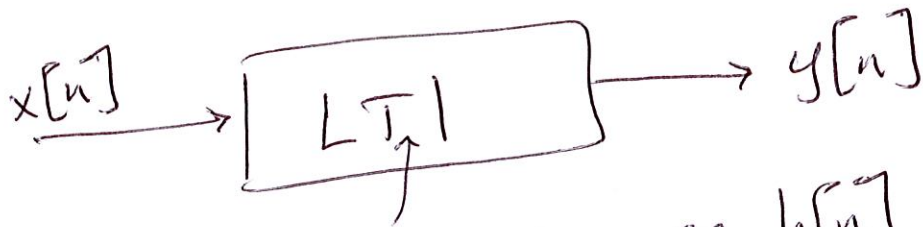


Linear and Time Invariant (LTI) SystemsConvolution Sum (DT) and Integral (CT)

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

Example 2.3

$$x[n] = \alpha^n u[n]$$

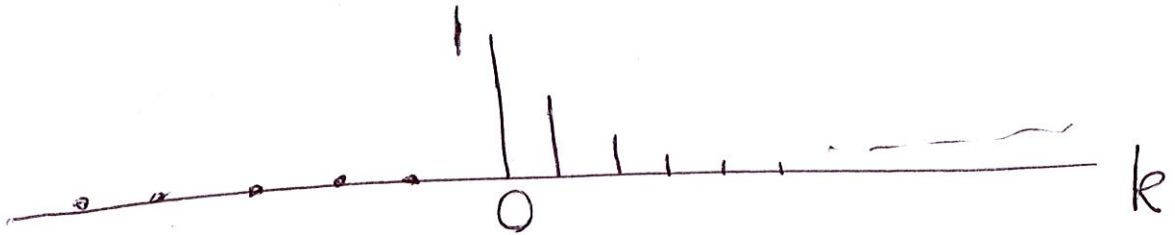
For example,

$$\alpha = \frac{1}{2}, \frac{1}{3}$$

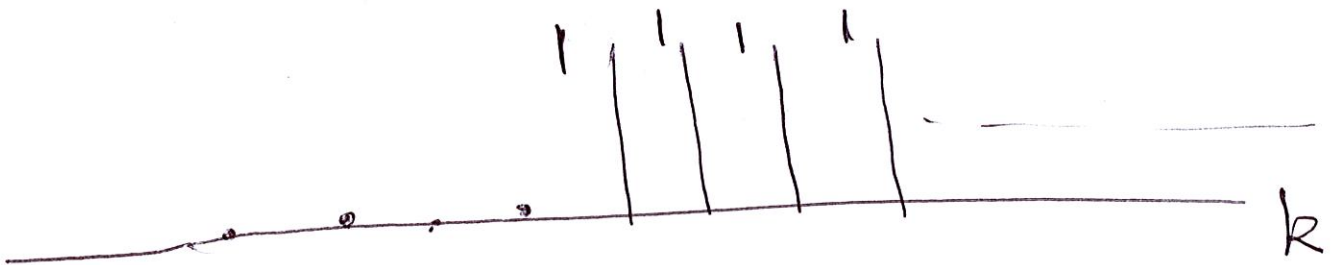
$$h[n] = u[n]$$

(2)

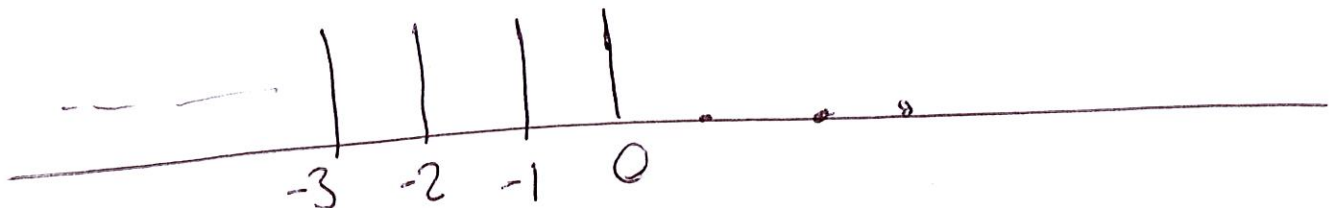
$$x[k] = \alpha^k u[k]$$



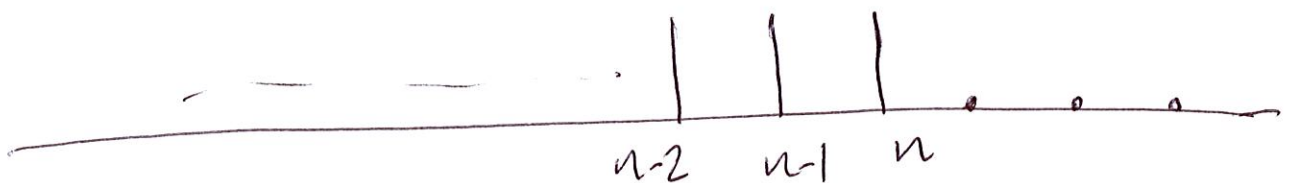
$$h[k]$$



$$h[-k]$$



$$h[n-k]$$



$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

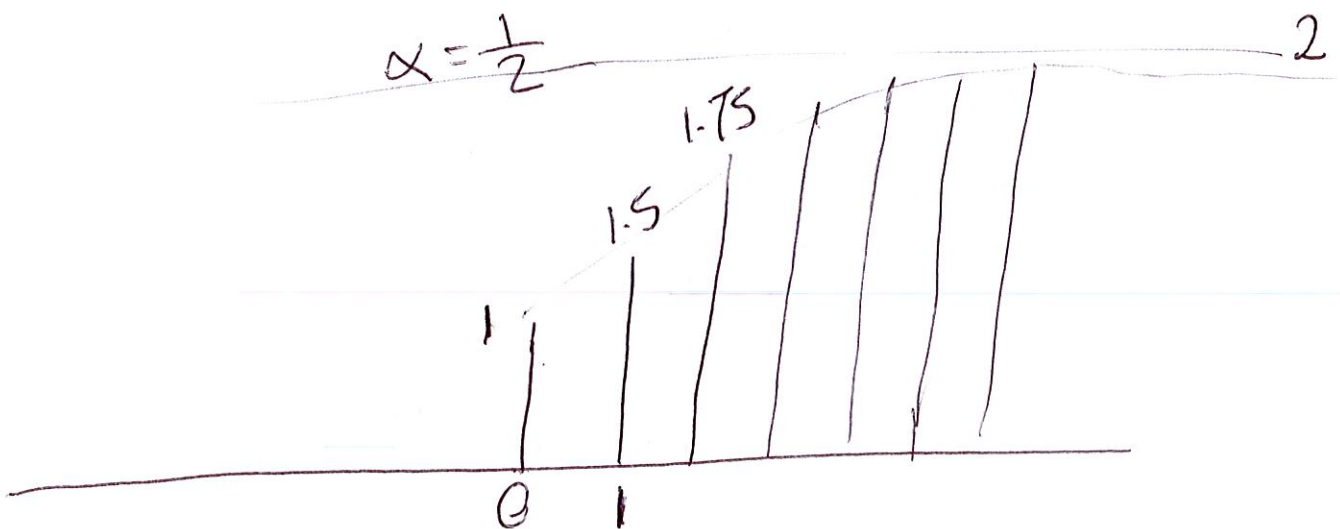
(3)

$$\text{If } n < 0 \Rightarrow y[n] = 0$$

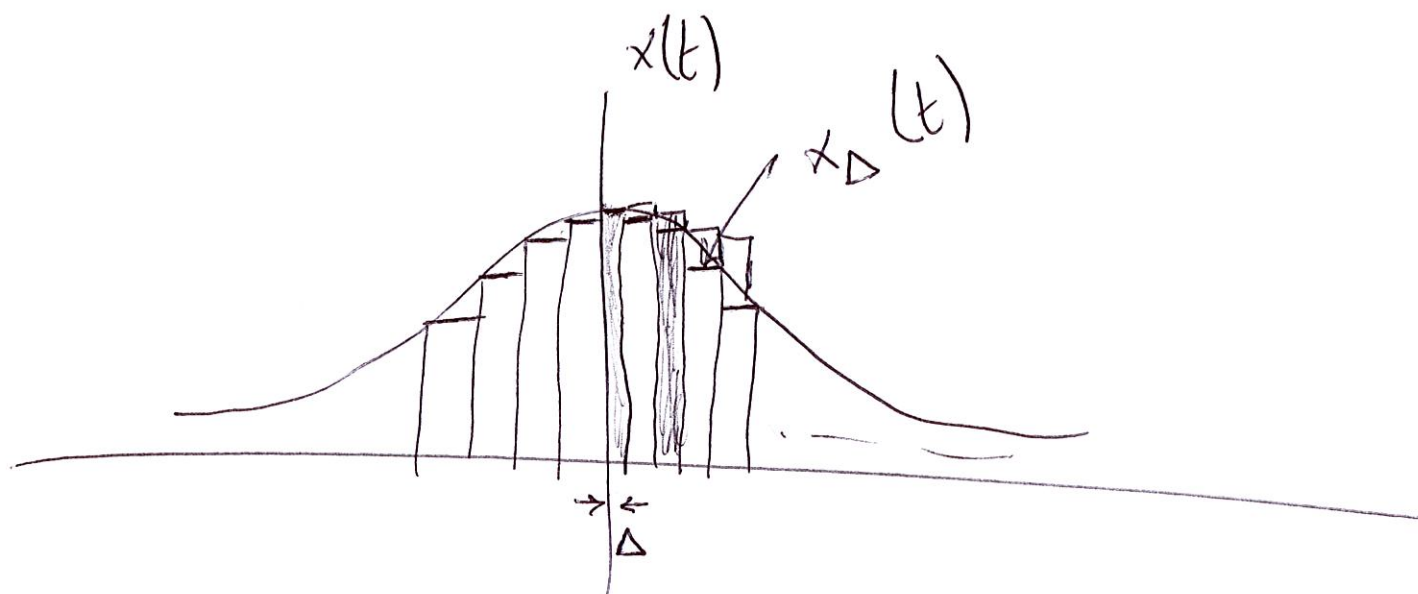
$$\text{If } n \geq 0 \Rightarrow y[n] = \sum_{k=0}^n \alpha^k$$

$$= \left(\frac{1 - \alpha^{n+1}}{1 - \alpha} \right)$$

$$\Rightarrow y[n] = \left(\frac{1 - \alpha^{n+1}}{1 - \alpha} \right) u[n]$$



(4)



$$\delta_{\Delta}(t) = \begin{cases} \frac{1}{\Delta}, & 0 \leq t \leq \Delta \\ 0 & \text{otherwise} \end{cases}$$

$$x_{\Delta}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta) \delta_{\Delta}(t - k\Delta) \Delta \quad (1)$$

$$x(t) = \lim_{\Delta \rightarrow 0} x_{\Delta}(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t-\tau) d\tau$$

(5)

Verification

$$\int_{-\infty}^{\infty} x(\tau) \delta(t-\tau) d\tau = \int_{-\infty}^{\infty} x(t) \delta(t-\tau) d\tau$$

$$= x(t) \int_{-\infty}^{\infty} \delta(t-\tau) d\tau$$

$$= x(t)$$

Go back to (1)

$$x_{\Delta}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta) \delta_{\Delta}(t-k\Delta) \Delta$$

$$x(t) = \lim_{\Delta \rightarrow 0} \sum_{k=-\infty}^{\infty} x(k\Delta) \delta_{\Delta}(t-k\Delta) \Delta$$

$$x(t) = \lim_{\Delta \rightarrow 0} \sum_{k=-\infty}^{\infty} x(k\Delta) \delta_{\Delta}(t-k\Delta) \Delta$$

(6)

$$y(t) = \lim_{\Delta \rightarrow 0} \sum_{k=-\infty}^{\infty} x(k\Delta) h(t-k\Delta) \Delta$$

$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$= x(t) * h(t)$$

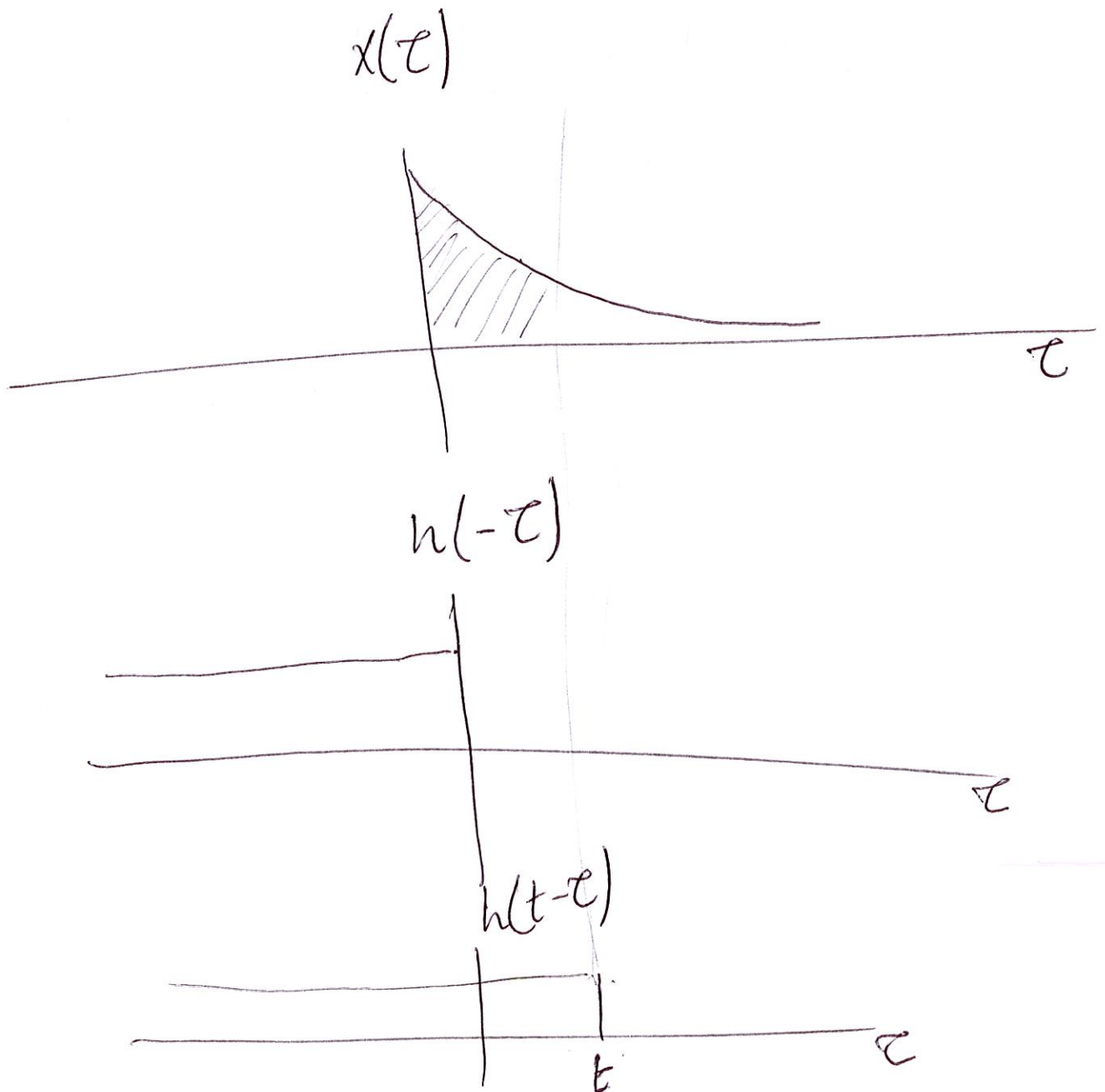
Example 2-6

(7)

$$x(t) = e^{-at} u(t), \quad a > 0$$

$$h(t) = u(t)$$

$$\int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

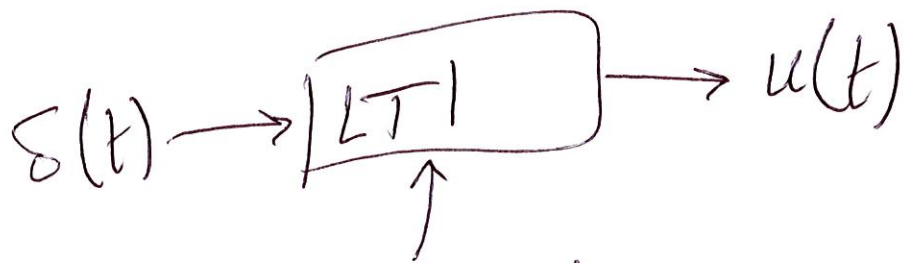
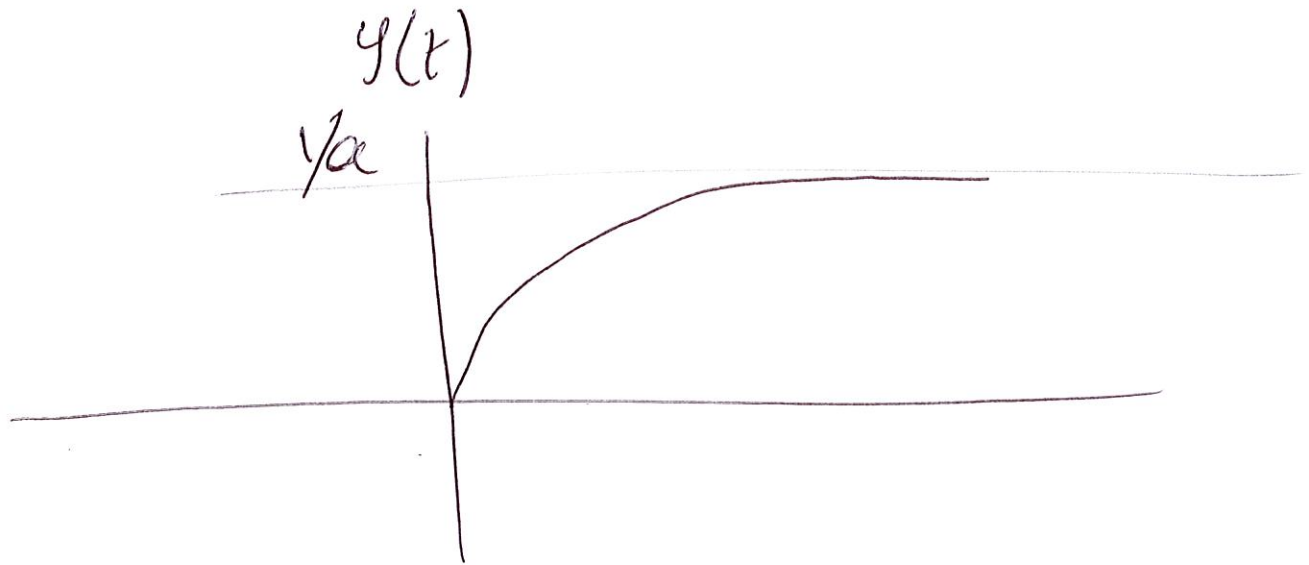


(8)

$$\underline{t < 0} \quad y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$
$$= 0$$

$$\underline{t \geq 0} \quad y(t) = \int_{\tau=-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$
$$= \int_0^t e^{-a\tau} d\tau \quad u(t)$$
$$= -\frac{1}{a} e^{-a\tau} \Big|_0^t \quad u(t)$$
$$= \frac{1}{a} (1 - e^{-at}) \quad u(t)$$

(9)



Causal
Memoryless? No

Stable? Take $u(t)$ as input, the output is unbounded, so the system is unstable

