ECE 301 - Lecture # 35

Examples on Laplace Transform

Example 9.1

$$x(t) = e^{-at} u(t) ,$$

If $a > 0$, LPF Approximation

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt = \frac{1}{j\omega + a}$$

For any value of a

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$X(s) = \int_0^\infty e^{-at}\, e^{-st}\, dt$$

$$= \int_0^\infty e^{-(s+a)t}\, dt$$

$s = \sigma + j\omega$

$$X(s) = \int_0^\infty e^{-(\sigma+a)t}\, e^{-j\omega t}\, dt$$

$$= \mathcal{F}\{\underbrace{x(t)\, e^{-\sigma t}}_{e^{-(a+\sigma)t}\, u(t)}\}$$

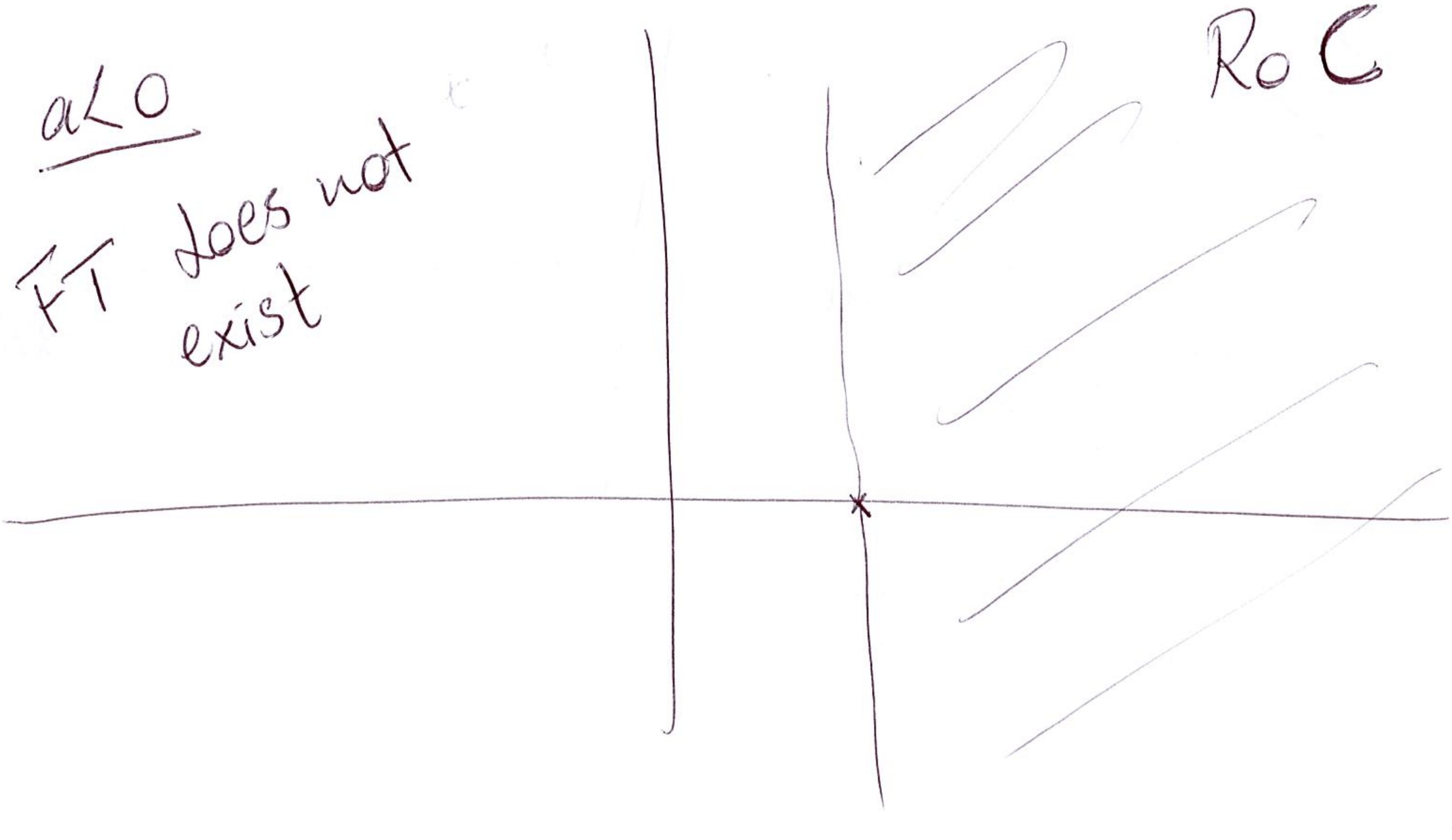
FT will exist only if $\sigma + a > 0$ $\Leftrightarrow$ $\sigma > -a$

$$X(s) = \frac{1}{s+a}, \quad \underbrace{Re\{s\} > -a}_{\text{Region of convergence of the Laplace Transform}}$$

$a > 0$

If the RoC contains the $j\omega$-axis, then the Fourier Transform exists

$a < 0$



Example 9.2

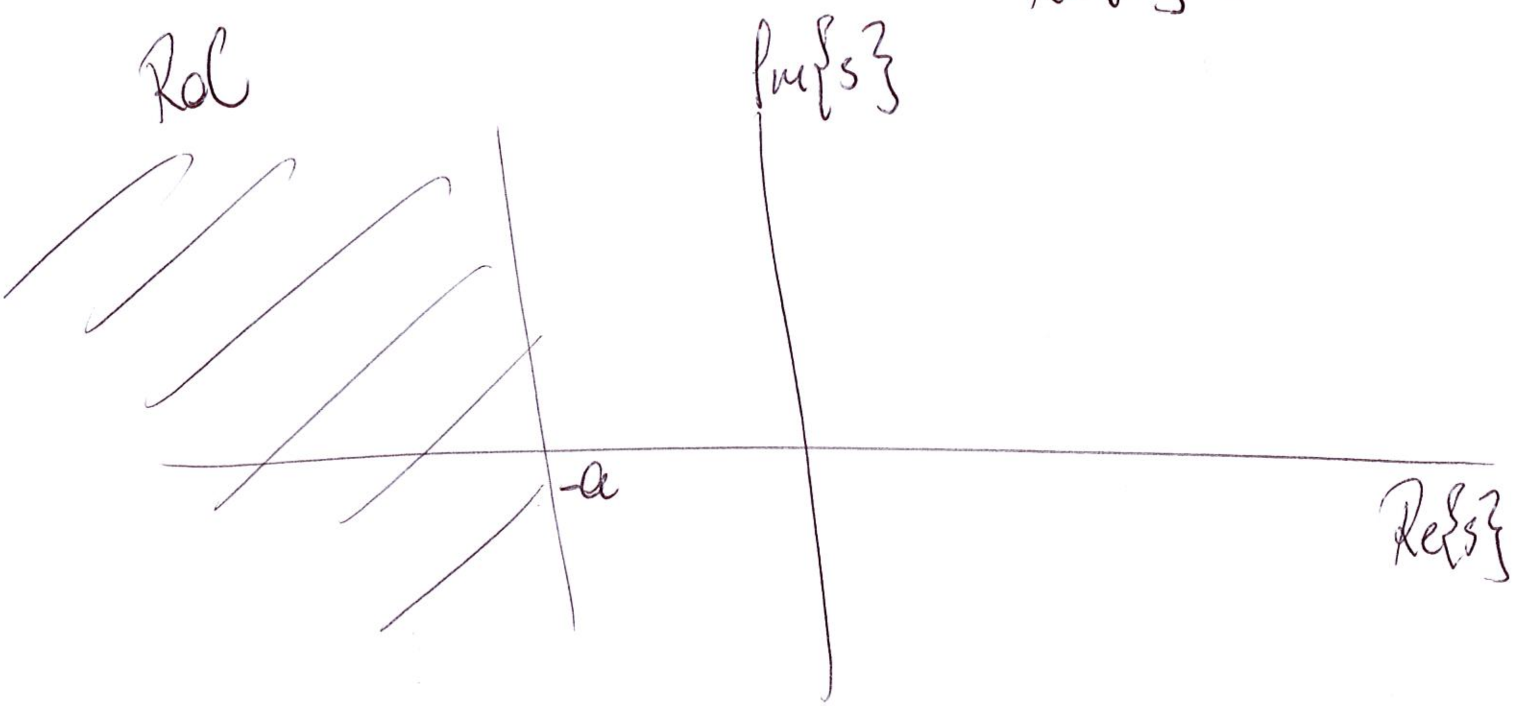
$$x(t) = -e^{-at}\, u(-t)$$

$$X(s) = -\int_{-\infty}^{0} e^{-at}\, e^{-st}\, dt$$

$$= -\int_{-\infty}^{0} e^{-(s+a)t}\, dt = \frac{1}{s+a},$$

$$Re\{s\} < -a$$

RoC



Example 9.3

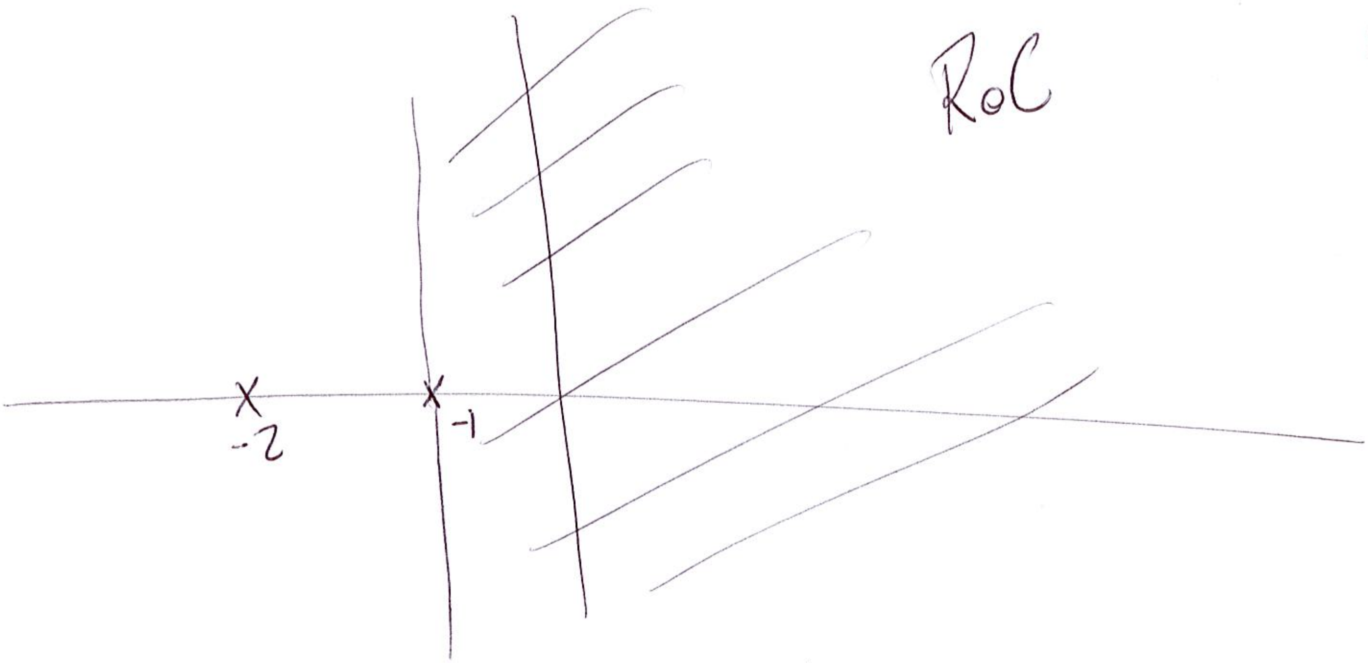
$$x(t) = 3e^{-2t}\,u(t) - 2e^{-t}\,u(t)$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} \left(3e^{-2t} - 2e^{-t}\right) e^{-st}\, dt$$

$$= \underbrace{\int_{0}^{\infty} 3e^{-(s+2)t}\, dt}_{\substack{\updownarrow \mathcal{L} \\ \frac{3}{s+2},\ Re\{s\}>-2}} - \underbrace{\int_{0}^{\infty} 2e^{-(s+1)t}\, dt}_{\substack{\updownarrow \mathcal{L} \\ \frac{2}{s+1},\ Re\{s\}>-1}}$$

$$= \frac{3}{s+2} - \frac{2}{s+1} = \frac{s-1}{s^2+3s+2}, \quad Re\{s\}>-1$$



FT exists

Example 9.4

$$x(t) = e^{-2t}\, u(t) + e^{-t} \cos(3t)\, u(t)$$

$$x(t) = \left[e^{-2t} + \frac{1}{2} e^{-(1-3j)t} + \frac{1}{2} e^{-(1+3j)t} \right] u(t)$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-2t} e^{-st}\, dt + \frac{1}{2} \int_{0}^{\infty} e^{-(1-3j)t} e^{-st}\, dt$$

$$+ \frac{1}{2} \int_{0}^{\infty} e^{-(1+3j)t} e^{-st}\, dt$$

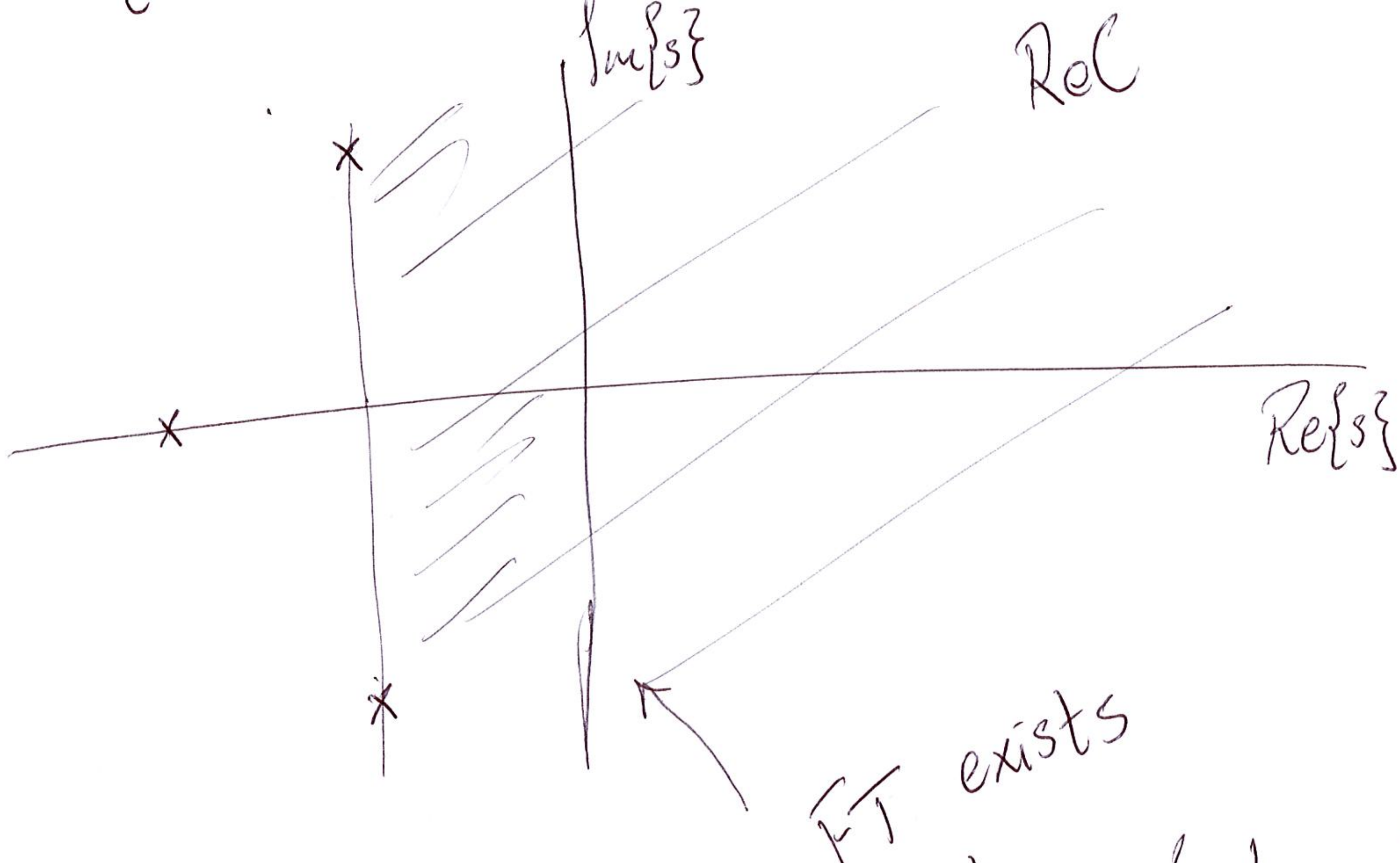
$$= \int_{0}^{\infty} e^{-(s+2)t}\, dt + \frac{1}{2} \int_{0}^{\infty} e^{-(s+1-3j)t}\, dt$$

$$+ \frac{1}{2} \int_{0}^{\infty} e^{-(s+1+3j)t}\, dt$$

$$e^{-2t}\,u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+2}, \quad Re\{s\} > -2$$

$$e^{-(1-3j)t}\,u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+(1-3j)}, \quad Re\{s\} > -1$$

$$e^{-(1+3j)t}\,u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+(1+3j)}, \quad Re\{s\} > -1$$



$$X(s) = \frac{1}{s+2} + \frac{1}{2}\left(\frac{1}{s+(1-3j)}\right) + \frac{1}{2}\left(\frac{1}{s+(1+3j)}\right)$$

$$, \quad Re\{s\} > -1$$

Example 9.5

$$x(t) = \delta(t) - \frac{4}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(t)$$

$$\mathcal{L}\{\delta(t)\} = \int \delta(t) e^{-st} dt = 1$$

RoC of $\mathcal{L}\{\delta(t)\}$ is the entire plane

$$X(s) = 1 - \frac{4}{3} \frac{1}{s+1} + \frac{1}{3} \frac{1}{s-2}, \quad Re\{s\} > 2$$