ECE 301 - Lecture # 34

Final

10 Problems

1 Similar to HW 1
1 " " HW 2
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2 " " HW 5
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1 " " HW 7
1 Laplace Transform (from notes)

Example 7.4 — Sampling of DT-Signals

$$X(e^{j\omega}) = 0 \quad \text{for } \frac{2\pi}{9} \leq |\omega| \leq \pi$$

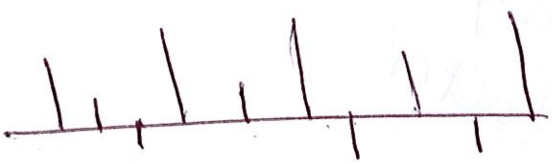
What is the largest sampling period that guarantee perfect reconstruction?

We find largest N_s such that

$$\frac{2\pi}{N_s} \geq 2\left(\frac{2\pi}{9}\right) \Rightarrow N_s \leq \frac{9}{2}$$

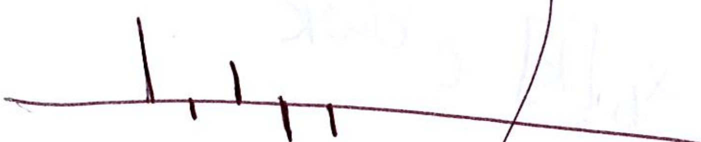
$$\Rightarrow \text{largest } N_s = 4 \qquad \omega_s = \frac{2\pi}{4} = \frac{\pi}{2}$$

Discrete-time Decimation and Interpolation



Downsampling

Upsampling



Used for efficient representation of Sampled signals (Getting rid of Zeros)

What happens in Frequency Domain?

$x_p[n]$: Sampled signal

$x_b[n]$: Down-sampled signal

$x_b[n] = x_p[nN]$ ← Downsampling factor

Determining $X_b(e^{j\omega})$

$$X_b(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x_b[k]\, e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} x_p[kN]\, e^{-j\omega k}$$

Let $n = kN$ $\quad k = \frac{n}{N}$

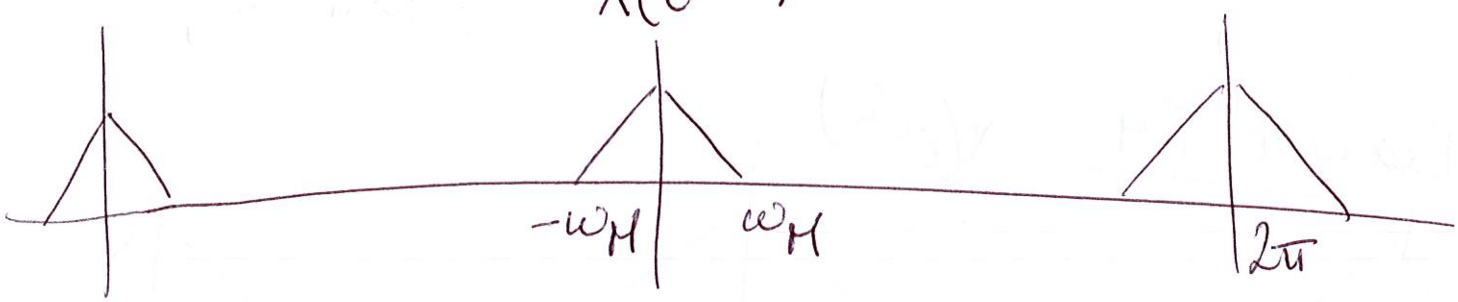
$$X_b(e^{j\omega}) = \sum_{n:\ \text{integer multiple of } N} x_p[n]\, e^{-j\omega \frac{n}{N}}$$

But we know that $x_p[n] = 0$ when n is not an integer multiple of N

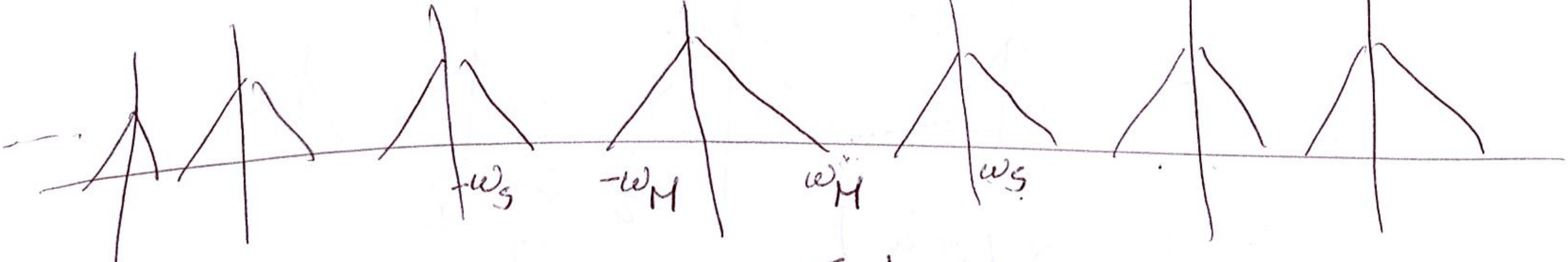
$$\Rightarrow X_b(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_p[n]\, e^{-j\frac{\omega}{N} n}$$

$$= X_p(e^{j\omega/N})$$

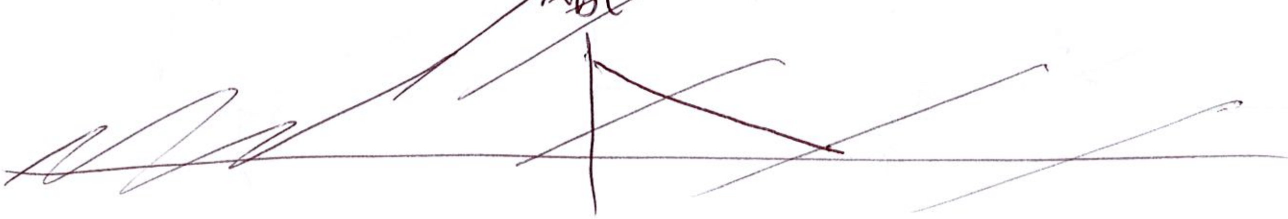
$X(e^{j\omega})$



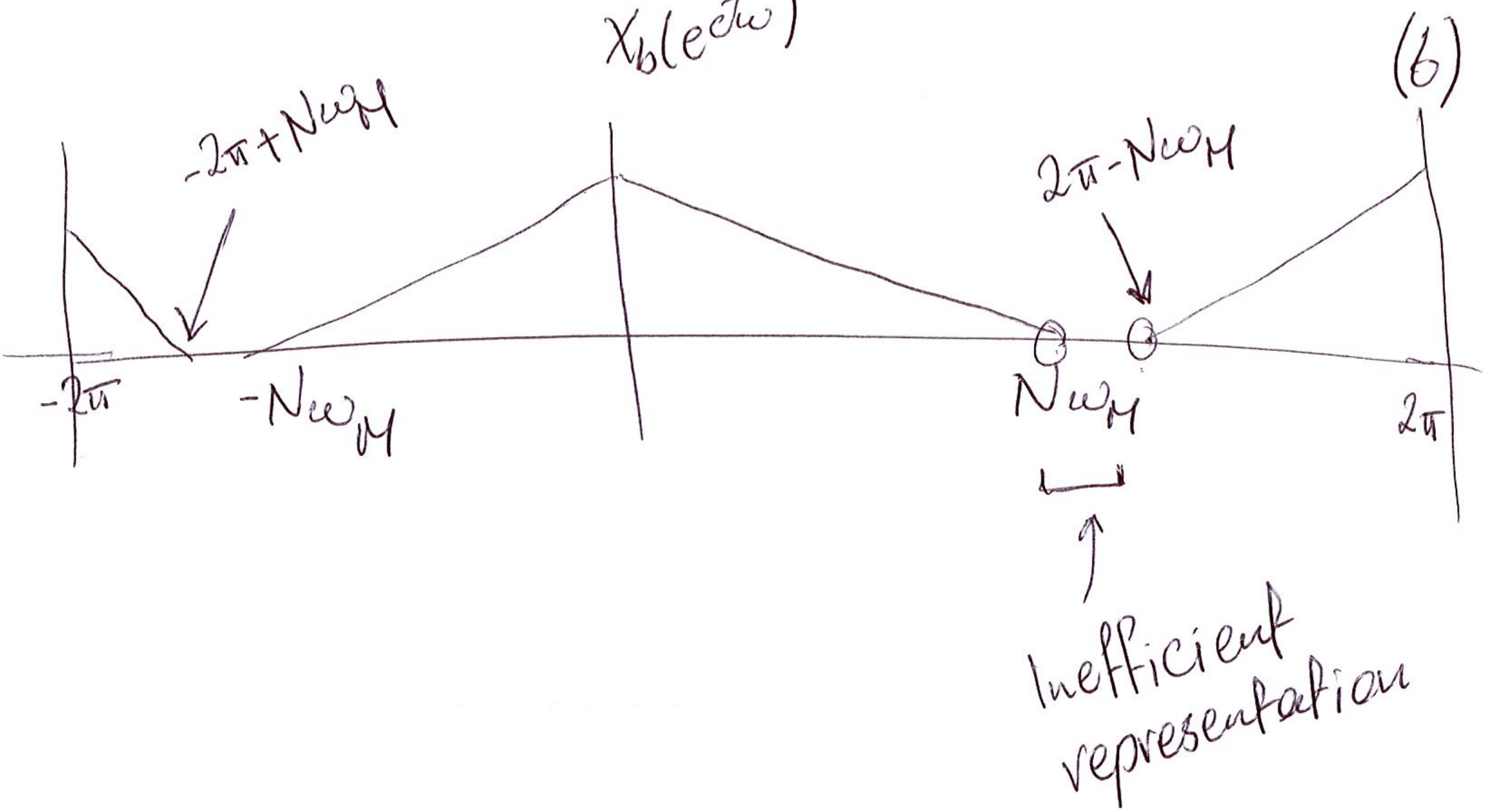
$X_p(e^{j\omega})$



~~$X_b(e^{j\omega})$~~

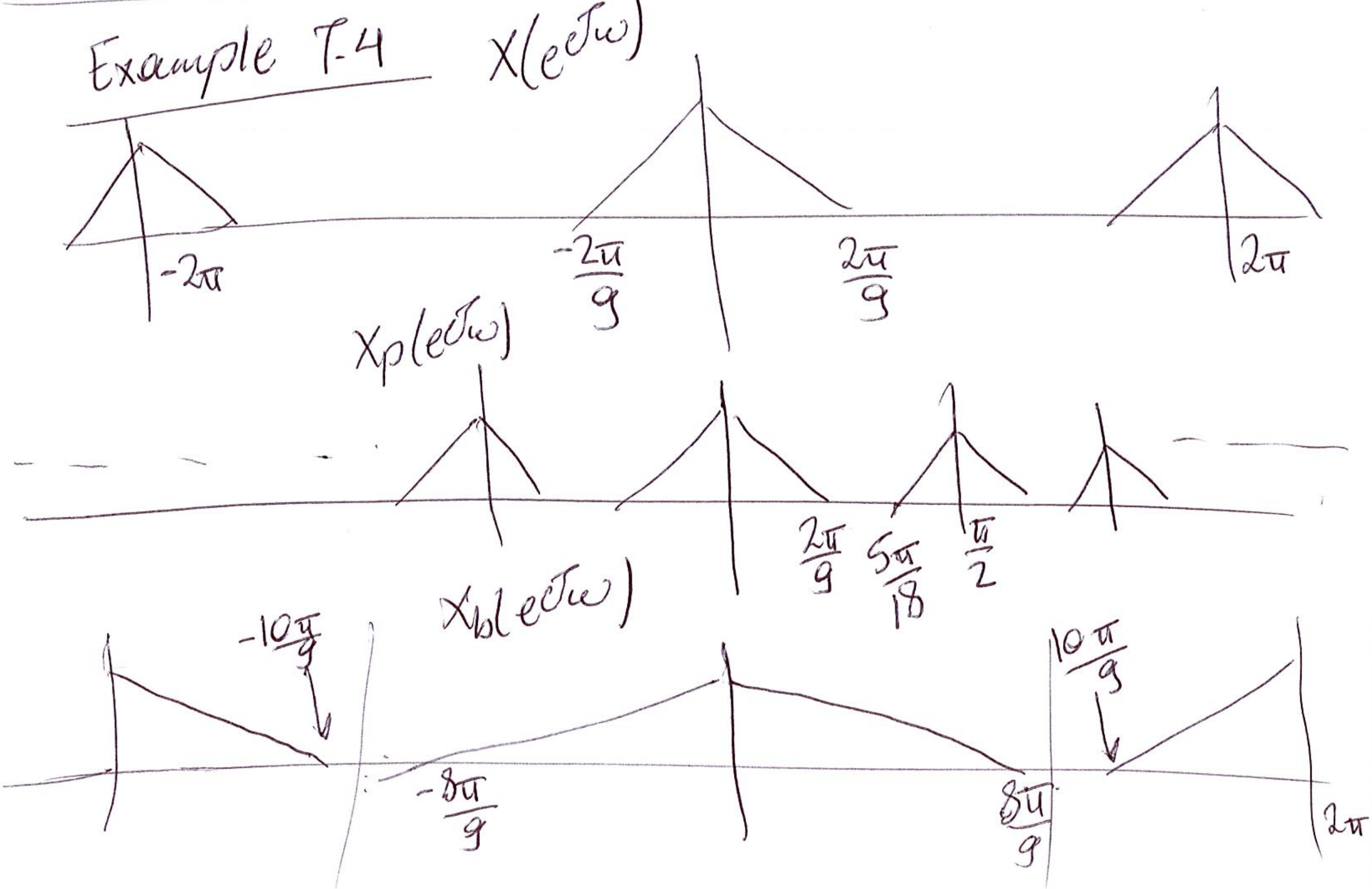


(b)



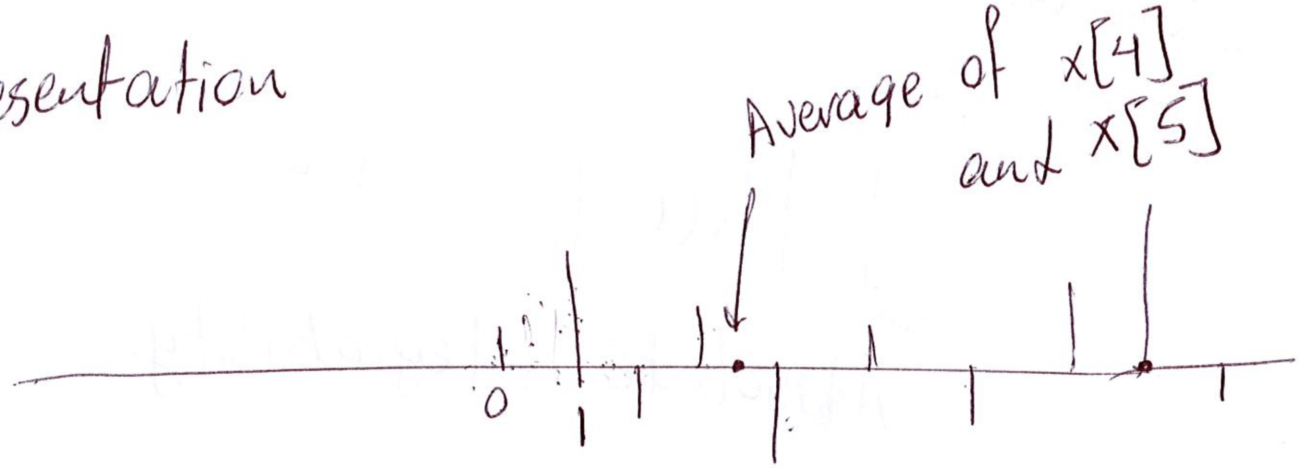
$X_b(e^{j\omega})$
$-2\pi + N\omega_M$
$2\pi - N\omega_M$
-2π
$-N\omega_M$
$N\omega_M$
2π
Inefficient representation

Example T-4



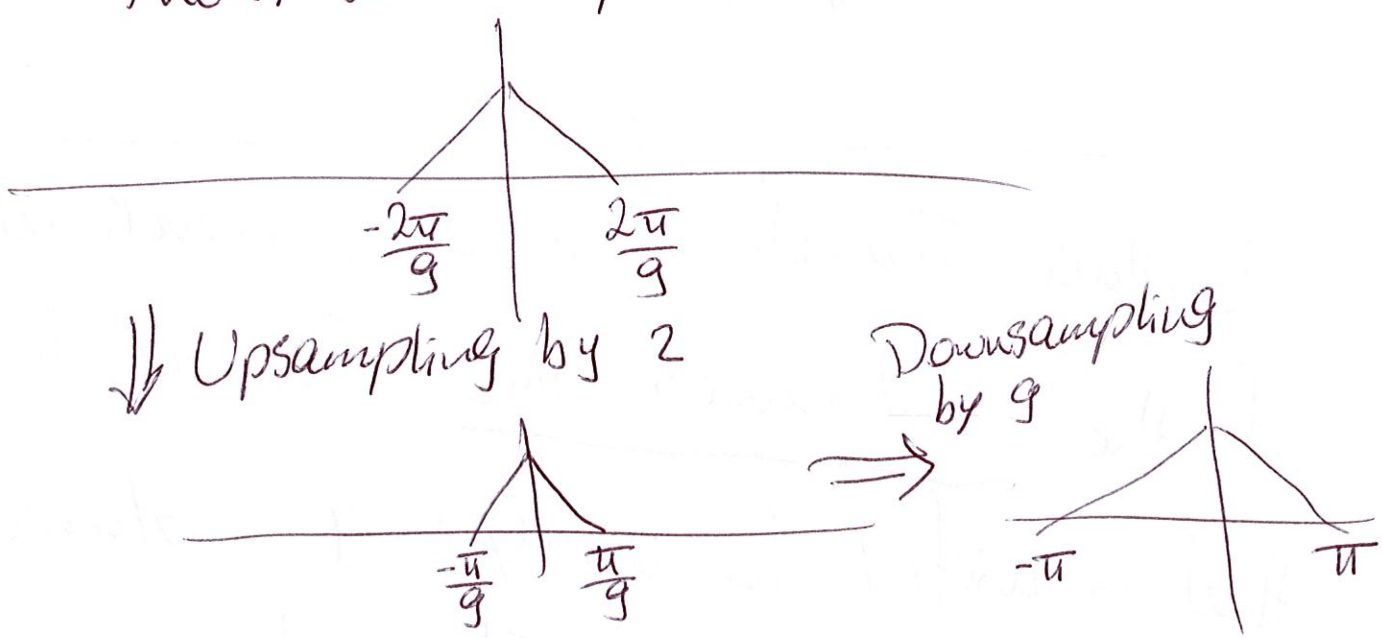
$X(e^{j\omega})$
-2π
$-\frac{2\pi}{9}$
$\frac{2\pi}{9}$
2π
$X_p(e^{j\omega})$
$\frac{2\pi}{9}$
$\frac{5\pi}{18}$
$\frac{\pi}{2}$
$X_b(e^{j\omega})$
$-\frac{10\pi}{9}$
$-\frac{8\pi}{9}$
$\frac{8\pi}{9}$
$\frac{10\pi}{9}$
2π

In this example, we need to sample every $\frac{9}{2}$ to obtain the most efficient representation



Same thing as

- First, upsampling by 2.
- Then, downsampling by 9.



Introduction to Laplace Transform

- Dirichlet Condition for Fourier Transform

$$\int_{-\infty}^{\infty} |h(t)| < +\infty$$

Absolute Integrability

- Condition for stability of LTI Systems

$$\int_{-\infty}^{\infty} |h(t)| < +\infty$$

Laplace Transform is a generalization of the <u>Continuous-Time</u> Fourier Transform, that is useful for analyzing unstable LTI Systems

Laplace Transform

$$H(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt$$

$s = \alpha + j\omega$

$\alpha = 0 \Rightarrow$ we get the Fourier Transform

$$X(s) = \int x(t)\, e^{-st}\, dt$$

$$X(\alpha + j\omega) = \int x(t)\, e^{-(\alpha + j\omega)t}\, dt$$

$$= \int \underbrace{x(t)\, e^{-\alpha t}}\, e^{-j\omega t}\, dt$$

$$= \mathcal{F}\left\{ x(t)\, e^{-Re\{s\}t} \right\}$$