

Linear and Time-Invariant Systems

Example on linearity

$$y[n] = 2x[n] + 3$$

Incrementally linear

Non-linear

Not additive

$$y_1[n] = 2x_1[n] + 3$$

$$y_2[n] = 2x_2[n] + 3$$

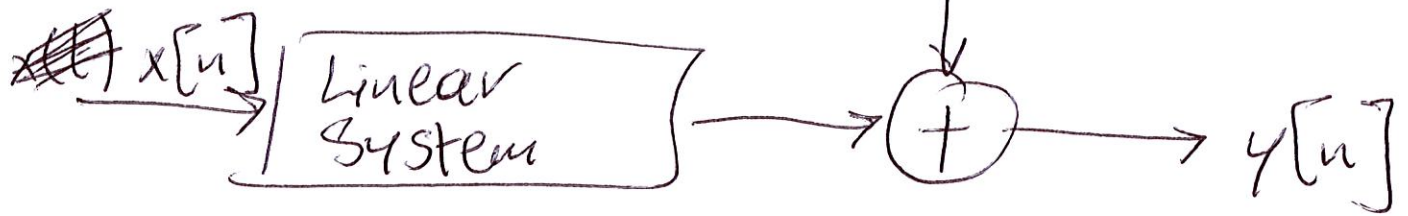
$$x_3[n] = x_1[n] + x_2[n]$$

$$y = 2x_3[n] + 3$$

$$2x_1[n] + 2x_2[n] + 3$$

$$y_1[n] + y_2[n]$$

Zero-Input Response (2)



Incrementally linear system

Difference between any two output signals is a linear function of the difference between the corresponding input signals

$$y_1[n] = 2x_1[n] + 3$$

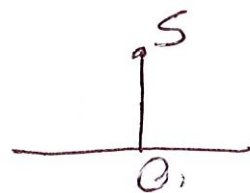
$$y_2[n] = 2x_2[n] + 3$$

$$y_1[n] - y_2[n] = 2(x_1[n] - x_2[n])$$

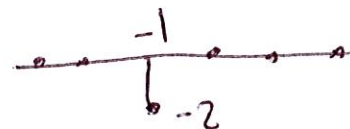
Let $x[n]$ be a DT-signal

(3)

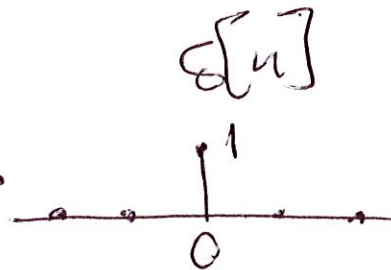
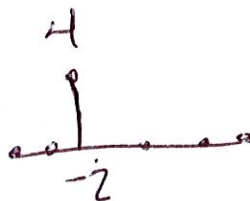
$$x[0] \delta[n] = \begin{cases} x[0], & n=0 \\ 0, & n \neq 0 \end{cases}$$



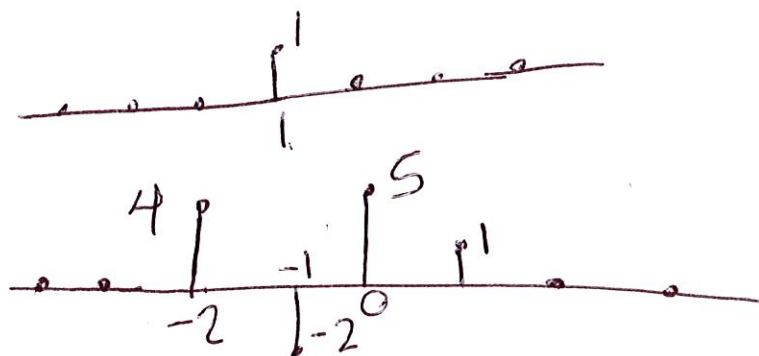
$$x[-1] \delta[n+1] = \begin{cases} x[-1], & n=-1 \\ 0, & n \neq -1 \end{cases}$$



$$x[-2] \delta[n+2] = \begin{cases} x[-2], & n=-2 \\ 0, & n \neq -2 \end{cases}$$



$$x[1] \delta[n-1] = \begin{cases} x[1], & n=1 \\ 0, & n \neq 1 \end{cases}$$



$$x[0] \delta[n] + x[-2] \delta[n+2] + x[-1] \delta[n+1] + x[1] \delta[n-1] = x[n]$$

(4)

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k]$$

Linear system

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h_k[n]$$

Response of the system to $\delta[n-k]$

Time-Invariant System

$$h_k[n] = h_0[n-k] \triangleq h[n-k]$$

LTI

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

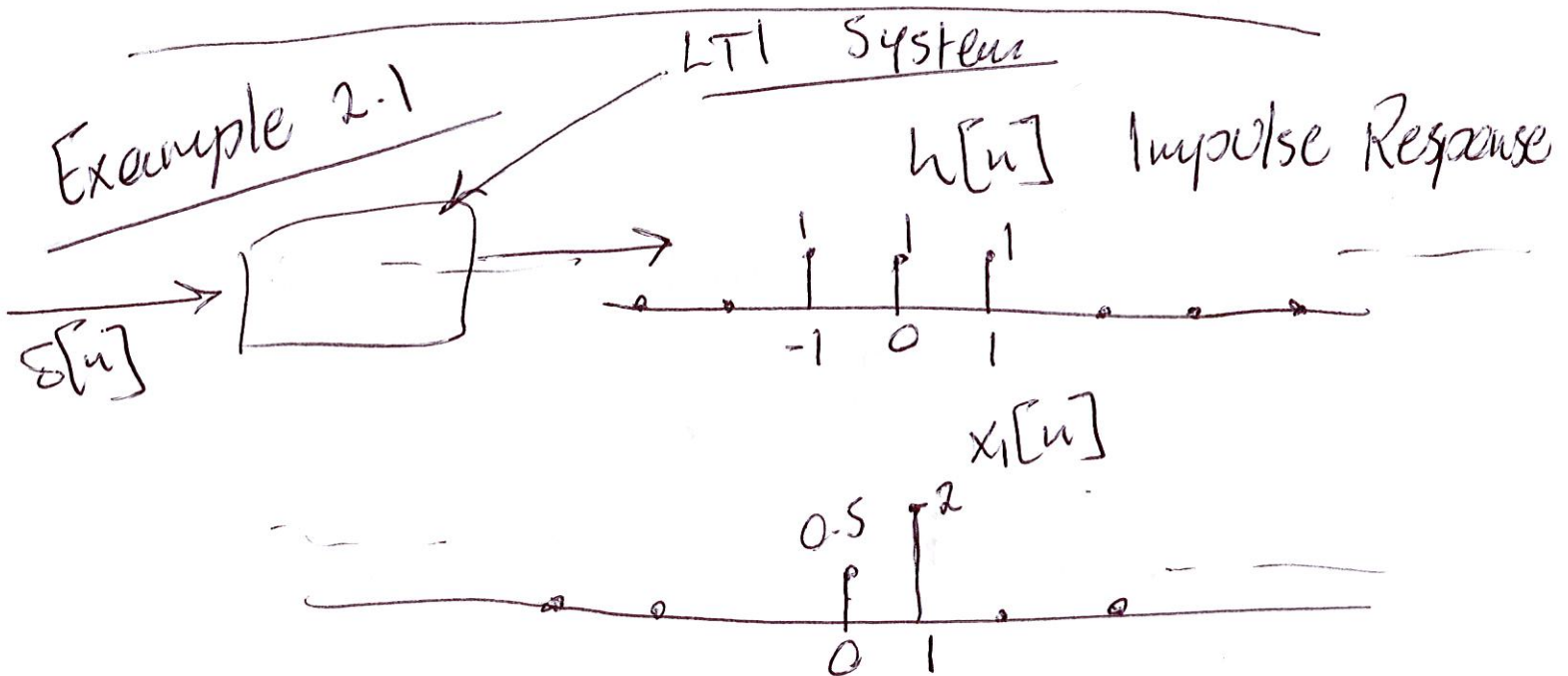
$h[n]$
Impulse Response

for any LTI system

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$= x[n] * h[n]$$

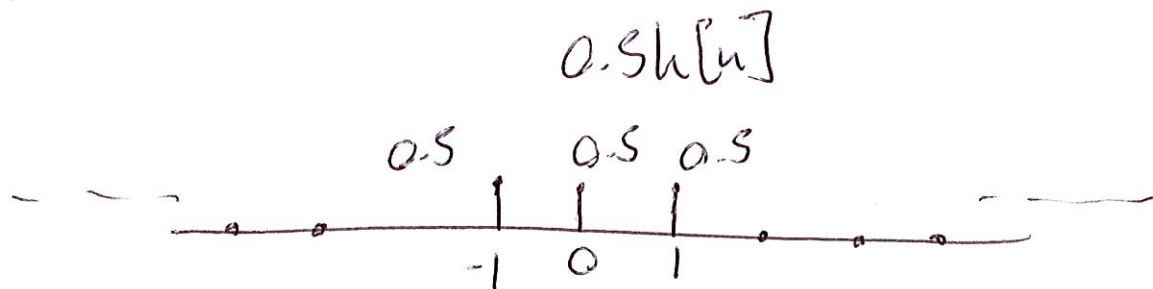
Convolution Sum



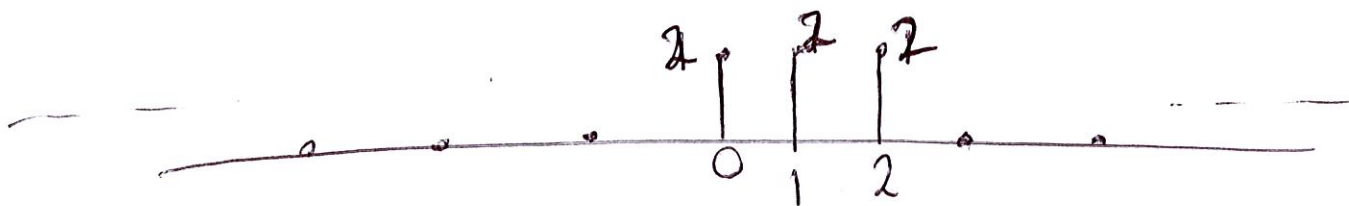
$$y_1[n] = \sum_{k=-\infty}^{\infty} x_1[k] h[n-k] = x_1[0] h[n] + x_1[1] h[n-1]$$

$$y_1[n] = 0.5 h[n] + 2 h[n-1]$$

(6)



$2h[n-1]$

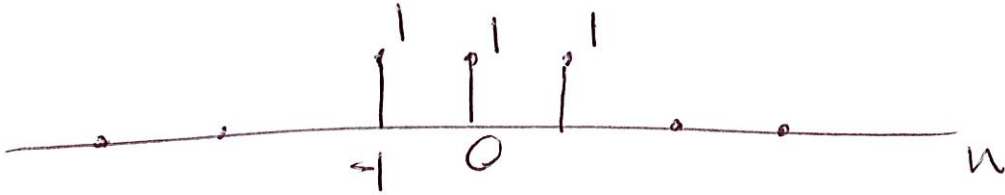


$y_1[n]$

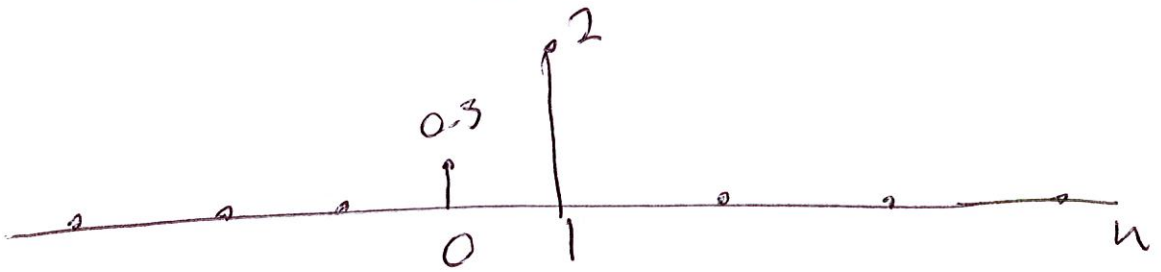


(7)

$h[n]$

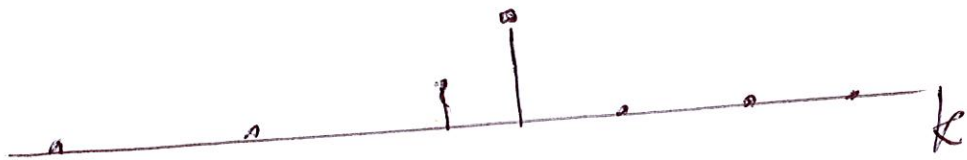


$x_1[n]$

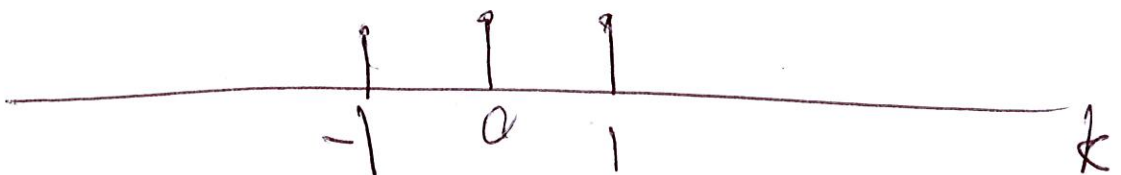


$$y_1[n] = \sum_{k=-\infty}^{\infty} x_1[k] h[n-k]$$

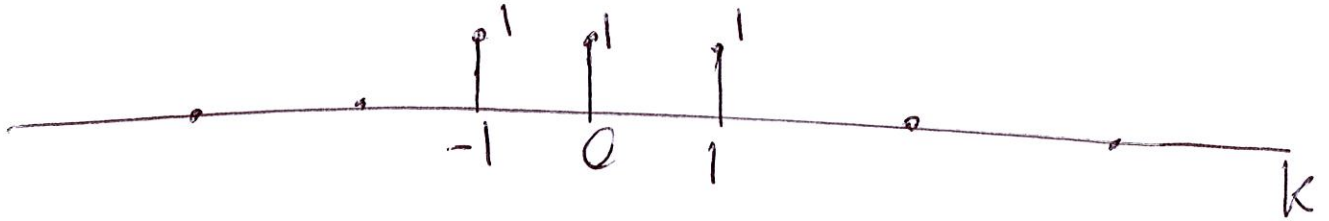
$x_1[k]$



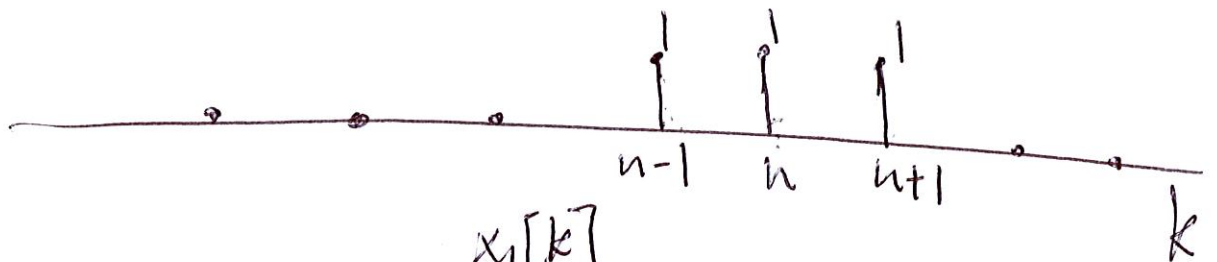
$h_1[k]$



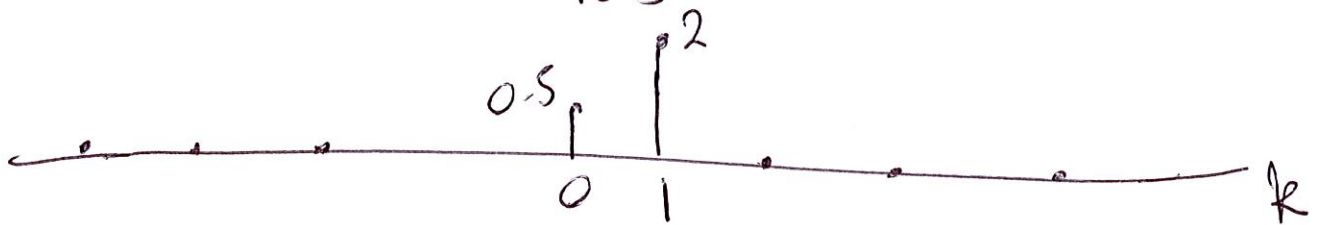
$$h_x[-k]$$



$$h_x[n-k]$$



$$x_1[k]$$



$$\sum_{k=-\infty}^{\infty} x_1[k] h_x[n-k] =$$

$$\left\{ \begin{array}{l} 0 \\ 2 \\ 2.5 \\ 0.5 \\ 0 \end{array} \right.$$

$$n \geq 3$$

$$n = 2$$

$$n = 1, 0$$

$$n = -1$$

$$n = -2$$

(9)

$$y_1[n] = \sum_{k=-\infty}^{\infty} x_1[k] h[n-k]$$

