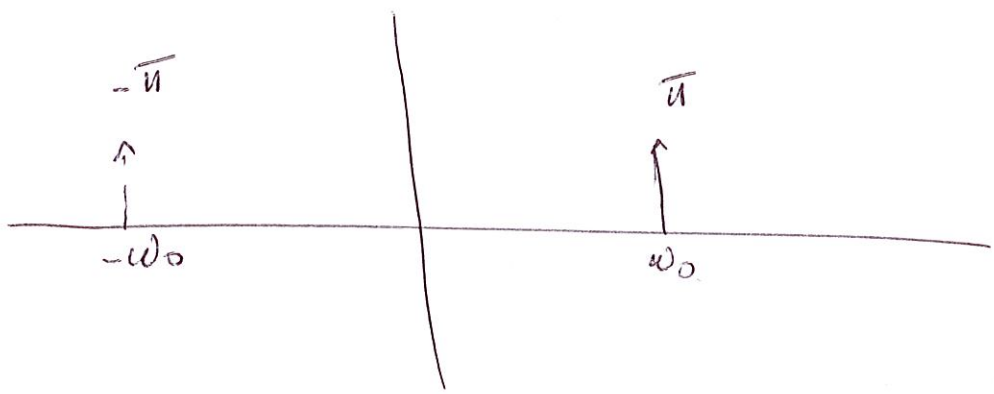
ECE 301 - Lecture # 33

Aliasing

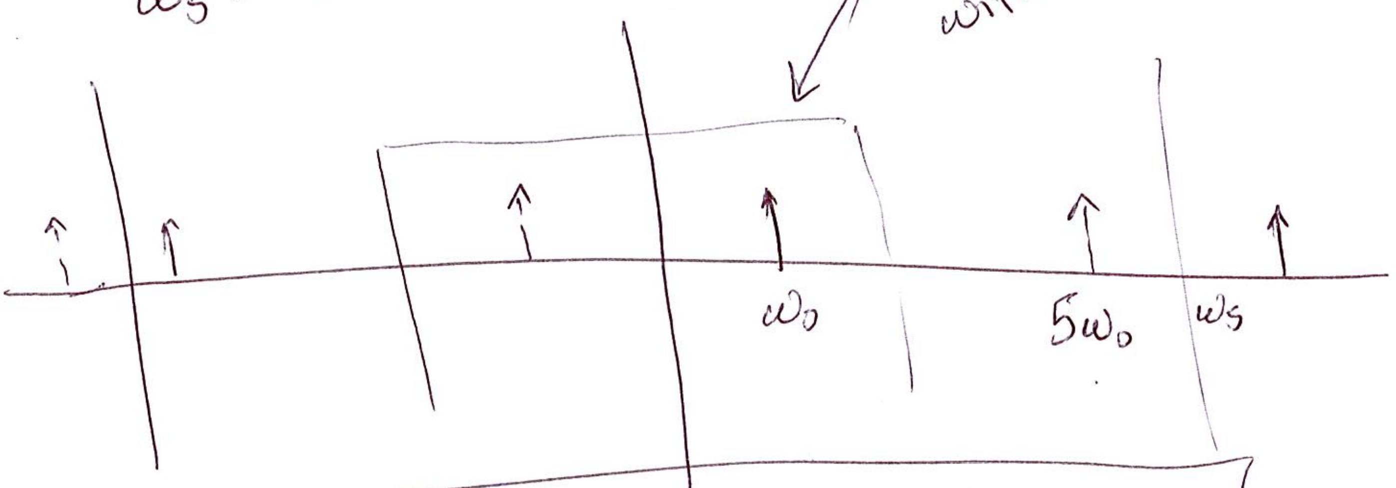
$x(t) = \cos \omega_0 t$



Sampling rate ω_s

$\omega_s = 6\omega_0$

LPF to reconstruct with $\omega_0 < \omega_c < 5\omega_0$



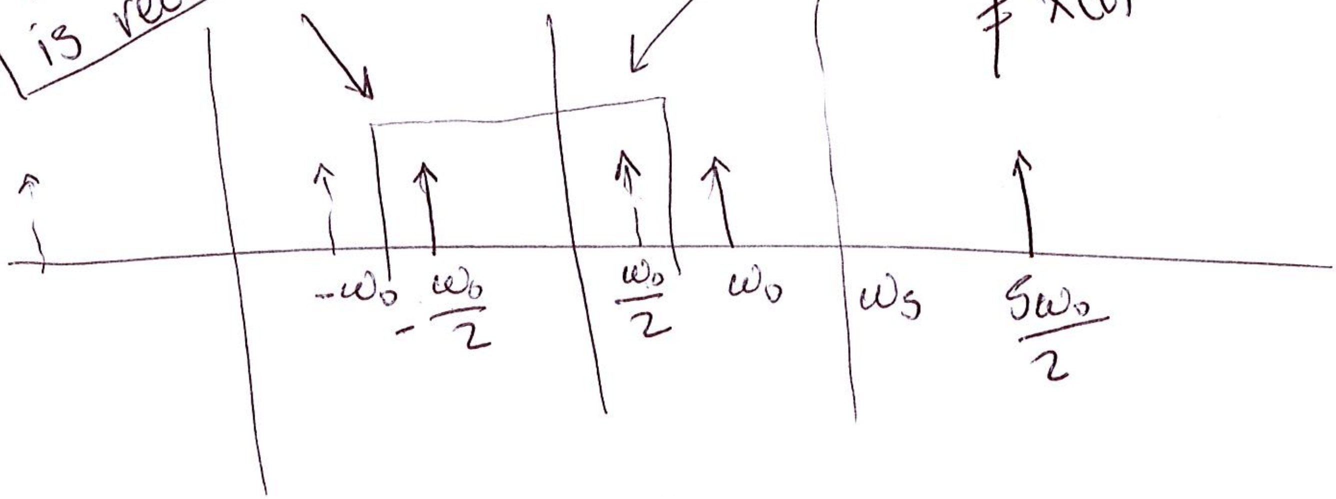
$$\boxed{\text{Same phenomenon as long as } \omega_s > 2\omega_0}$$

Undersampling

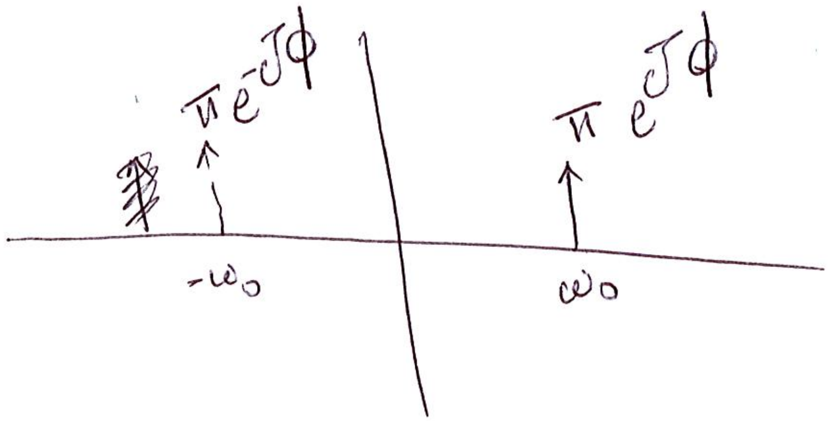
$\omega_s = \frac{3}{2}\omega_0$

Phase is reversed

$x_r(t) = \cos\frac{\omega_0}{2}t \neq x(t)$

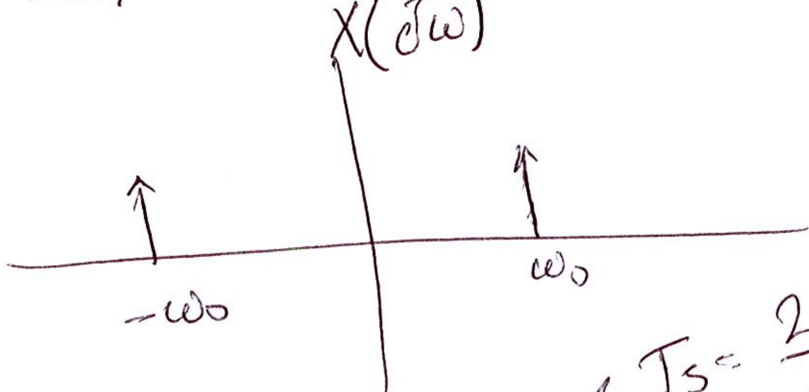


$x(t) = \cos(\omega_0 t + \phi)$



what happens if $\omega_s = 2\omega_0$?

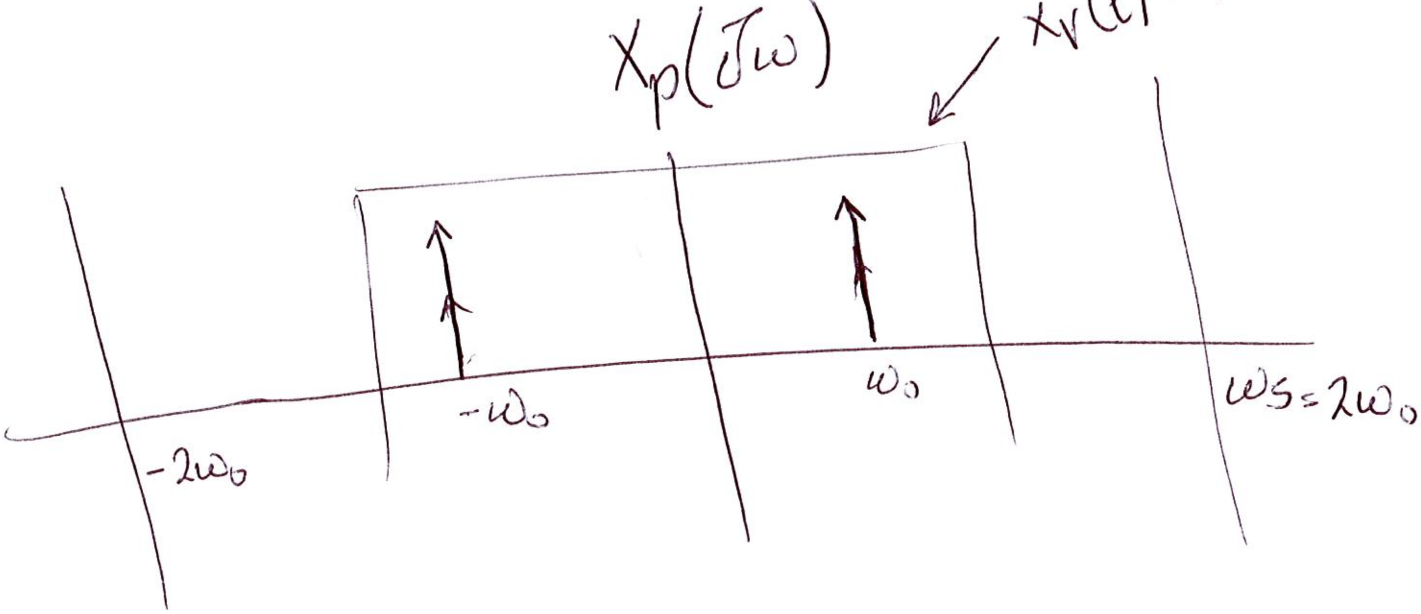
$x(t) = \cos \omega_0 t$



$T_s = \frac{2\pi}{\omega_s} = \frac{\pi}{\omega_0}$

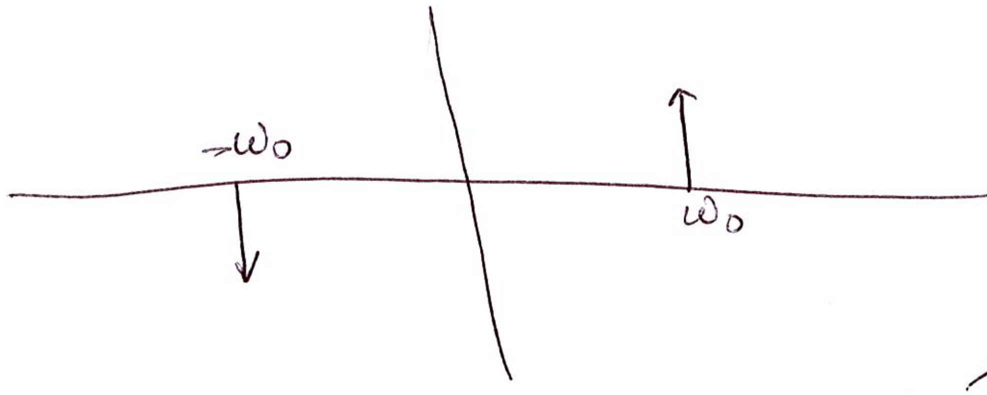
$x_p(t) = x(t)\, p(t)$

$x_r(t) = 2 \cos \omega_0 t$



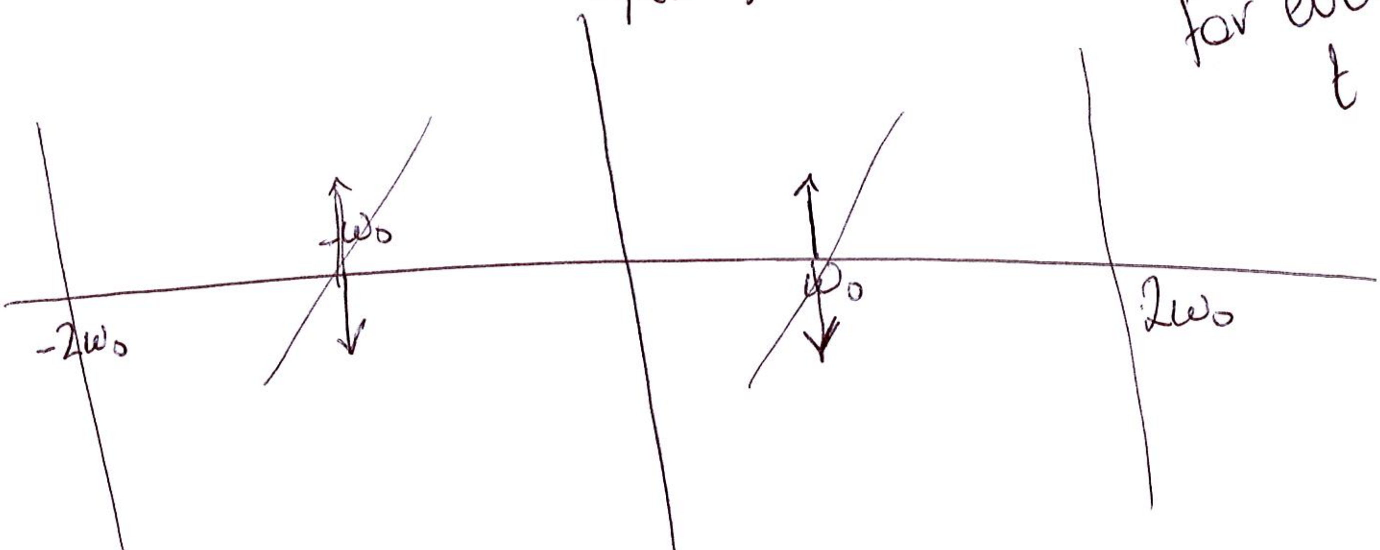
$x(t) = \sin \omega_0 t$

$|X(j\omega)|$

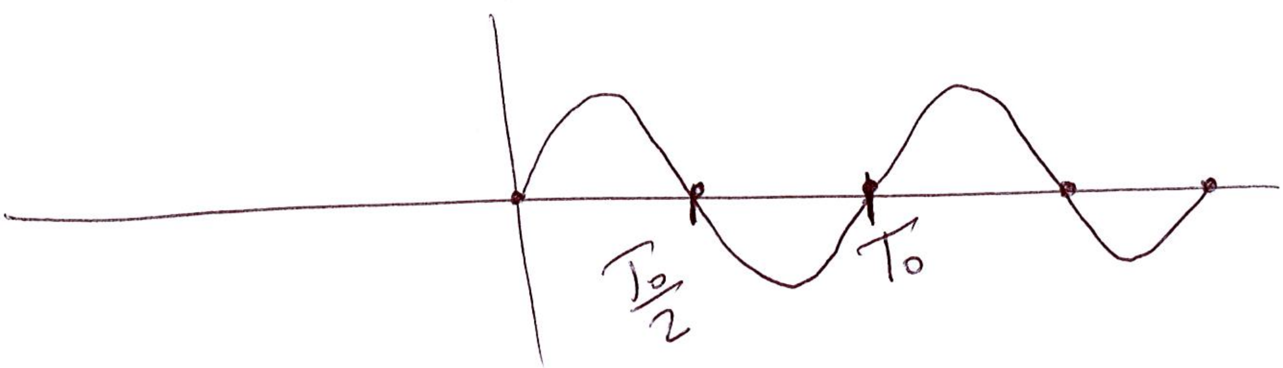


If $\omega_s = 2\omega_0 \Longrightarrow T_s = \frac{1}{2} T_0$

$X_p(j\omega) = 0$ $\quad$ $x_p(t) = 0$ for every t



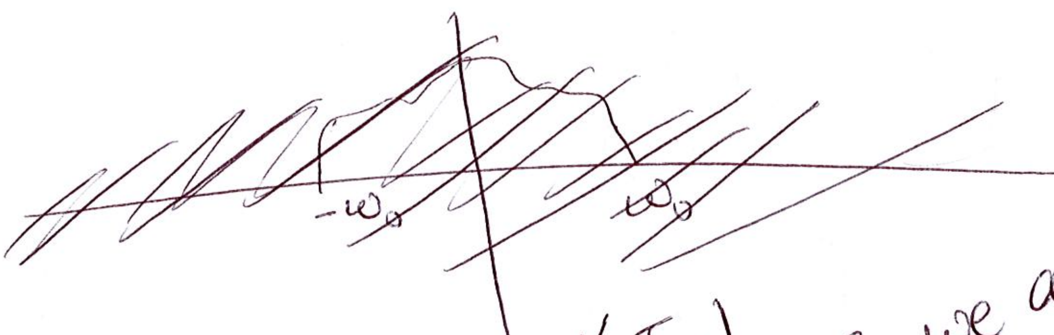
$x(t)$



$\boxed{\omega_s = 2\omega_0}$ does not always guarantee perfect reconstruction.

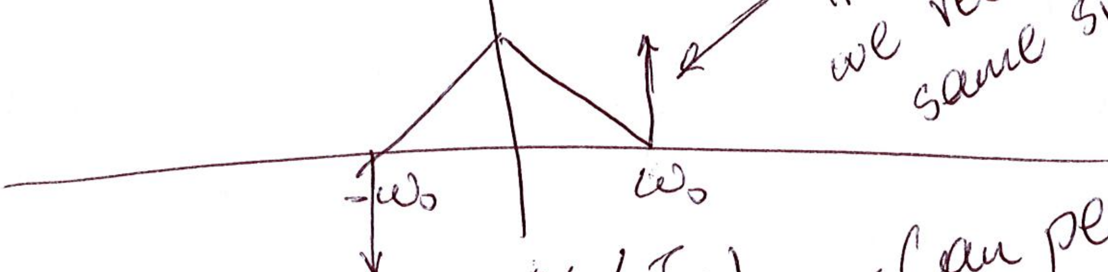
Depends on what? On the coefficients of the harmonics at ω_0 and $-\omega_0$

~~$X(j\omega)$~~



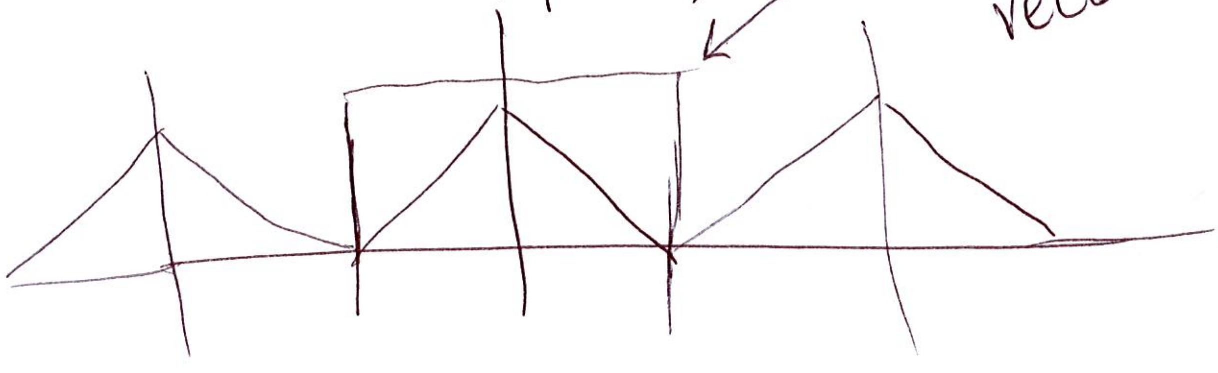
$X(j\omega)$

If we add $\sin \omega_0 t$, we reconstruct the same signal

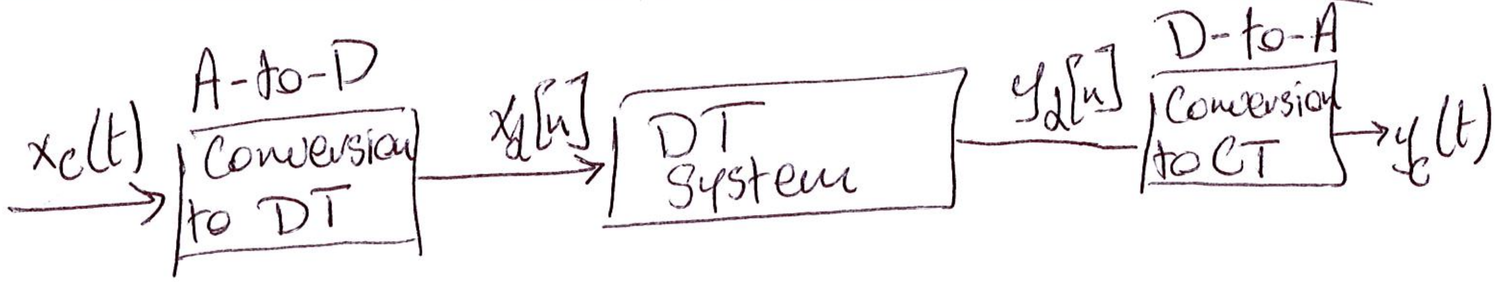


$X_p(j\omega)$

Can perfectly reconstruct



Discrete-time Processing of Continuous-time signals



$$x_d[n] = x_c(nT_s)$$

$X_c(j\omega) \Rightarrow X_p(j\omega)$ ✓ $\qquad X_d(e^{j\omega})$?

$$x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT_s)\,\delta(t-nT_s)$$

$$\mathcal{F}\{\delta(t-nT_s)\} = e^{-j\omega n T_s}$$

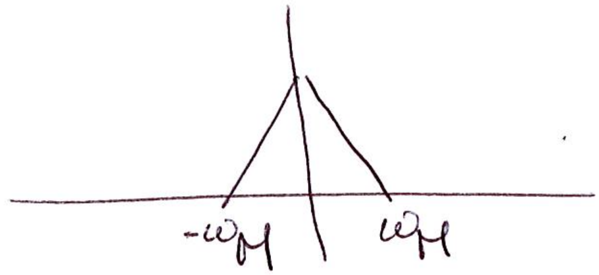
$$X_p(j\omega) = \sum_{n=-\infty}^{\infty} x_c(nT_s)\, e^{-j\omega n T_s}$$

$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_d[n]\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT_s)\, e^{-j\Omega n}$$

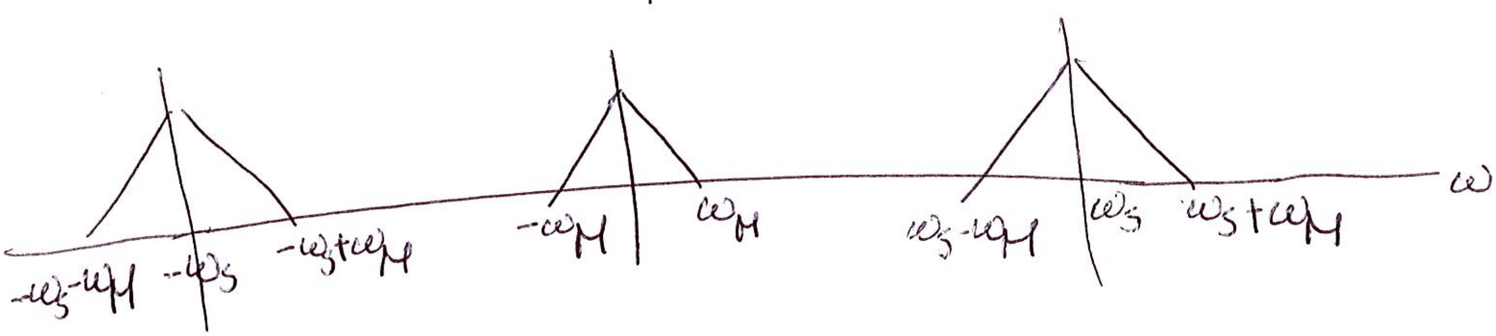
$$\Longrightarrow \boxed{\Omega = \omega T_s}$$

$X_c(j\omega)$

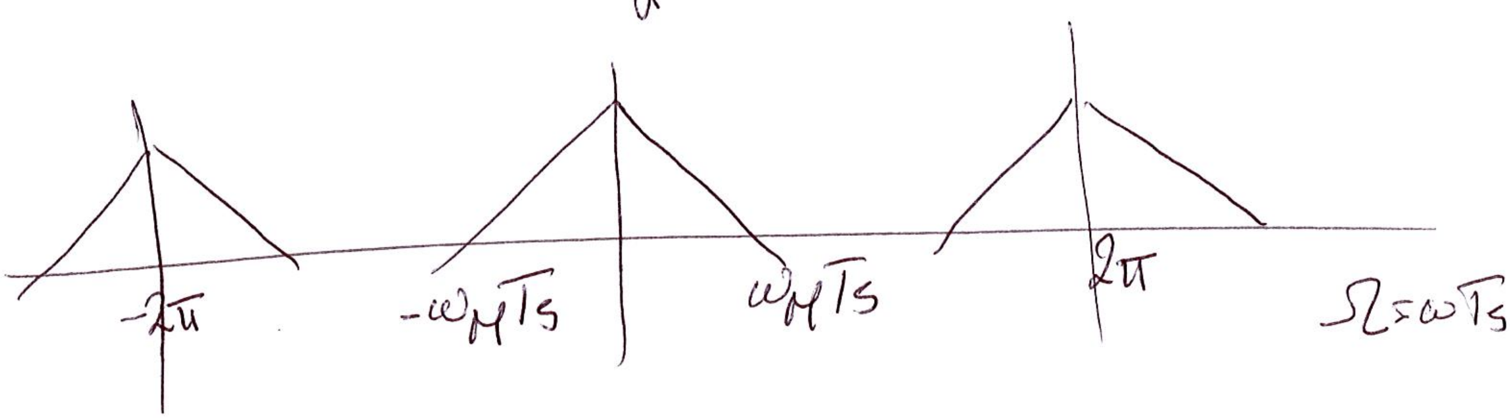


$\omega_s > 2\omega_M$

$X_p(j\omega)$



$X_d(e^{j\Omega})$



Sampling of Discrete-time Signals

$$x_p[n] = \begin{cases} x[n] & \text{if } n \text{ is a multiple of } N_s \\ 0 & \text{otherwise} \end{cases}$$

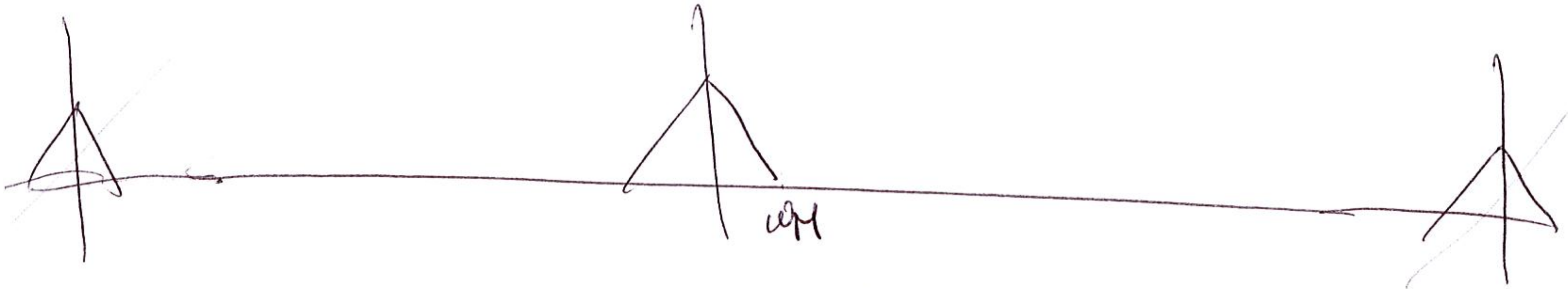
$$x_p[n] = x[n]\, p[n] = \sum_{k=-\infty}^{\infty} x[kN_s]\, \delta[n-kN_s]$$

$$X_p(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} P(e^{j\theta})\, X\left(e^{j(\omega-\theta)}\right) d\theta$$

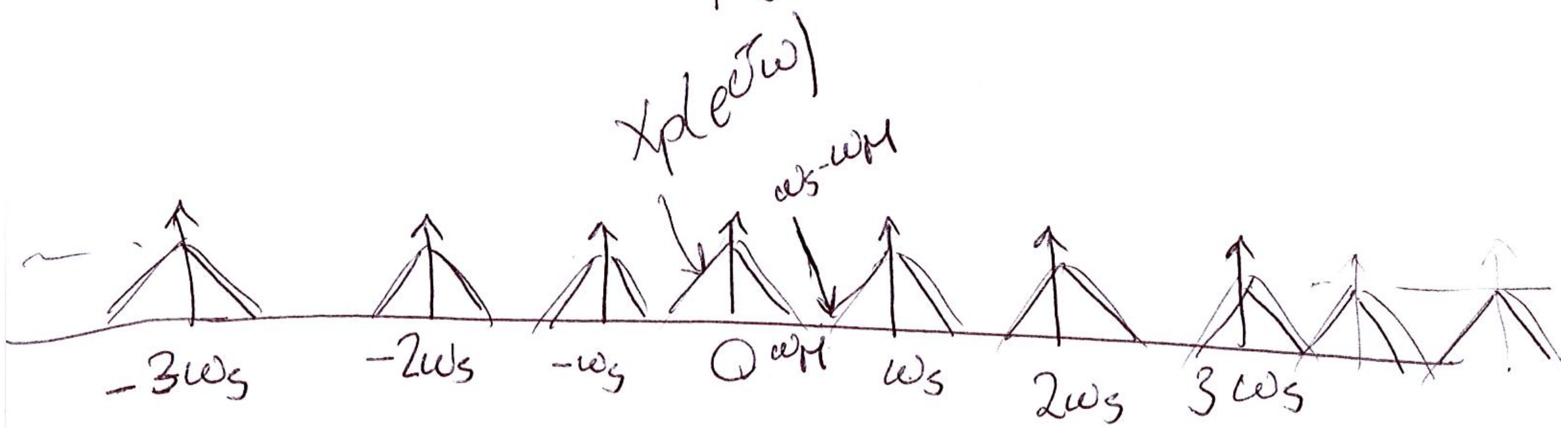
$$P(e^{j\omega}) = \frac{2\pi}{N_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

$$X_p(e^{j\omega}) = ?$$

$X(e^{j\omega})$



$P(e^{j\omega})$



$$X_p(e^{j\omega}) = \frac{1}{N_s} \sum_{k=0}^{N_s-1} X\left(e^{j(\omega - k\omega_s)}\right)$$

we need $\omega_s > 2\omega_M$ to guarantee perfect reconstruction

LPF