ECE 301 - Lecture # 5

Set of Harmonically related discrete-time complex exponentials

$e^{j\omega_0 n}$ different from $e^{j\omega_0 t}$

because $e^{j2\pi n}$ is always 1

but $e^{j2\pi t}$ is not

Say $\phi_k[n] = e^{j\omega_k n}$ $\quad \omega_k = k\,\omega_0$

$= k\,\frac{2\pi}{N}$

k^{th} element in the harmonically related set

fundamental period

$$\phi_1[n] = e^{j\frac{2\pi}{N}n}$$

$$\phi_2[n] = e^{j\frac{4\pi}{N}n}$$

$$\vdots$$

$$\phi_{N+1}[n] = e^{j(N+1)\frac{2\pi}{N}n}$$

$$= \cancel{e^{jN\frac{2\pi}{N}n}}\; e^{j\frac{2\pi}{N}n}$$

$$= \phi_1[n]$$

$$\phi_{N+2}[n] = e^{j(N+2)\frac{2\pi}{N}n}$$

$$= \cancel{e^{jN\frac{2\pi}{N}n}}\; e^{j4\pi n}$$

$$= \phi_2[n]$$

For a fundamental frequency $\omega_0 = \frac{2\pi}{N}$,

there are only N harmonically related

discrete-time complex exponentials

$\phi_0[n] = 1 \qquad \phi_1[n] = e^{j\frac{2\pi}{N}n}$

$\phi_2[n] = e^{j\frac{4\pi}{N}n}$ - - - - - - - $\phi_{N-1}[n] = e^{j(N-1)\frac{2\pi}{N}n}$

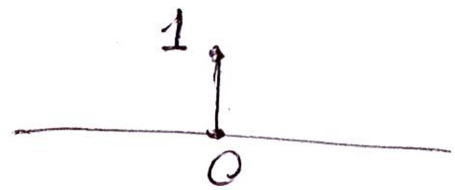
$\phi_N[n] = \phi_0[n], \quad \phi_{N+1}[n] = \phi_1[n]$ - - - - -

$\phi_{-1}[n] = \phi_{N-1}[n]$

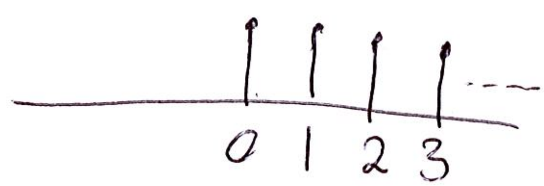
Unit Impulse and Unit Step Functions

Discrete-Time

$$\delta[n] = \begin{cases} 0 , & n \neq 0 \\ 1 , & n = 0 \end{cases}$$



$$u[n] = \begin{cases} 0 , & n < 0 \\ 1 , & n \geq 0 \end{cases}$$



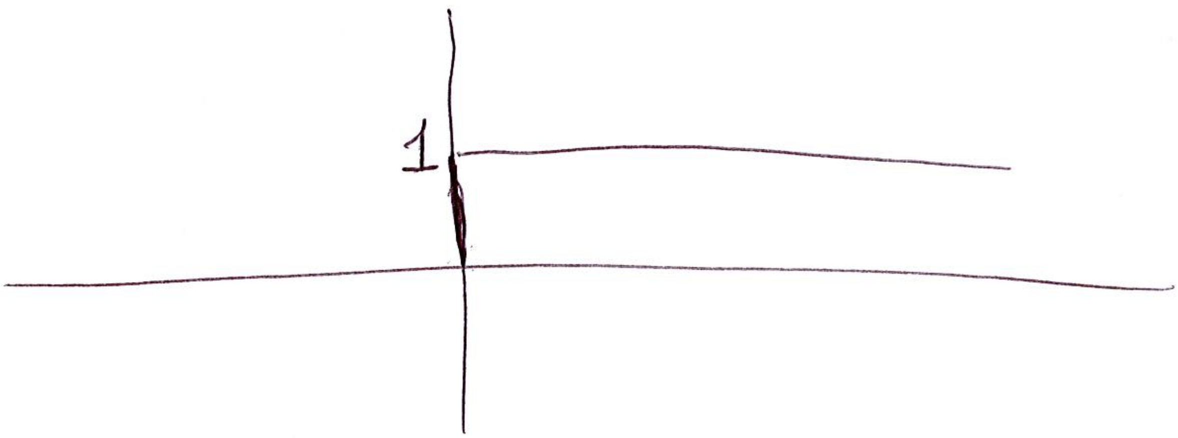
$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

$$= \sum_{k=0}^{\infty} \delta[n-k]$$

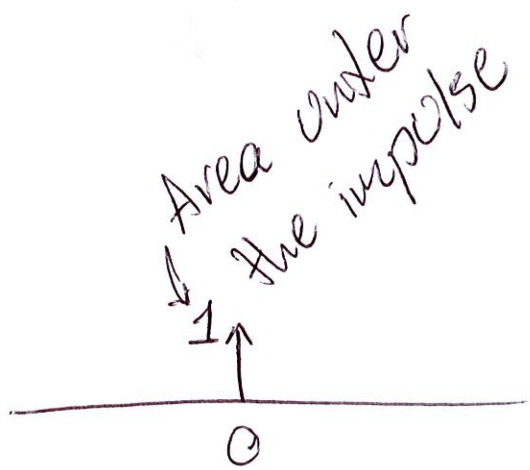
Continuous-Time

$$u(t) = \begin{cases} 0 & , \quad t < 0 \\ 1 & , \quad t \geq 0 \end{cases}$$

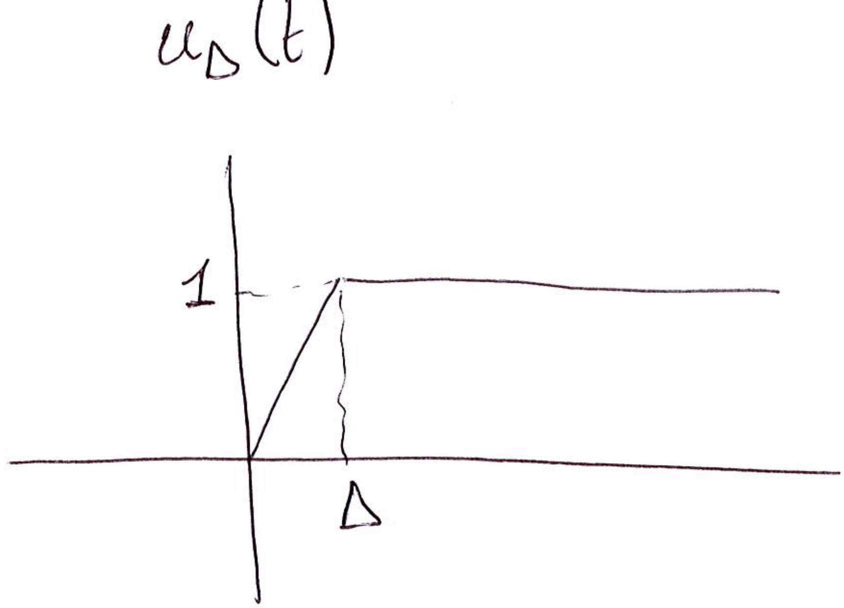


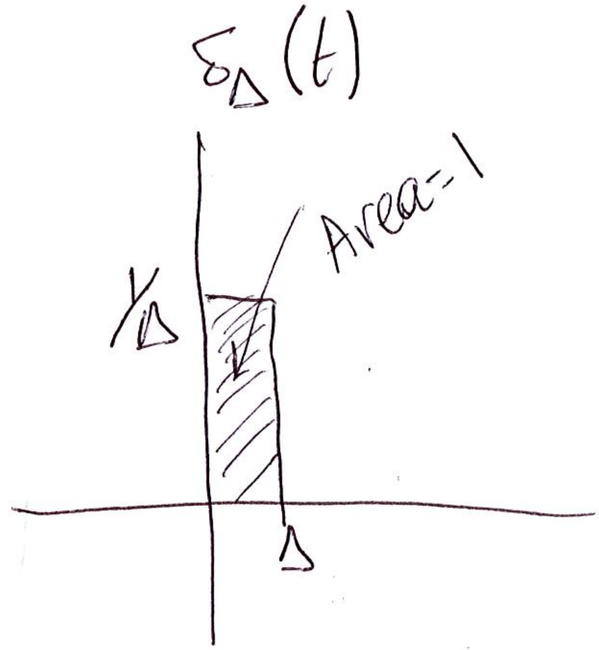
$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$$\delta(t) = \frac{du(t)}{dt}$$



Approximation





$$u(t) = \lim_{\Delta \to 0} u_\Delta(t)$$

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

$$\frac{d\, u_\Delta(t)}{dt}$$

Basic System Properties

Memoryless

Examples

$$y[n] = \left(2x[n] - x^2[n]\right)^2$$

Memoryless System

$$y[n] = (n-1)\,x[n]$$

$$y[n] = x[n-1] \rightarrow \text{Not memoryless}$$

$$y[n] = x[n+1]$$

Definition

We say that a system is memoryless if at every point in time, the output depends only on the value of the input at the same point

Causality

Examples

$y[n] = x[n]$ Causal

$y[n] = x[n-1]$ Causal

$y[n] = x[n+1]$ not causal

$y[n] = (n+1)\,x[n-1]$ Causal

Definition

Output at any point in time depends only on current and past input values

Invertibility

Is there an inverse system?

$$y(t) = 2\,x(t) \qquad \text{System}$$

$$\tilde{y}(t) = \frac{\tilde{x}(t)}{2} \qquad \text{Inverse system}$$

$$y[n] = \sum_{k=-\infty}^{n} x[k] \qquad \text{Accumulator}$$

$$y[n-1] = \sum_{k=-\infty}^{n-1} x[k]$$

$$y[n] - y[n-1] = x[n]$$

$$\tilde{y}[n] = \tilde{x}[n] - \tilde{x}[n-1] \qquad \text{Differentiator}$$

$y(t) = x^2(t)$ Not invertible

Definition

If For every distinct input value, there is a distinct output value, then the system is invertible