ECE 301 - Lecture # 6

System Properties

* Memoryless Systems
* Invertibility

- Causality

If two inputs are identical up to a given point in time, then their corresponding outputs are also identical up to the same point in time

Examples

$y[n] = x[n] - x[n-2]$ Causal

$y[n] = x[n-4] + x[n+1]$ Not Causal

$y[n] = x[-n]$ Not Causal

$y(t) = x(t-1)\cos(t+1)$ Causal

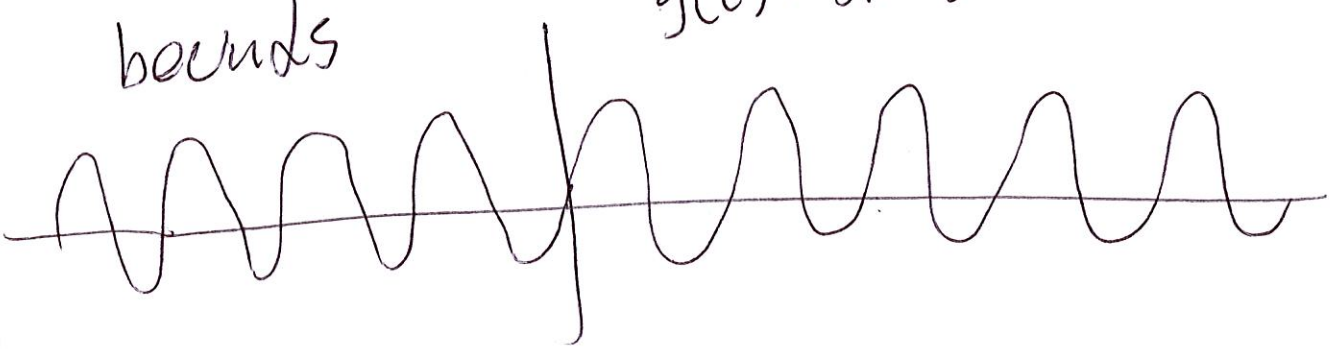
- Stability

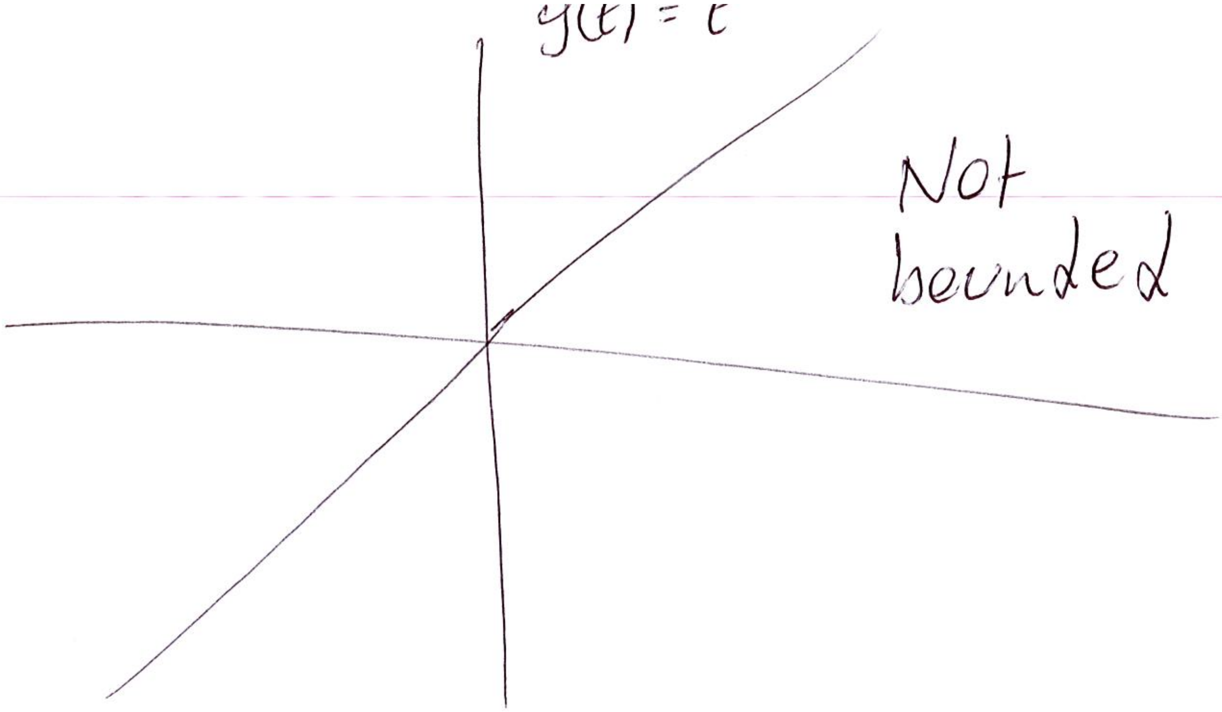
We say that a system is stable if for every ~~boote~~ bounded input, the output is bounded

What is a bounded signal?

If it has minimum and maximum bounds

$y(t) = \sin t$ is bounded





Not bounded

Examples

$y(t) = 1000\, x(t)$ Stable system

Given any bounded $x(t)$, let's say the minimum bound is B_{min} and the maximum bound is B_{max}

then is $y(t)$ bounded? Yes, and it has $\tilde{B}_{min} = 1000\, B_{min}$

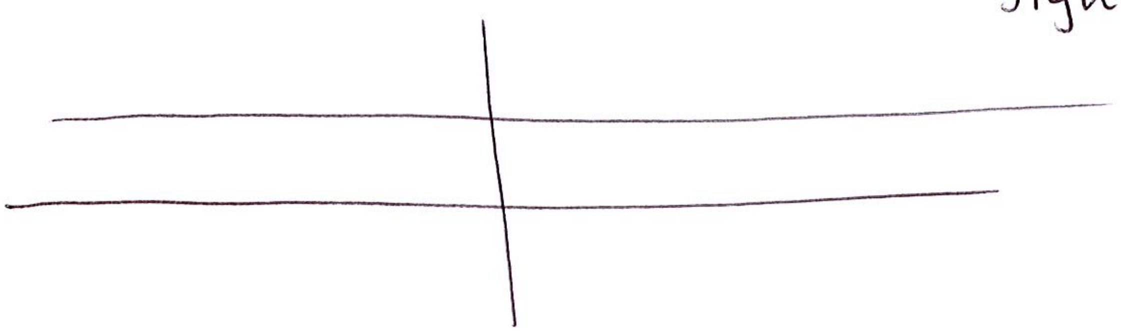
and $\tilde{B}_{max} = 1000\, B_{max}$

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$ Accumulator

Is the accumulator stable? No

$x(t) = |\sin(t)| \Longrightarrow y(t)$ is unbounded

$x(t) = 1 \Longrightarrow y(t)$ unbounded signal



Time Invariance

A system is time invariant if a time shift in the input signal results in the same time shift in the output signal

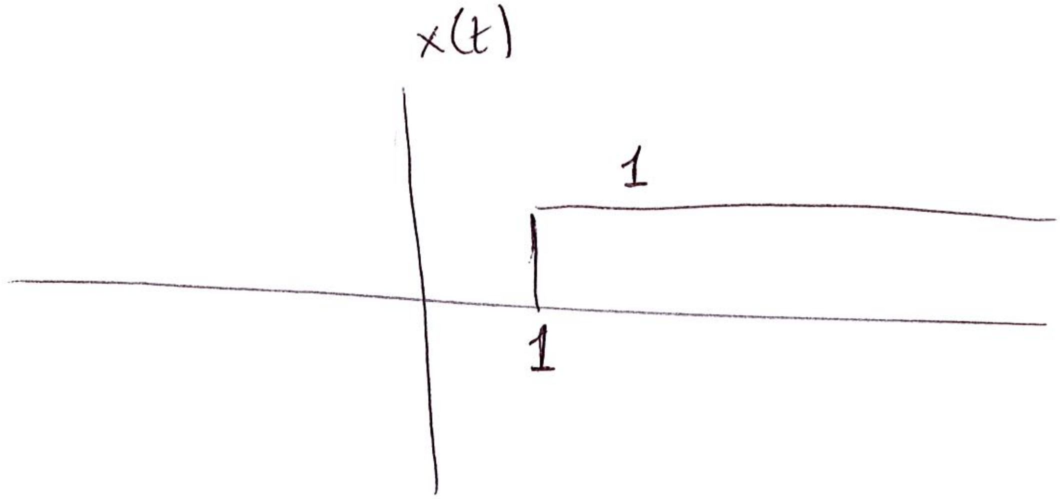
Examples

$y(t) = x(t)$ Time Invariant

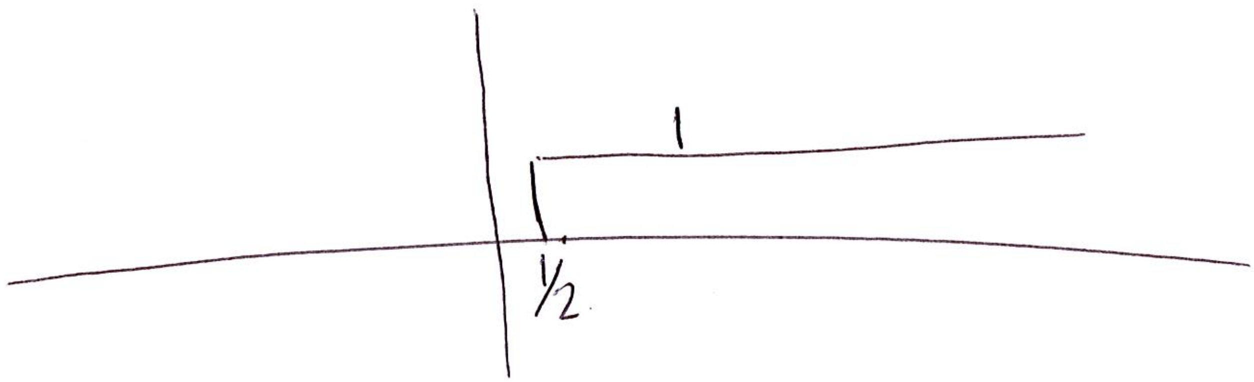
$y(t) = x(t-5)$ Time Invariant

$y(t) = \sin(x(t))$ Time Invariant

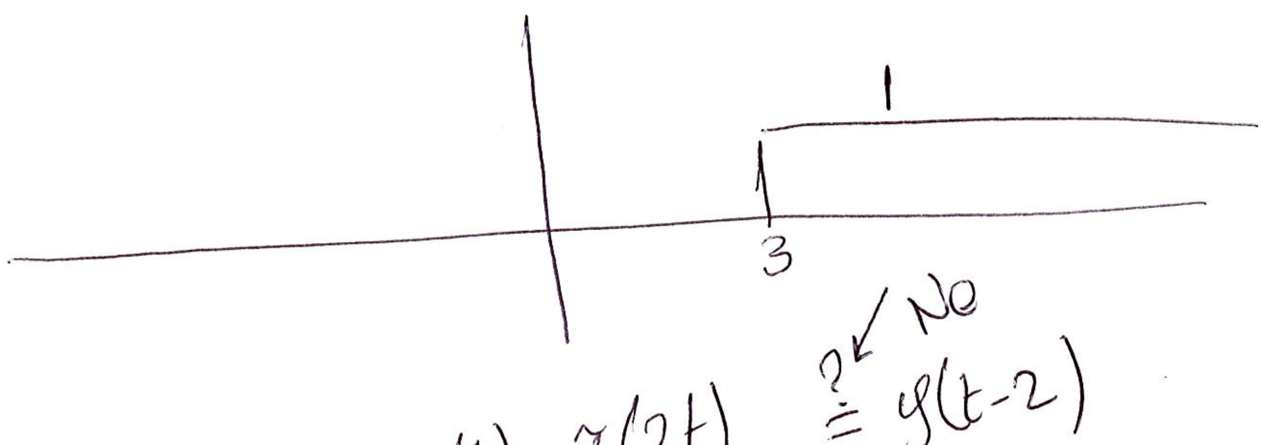
$y(t) = x(2t)$ Time Variant



$y(t) = x(2t)$

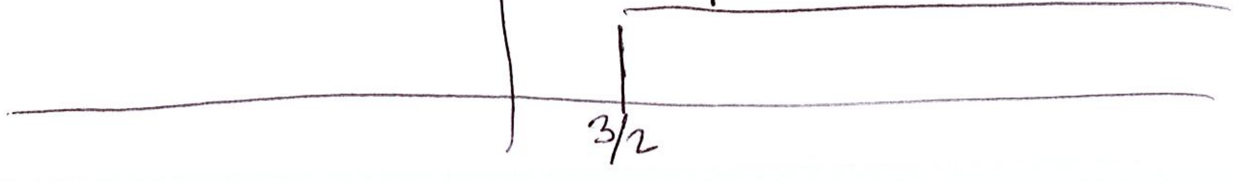


$\tilde{x}(t) = x(t-2)$



No

$\tilde{y}(t) = \tilde{x}(2t) \overset{?}{=} y(t-2)$



$y(t) = t\ x(t)$ (7)

Is this system Time Invariant?

No

Why? - Time shift in the input signal does ~~ue~~ not result in the same time shift in the output signal

- The dependence of the output on the input changes with time

Linearity

Additivity

1. Response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$

2. Response to $a\, x_1(t)$ is $a\, y_1(t)$

Complex constan

Homogeneous Systems

If the system is linear, then

if we have $x[n] = \sum_k a_k x_k[n]$

then the output $y[n] = \sum_k a_k y_k[n]$.

$y(t) = t\, x(t)$ Linear system

$y_1(t) = t\, x_1(t)$

$y_2(t) = t\, x_2(t)$

yes

1. Is it additive? $x_3(t) = x_1(t) + x_2(t)$

$$y_3(t) = t\, x_3(t) = t\left(x_1(t) + x_2(t)\right)$$
$$= t\, x_1(t) + t\, x_2(t)$$
$$= y_1(t) + y_2(t)$$

yes

2. Is it homogeneous? $x_3(t) = a\, x_1(t)$

$$y_3(t) = t\, x_3(t) = t\, a\, x_1(t)$$
$$= a\, y_1(t)$$

$y(t) = x^2(t)$ Not linear

$y_1(t) = x_1^2(t)$

$y_2(t) = x_2^2(t)$

1. Is it additive? NO

$x_3(t) = x_1(t) + x_2(t)$

$y_3(t) = x_3^2(t) = \left(x_1(t) + x_2(t)\right)^2$

$\neq \underbrace{x_1^2(t)}_{y_1(t)} + \underbrace{x_2^2(t)}_{y_2(t)}$

Not for all inputs

$y[n] = Re\{x[n]\}$ $\boxed{\text{Non-Linear System}}$ $y_1[n] = Re\{x_1[n]\}$ (11)

$y_2[n] = Re\{x_2[n]\}$

1. Is it additive? Yes $x_3[n] = x_1[n] + x_2[n]$

$$y_3[n] = Re\{x_3[n]\} = Re\{x_1[n] + x_2[n]\}$$

$$= Re\{x_1[n]\} + Re\{x_2[n]\}$$

$$= y_1[n] + y_2[n]$$

2. Is it homogeneous? No $x_3[n] = a\, x_1[n]$

Complex constant

$x_1[n] = r[n] + j\, s[n]$

$a = j$

$y_1[n] = r[n]$

$$y_3[n] = Re\{x_3[n]\} = Re\{j\, r[n] - s[n]\} = -s[n]$$

$$\neq j\, y_1[n]$$