{x(t)\} * \mathcal{F}\{p(t)\}$$



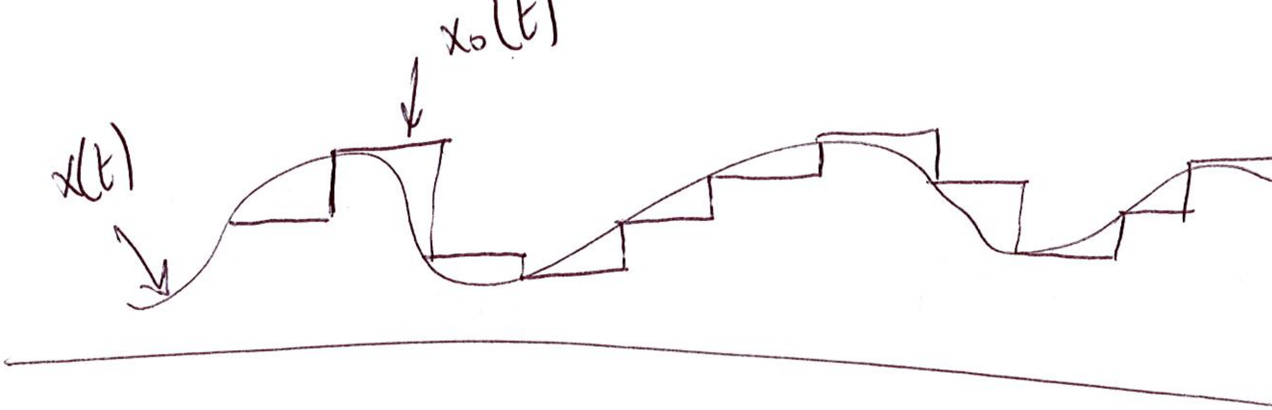
$\boxed{\omega_s > 2\omega_M}$

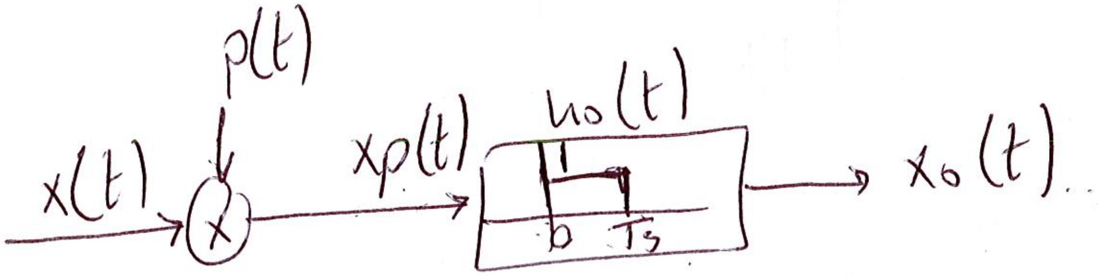


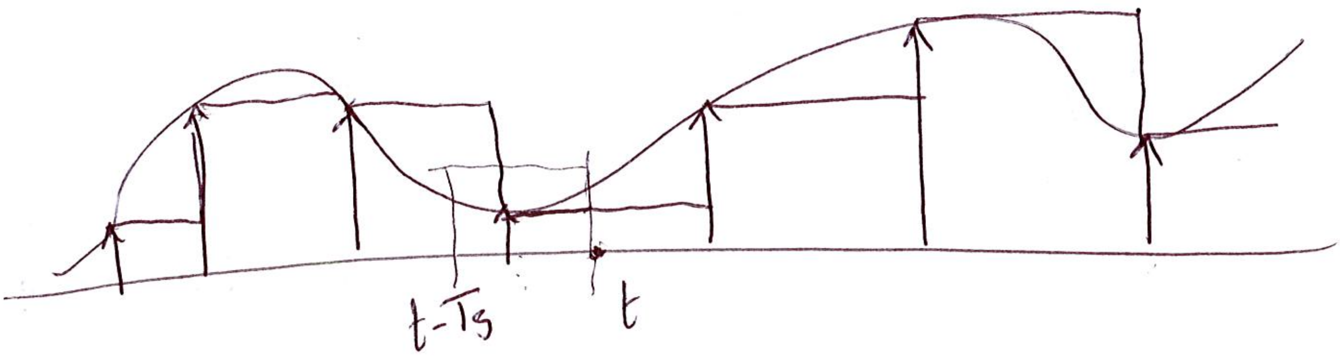
$\omega_M < \omega_c < \omega_s - \omega_M$



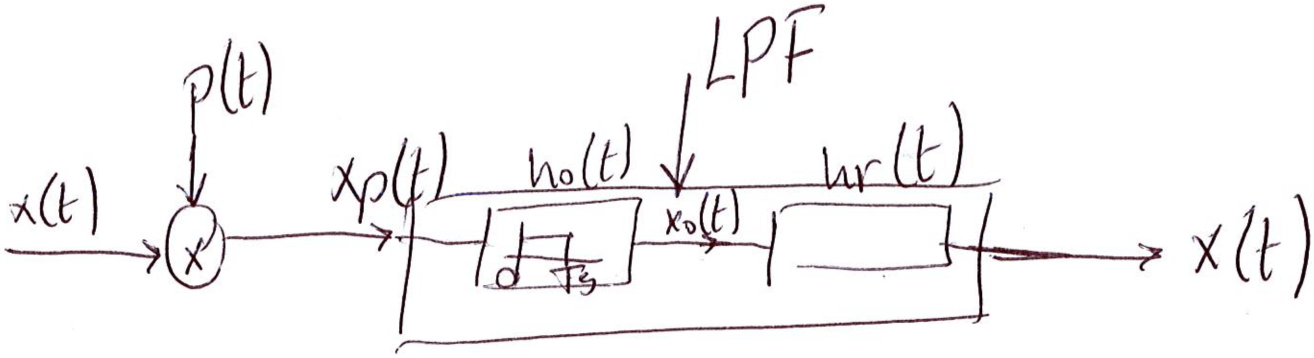
Sampling with a Zero-order Hold







$\boxed{\text{How to reconstruct?}}$



$$H_0(j\omega)\, H_r(j\omega) = H_{LPF}(j\omega)$$

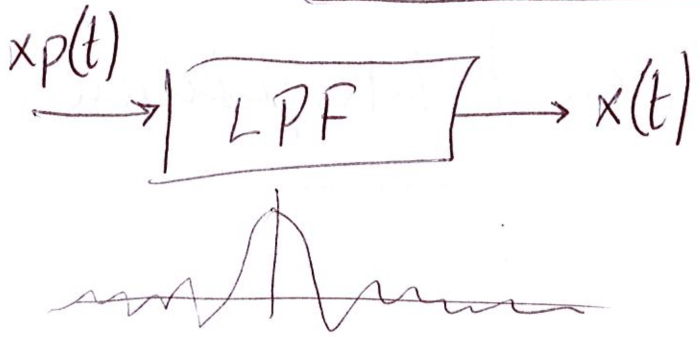
$$H_r(j\omega) = \frac{H_{LPF}(j\omega)}{H_0(j\omega)}$$

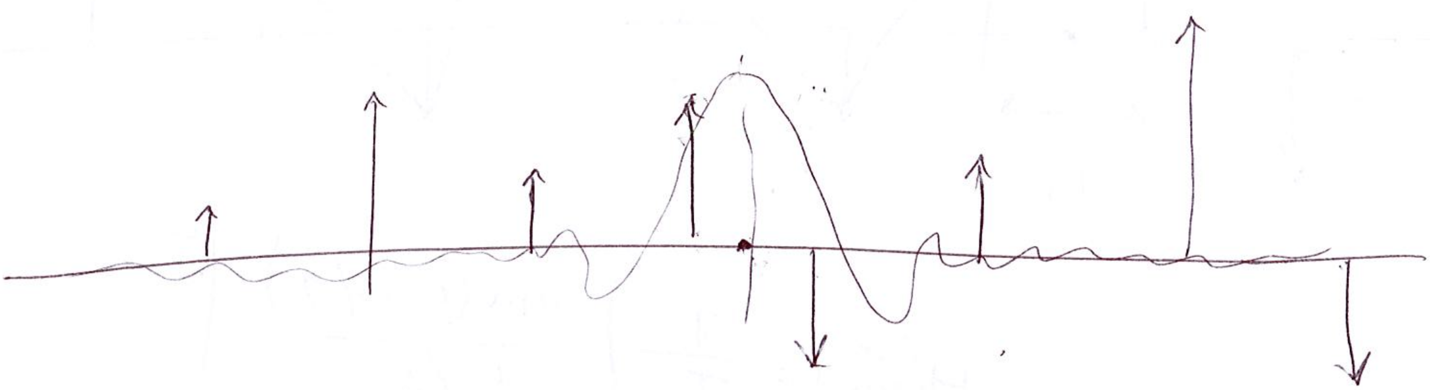
$$H_0(j\omega) = e^{-j\omega \frac{T_s}{2}} \left[\frac{2 \sin(\omega T_s/2)}{\omega} \right]$$

$$\omega_c = 2\omega_M$$

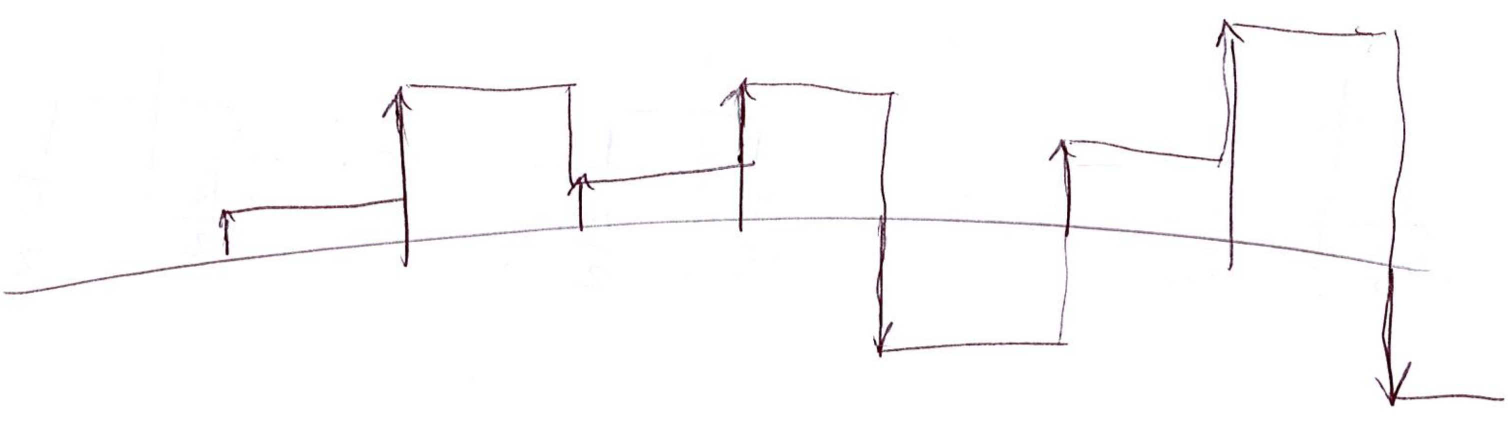
$$H_r(j\omega) = \frac{\omega\, e^{j\omega T_s/2}\, H_{LPF}(j\omega)}{2 \sin(\omega T_s/2)}$$

Reconstruction as Interpolation (5)

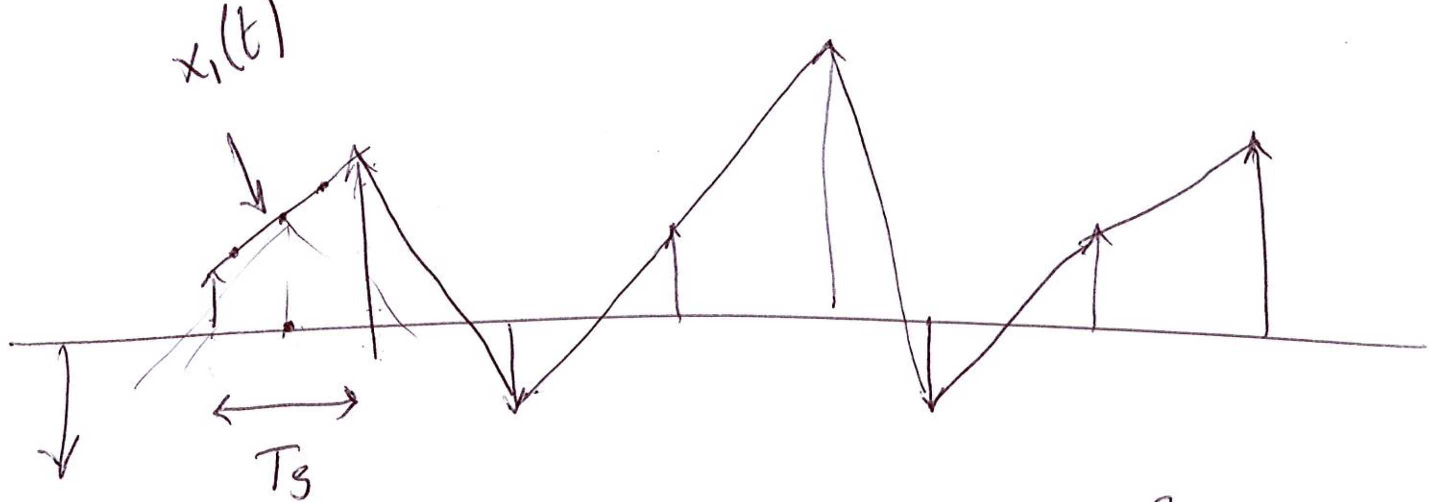




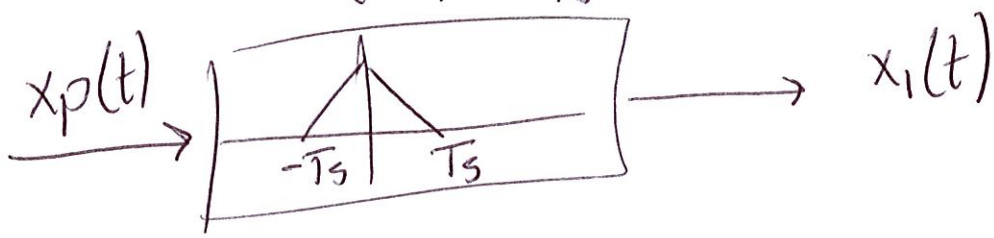
Zero-order Hold for reconstruction

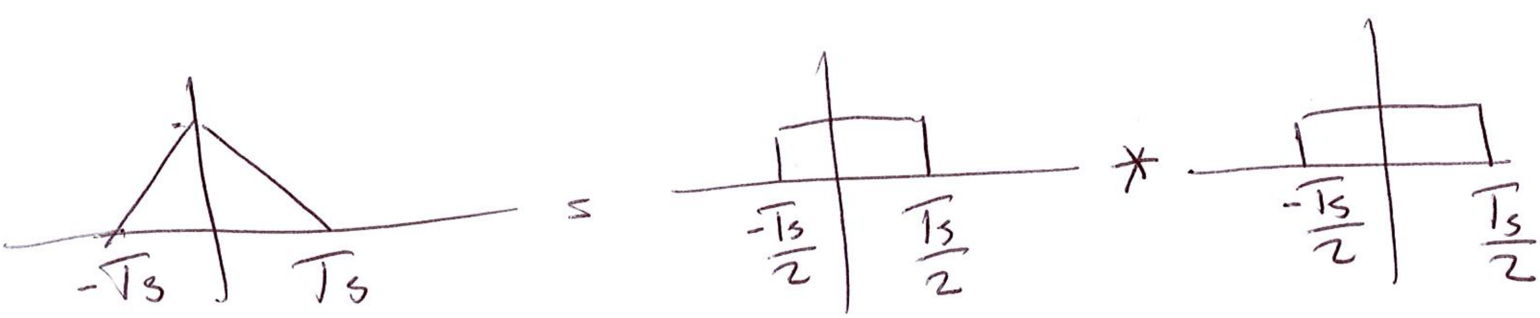


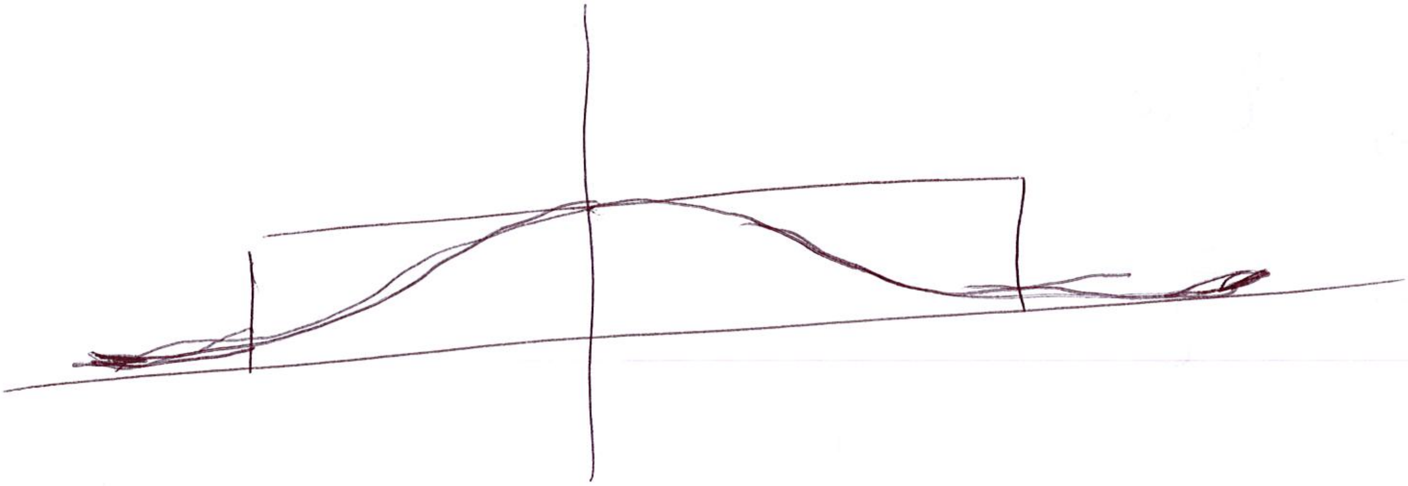
First-order (Linear) Interpolation



$$H(j\omega) = \frac{1}{T_s}\left[\frac{\sin(\omega T_s/2)}{\omega/2}\right]^2$$



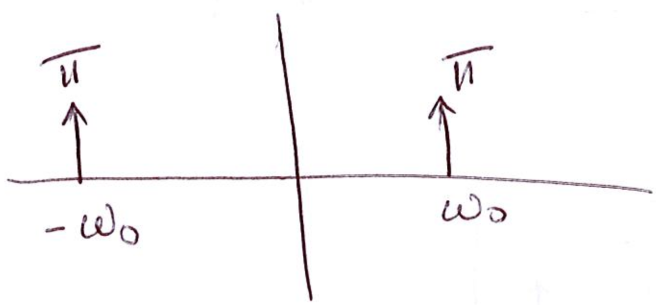


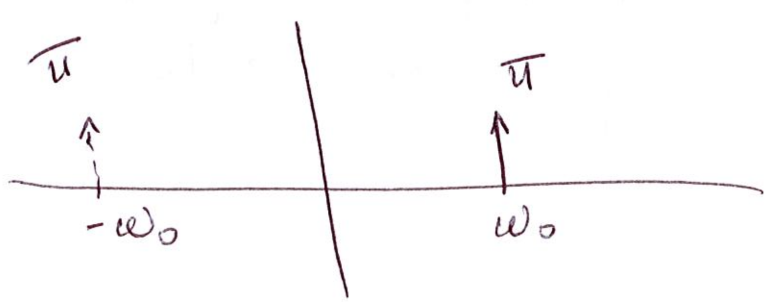


what would happen if we under-sample?

Sub-Nyquist Sampling

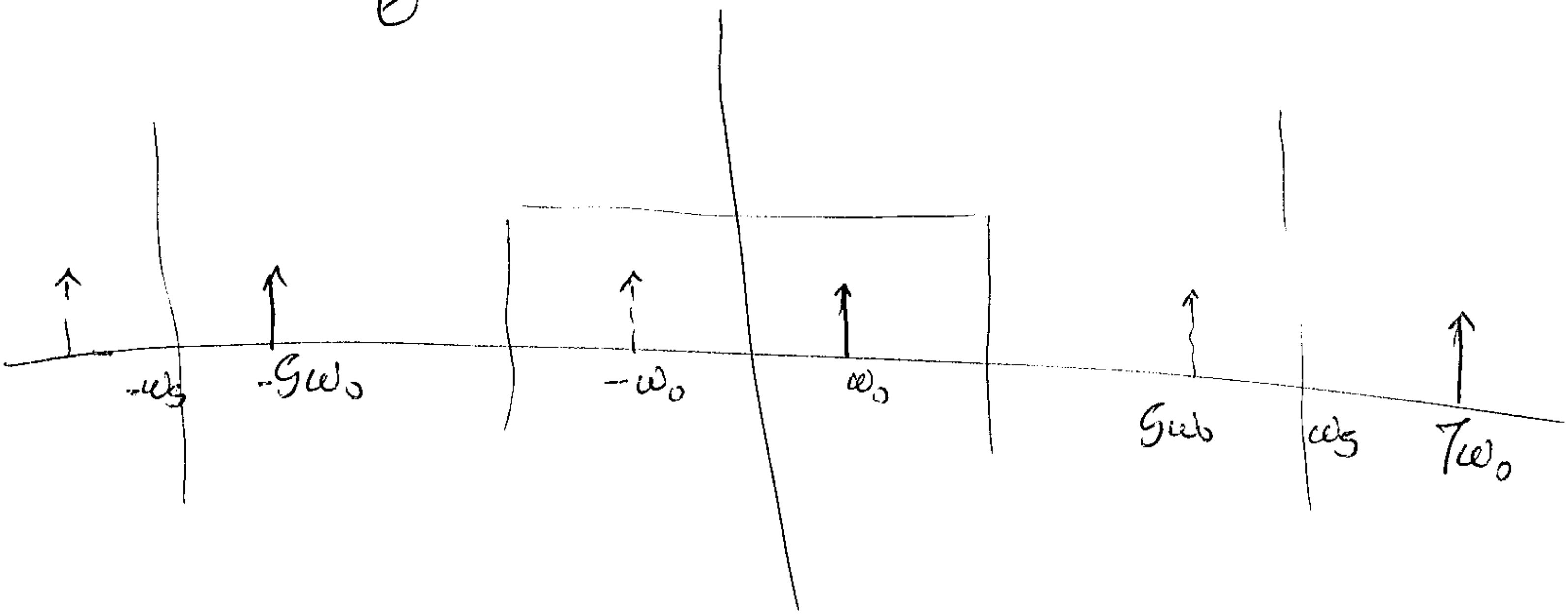
$x(t) = \cos \omega_0 t$



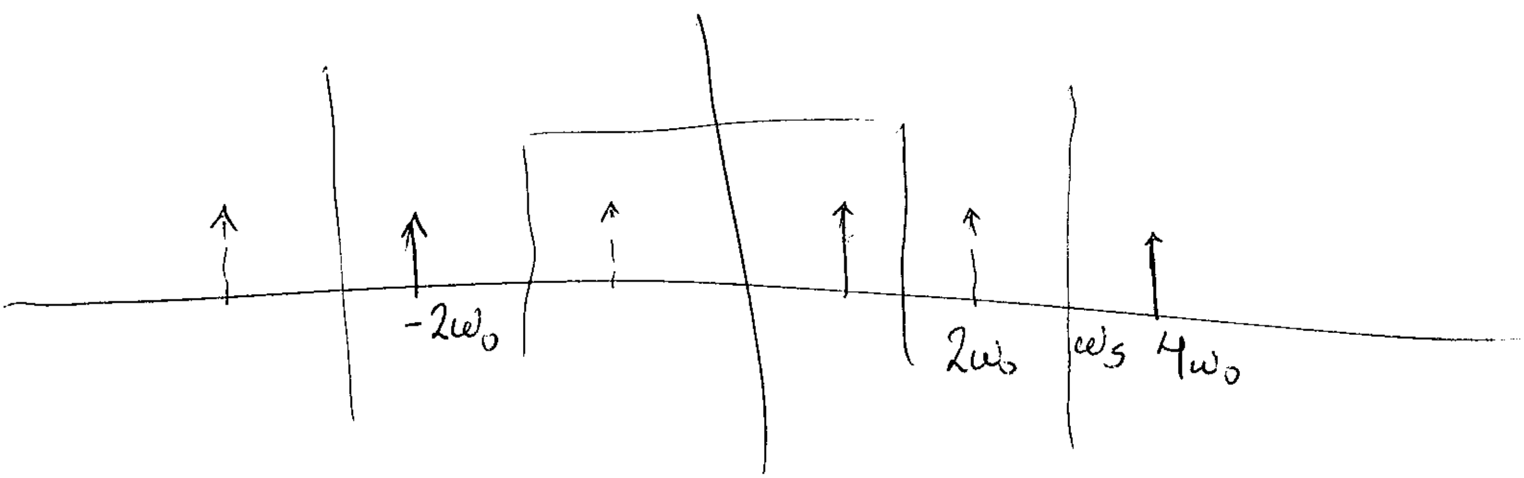


$$\omega_s = 6\,\omega_0$$

$$\omega_0 = \frac{\omega_s}{6}$$



$$\omega_s = 3\,\omega_0 \qquad \omega_0 = \frac{\omega_s}{3}$$

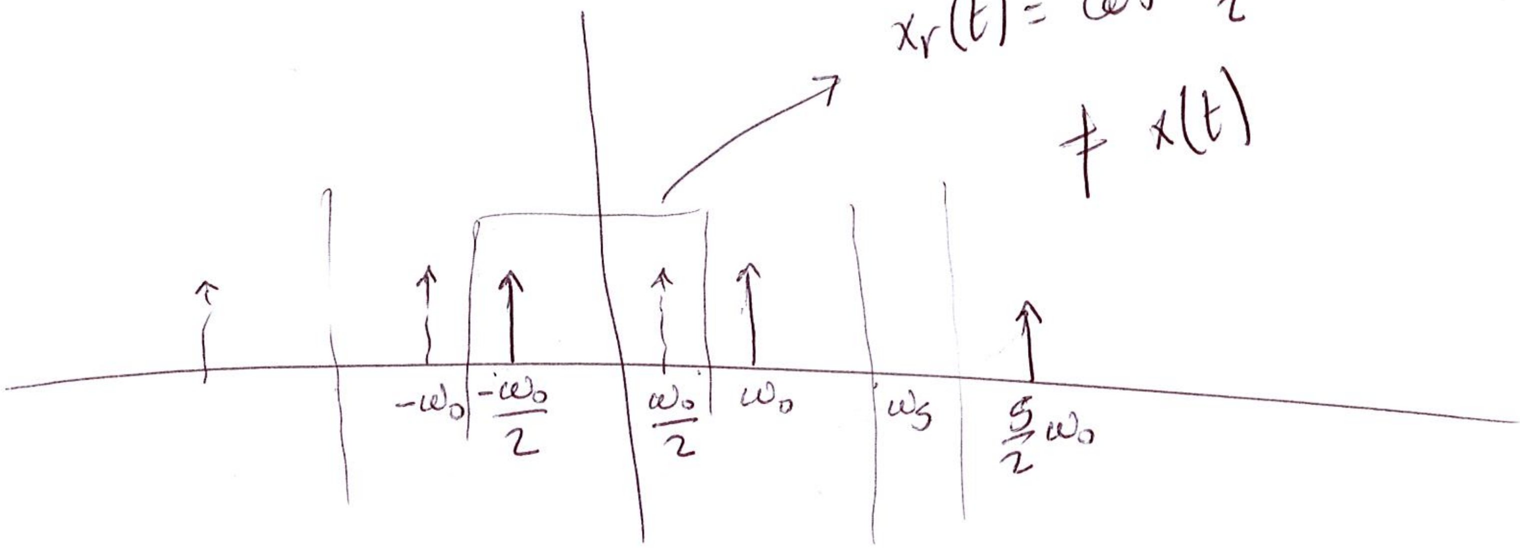


(g)

$\omega_0 = \frac{2\omega_s}{3}$ $\qquad \omega_s = \frac{3}{2}\omega_0$

$x_r(t) = \cos \frac{\omega_0}{2} t$

$\neq x(t)$



$\boxed{\text{Aliasing}}$