# Spring 2019 - Exam 1
# ECE 301: Signals and Systems

Prof. Aly El Gamal

Last Name :

First Name :

PUID :

**Instructions.**

- Write your full name, PUID on this page
- This is a 50 minutes exam containing 6 questions, 25 points each. Maximum score is 125.
- Please write clearly and legibly
- Your solutions must include detailed steps and/or explanations. Do not simply state the answer
- One A-4 sized crib sheet front and back is allowed.
- There should be 7 pages including the cover page.

| Question | Points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| Total | |

# Problem 1

a Consider $x(t)$ in Figure 1.

i (**3 Points**) Sketch $x(-2t+4)$

ii (**3 Points**) Sketch $x(\frac{t}{2}-2)$

iii (**3 Points**) Sketch $x(-t)$

iv (**4 Points**) Is $x(t)$ even? Is it odd? Justify your answer.

b (**6 Points**) For $x[n]$ in Figure 2, sketch $x[-3n+1]u[n-1]$

c (**6 Points**) How many different signals are in the set $\{e^{j\frac{5\pi nk}{2}}$, k is an integer$\}$? Justify your answer.

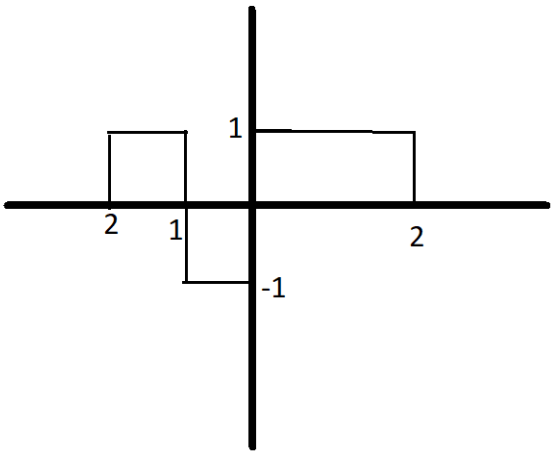


Figure 1: x(t)

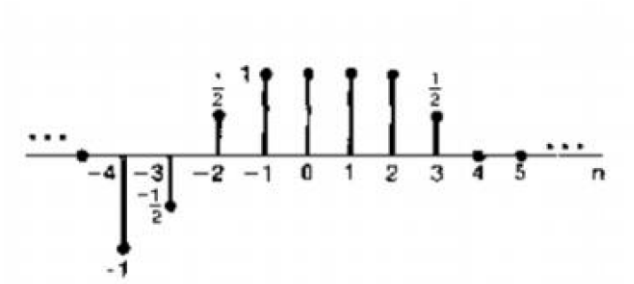


Figure 2: x[n]

# Problem 2

a (**12 Points**) For $x(t) = t^{-\frac{1}{2}}u(t)$, calculate $E_\infty$ and $P_\infty$.

b Are the following signals invertible? Justify your answers.

i (**3 Points**) $y[n] = x[n] - x[2]$

ii (**3 Points**) $y[n] = x[n]u[n]$

iii (**3 Points**) $y[n] = nx[n]$

iv (**4 Points**) $y[n] = \sum_{k=n+1}^{\infty} x[k]$

# Problem 3

Consider a Discrete-Time LTI system, where the output at each time $n$ is obtained by accumulating the values of the input from $n-3$ to $\infty$.

a (**5 Points**) What is the impulse response?

b (**10 Points**) What is y[n] when $x[n] = u[n+5] - u[n+7]$, where $u[n]$ indicates the unit step signal?

c (**10 Points**) Is this system

i Causal?

ii Memoryless?

iii Stable?

iv Invertible? If so, what is the impulse response of the inverse system.

# Problem 4

(**25 points**) Consider a linear system, where the response to an input signal $x[n] = \delta[n-k]$ is given by $y[n] = \delta[n-k] + \delta[n-k-2]$.
Is this system time invariant? What is the output to $x[n]$ in Figure 3 (all values that are not shown in the figure equal zero)?

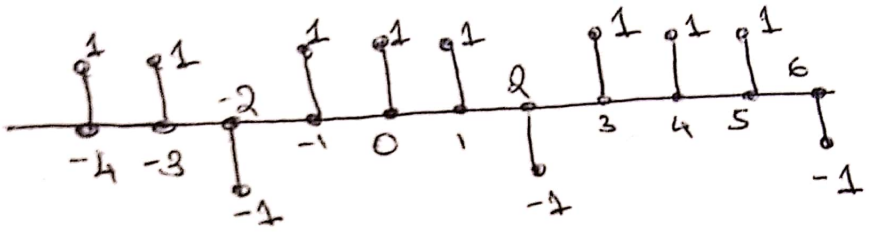


Figure 3: x[n]

# Problem 5

(**25 points**) Consider a system whose impulse response is $h(t) = u(t-4)$. What is the output $y(t)$, when the input signal is $x(t) = e^{-at}u(t)$?

# Problem 6

Consider the signal $\delta_\Delta(t)$, defined as

$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & \text{if } 0 \leq t \leq \Delta \\ 0 & \text{otherwise} \end{cases}$$

a (**10 points**) Sketch $\delta_{0.5}(t)$

b (**15 points** ) For x(t) in Figure 4, sketch $x(k\Delta)\delta_\Delta(t - k\Delta)$, for $k = 3$, $\Delta = 0.5$.

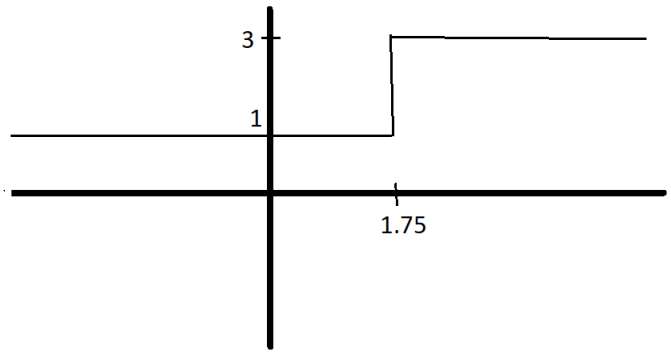


Figure 4: x(t)