Lecture - ~~W~~ Fri. Oct. 9

Exam Answers

Problem 1

(a) $x_1(t) = e^t$

$E_\infty = \infty \qquad P_\infty = \infty$

(b) $x_2(t) = \begin{cases} e^t & 0 \le t \le 5000 \\ 0 & \text{otherwise} \end{cases}$

$$E_\infty = \int_0^{5000} e^{2t}\,dt = \frac{1}{2} e^{2t}\Big|_0^{5000}$$

$$= \frac{e^{10000}}{2} - \frac{1}{2}$$

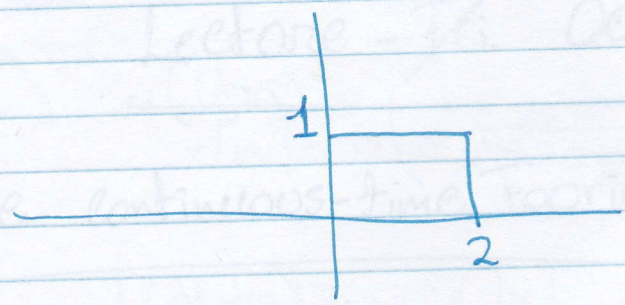
$P_\infty = 0$

(c) $x_3(t) = e^0 = 1$

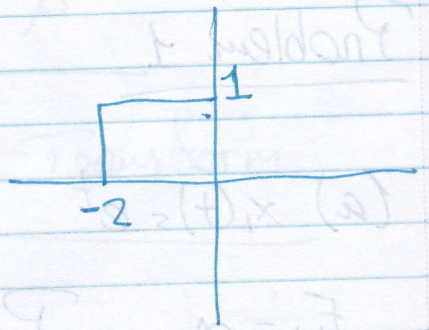
$E_\infty = \infty \qquad P_\infty = 1$

Problem 2

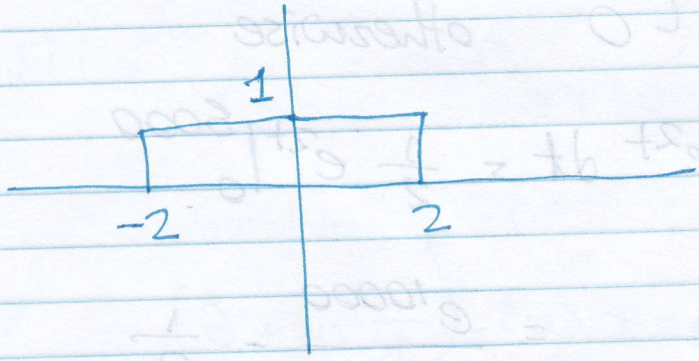
(a) $x(t-1)$



$x(t+1)$

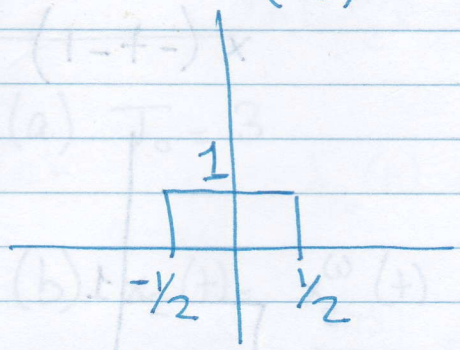


$x(t-1) + x(t+1)$

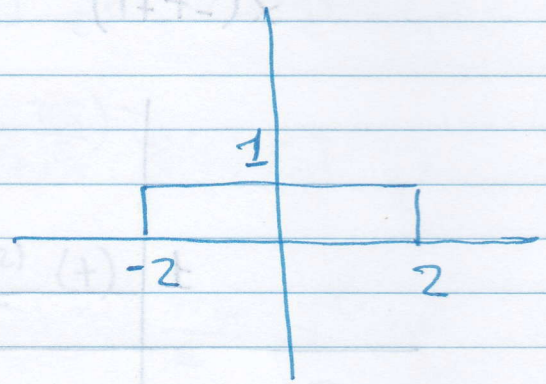


(b)

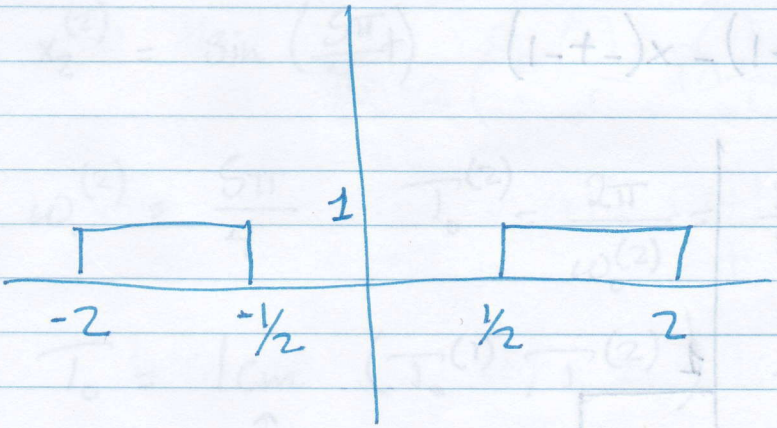
$x(2t)$



$x(t/2)$

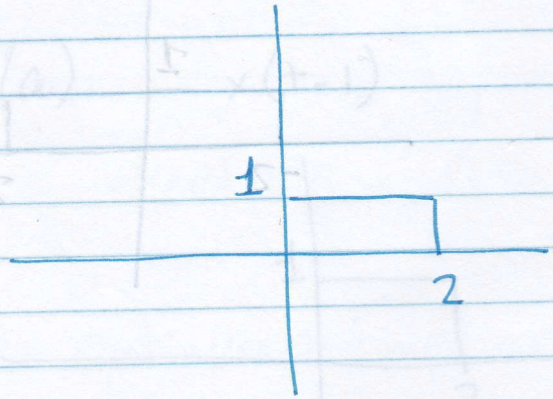


$x(2t) - x(t/2)$

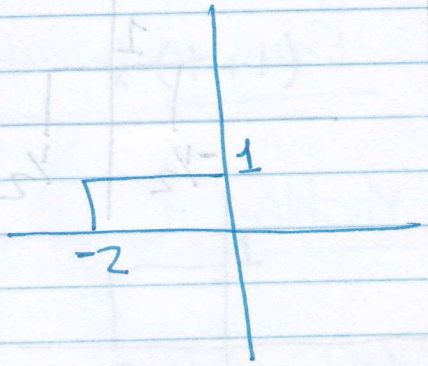


(c)

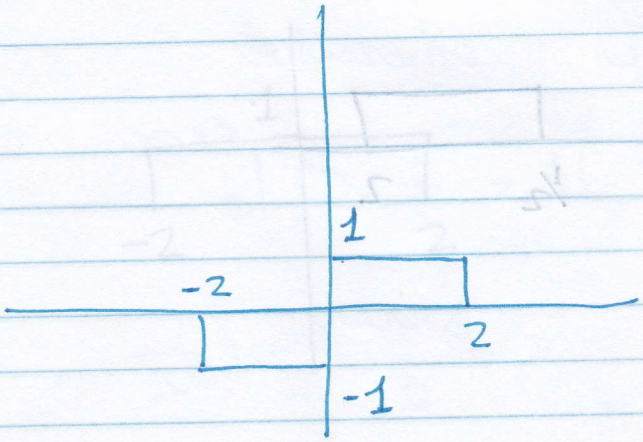
$x(-t+1)$



$x(-t-1)$



$x(-t+1) - x(-t-1)$



Problem 3

(a) $T_0 = 3$

(b) $x_2(t) = x_2^{(1)}(t) + x_2^{(2)}(t)$

$$x_2^{(1)} = \cos\left(\frac{10\pi}{3}t\right)$$

$$\omega_0^{(1)} = \frac{10\pi}{3} \qquad T_0^{(1)} = \frac{2\pi}{\omega_0^{(1)}} = \frac{3}{5}$$

$$x_2^{(2)} = \sin\left(\frac{5\pi}{4}t\right)$$

$$\omega_0^{(2)} = \frac{5\pi}{4} \qquad T_0^{(2)} = \frac{2\pi}{\omega_0^{(2)}} = \frac{8}{5}$$

$$T_0 = \underset{\uparrow}{\mathrm{lcm}}\left(T_0^{(1)}, T_0^{(2)}\right) = \frac{24}{5}$$

Least Common multiple

(c) $x_3[n] = x_3^{(1)}[n] + x_3^{(2)}[n]$

$x_3^{(1)}[n] = e^{j\left(\frac{2\pi}{3}n\right)}$

$\omega_0^{(1)} = \frac{2\pi}{3}$ $\quad T_0^{(1)} = \frac{2\pi}{\omega_0^{(1)}} = 3$

$x_3^{(2)}[n] = e^{j\left(\frac{3\pi}{4}\right)n}$

$\omega_0^{(2)} = \frac{3\pi}{4}$ $\quad T_0^{(2)} = \frac{2\pi}{\omega_0^{(2)}} = \frac{8}{3}$

$T_0 = \mathrm{lcm}\left(T_0^{(1)}, T_0^{(2)}\right) = 24$

(d) $x_4(t) = 1$

$T_0 = 0$

Problem 4

(a) $\lim_{\Delta \to 0} \delta_\Delta(t) * r_\Delta(t) = \delta(t)$

Unit Impulse

(b) $\lim_{\Delta \to 0} \delta_\Delta(t) * r_\Delta(t) * \delta_\Delta(t) * r_\Delta(t) = \delta(t)$

Note that $\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$

and $\lim_{\Delta \to 0} r_\Delta(t) = \delta(t)$

because $r_\Delta(t) = \delta_\Delta(t) * \delta_\Delta(t)$

and $\delta(t) * \delta(t) = \delta(t)$

Problem 5

(a) $x_2(t)$, $x_3(t)$

because $a_k = a_{-k}^*$ for these two signals

(b) $x_2(t)$

because $a_k = a_{-k}$ for this signal

Problem 6

(a) $h[n]=0$ for $n<0$

(b) $y[n]=x[n]*h[n]=h[n]*x[n]$

$$=\sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

if $h[k]=0$ for $k<\infty$

then $y[n]=\sum_{k=0}^{\infty} h[k]\,x[n-k]$

$\Rightarrow$ LTI System is Causal

Also, if LTI system is causal, then

$y[n]$ depends only on $x[n']$, where $n'\leq n$

$\Rightarrow y[n]=\sum_{k=0}^{\infty} h[k]\,x[n-k]$

$\Rightarrow h[k]=0$ for $k<0$