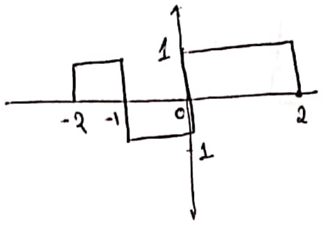
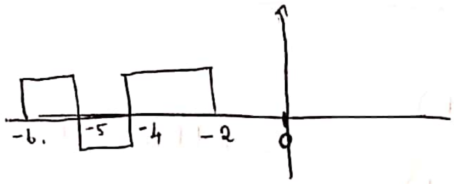
Exam - 1.

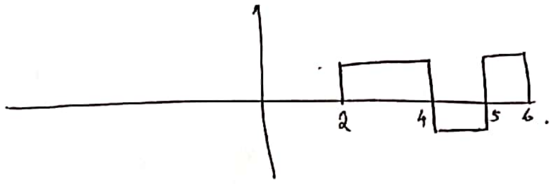
1. a). (i)



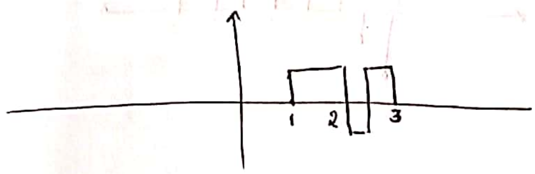
$x(t+4)$.



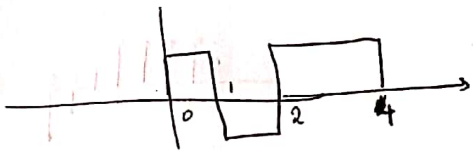
$x(-t+4)$.



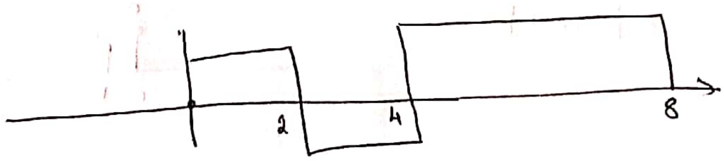
$x(-2t+4)$



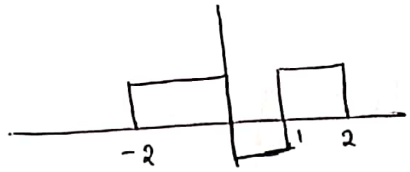
(ii) $x(t-2)$.



$x(t/2 - 2)$



(iii). $x(-t)$

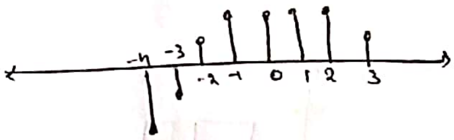


b).

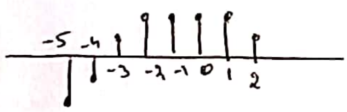
(iv). $x(t) \neq x(-t)$. $\therefore$ $x(t)$ is neither odd nor even.

$x(t) \neq -x(-t)$.

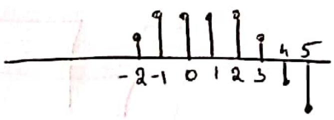
b).



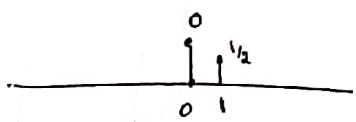
$x[n+1]$.



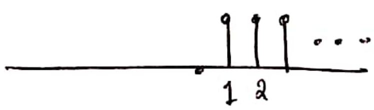
$x[-n+1]$.



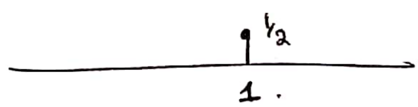
$x[-3n+1]$.



$u[n-1]$.



$x[-3n+1]\, u[n-1]$.



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c). $e^{j\frac{5\pi n k}{2}}$

$k = 0 \quad , 1$

$k = 1 \quad , e^{j\frac{5\pi n}{2}}$

$k = 2 \quad , e^{j 5\pi n}$

$k = 3 \quad , e^{j\frac{15\pi n}{2}}$

$k = 4 \quad , e^{j 10\pi n} = 1.$

$\therefore$ 4 different signals.

2. $x(t) = t^{-1/2} u(t).$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} |t^{-1/2} u(t)|^2 \, dt$$

$$= \int_{0}^{\infty} t^{-1} dt = \cancel{\left[\frac{t^2}{2}\right]_0^\infty} \; [\log t]_0^\infty = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} | t^{-1/2} u(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} t^{-1} dt$$

$$\cancel{= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{t^2}{2}\right]_0^T}$$

$$\cancel{= \lim_{T\to\infty} \frac{1}{T} \frac{T^2}{2} = \lim_{T\to\infty} \frac{T}{2} = \infty}$$

$$= \lim_{T\to\infty} \frac{1}{2T} [\log t]_0^T$$

$$= 0$$

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b). (i) $y[n] = x[n] - x[2]$.

$x[n] = 0 \quad \forall n \qquad y[n] = 0, \ \forall n.$

$x[n] = 2 \quad \forall n \qquad y[n] = 0, \ \forall n.$

$\therefore$ Not invertible.

(ii). $y[n] = x[n]\, u[n]$.

$x[n] = \delta[n] + \delta[n+1] \Rightarrow y[n] = \delta[n].$

$x[n] = \delta[n] \Rightarrow y[n] = \delta[n].$

$\therefore$ Not invertible.

(iii). $y[n] = n\, x[n]$.

$y[n] = \delta[n] \Rightarrow y[n] = 0.$

$y[n] = 2\,\delta[n] \Rightarrow y[n] = 0.$

$\therefore$ Not invertible.

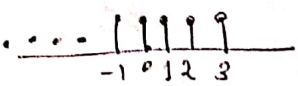
(iv) $y[n] = \sum_{k=n+1}^{\infty} x[k].$

Invertible.

Inverse system $\tilde{y}[n] = x[n-1] - x[n]$.

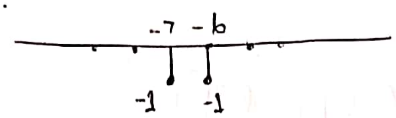
3. a) $y[n] = \sum_{k=n-3}^{\infty} x[k].$

$h[n] = \sum_{k=n-3}^{\infty} \delta[k].$



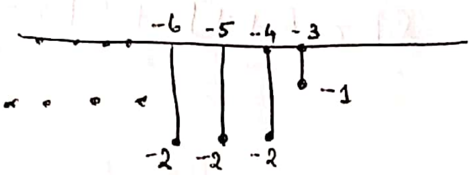
b) $x[n] = u[n+5] - u[n+7]$.

$x[n]$.



$y[n]$.

$y[n]$



c) (i) Not Causal. Depends on future input values

(ii) Not Memoryless

(iii). Not stable.

$x[n] = u[n]$. ~~$\lim_{n\to\infty}$~~

$y[n] \to \infty \quad \forall n.$

(iv). Invertible.

$\check{y}[n] = x[n+3] - x[n+4]$

4. If $x[n] = \delta[n-k]$.

$y[n] = \delta[n-k] + \delta[n-k-2]$.

System is time invariant.

~~because~~ for any.

~~consider~~ ~~let~~. $x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$.

$\because$ System is linear.

$y[n] = \sum_{k=-\infty}^{\infty} x[k] \left(\delta[n-k] + \delta[n-k-2]\right)$.

Now consider

$x_1[n] = x[n+N]$

$$\Rightarrow x_1[n] = \sum_{k=-\infty}^{\infty} x_1[k] \left(\delta[n-k]\right).$$

$$= \sum_{k=-\infty}^{\infty} x[k+N] \left(\delta[n-k]\right).$$

(Let $k_1 = k+N \Rightarrow k = k_1 - N$.)

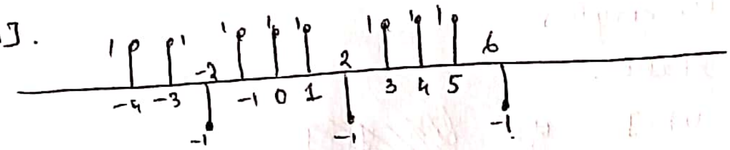
$$= \sum_{k_1=-\infty}^{\infty} x[k_1] \left(\delta[n-k_1+N]\right).$$

$$y_1[n] = \sum_{k_1=-\infty}^{\infty} x[k_1] \left(\delta[n-k_1+N] + \delta[n-k_1+N-2]\right).$$

$$= y[n+N].$$

$\therefore$ The system is time invariant.

$x[n]$.



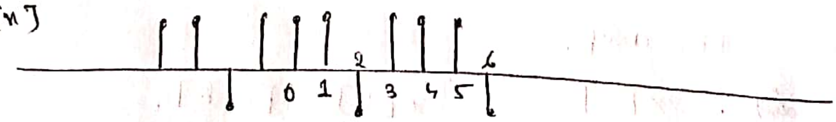
$\therefore$ The system is time invariant,

$h[n] = \delta[n] + \delta[n-2]$.

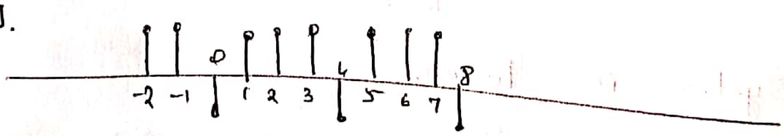
$\cancel{x[n] * \delta[n] = x[n]}$.

$\Rightarrow x[n] * h[n] = x[n] + x[n-2]$.

$x[n]$



$x[n-2]$.



$y[n]$

5.

$h(t) = u(t-4)$.

$x(t) = e^{-at} u(t)$.

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau.$$

$$= \int_{-\infty}^{\infty} e^{-a\tau} u(\tau)\, u(t-\tau-4)\, d\tau$$

$$= \int_{0}^{\infty} e^{-a\tau} u(t-\tau-4) \cdot d\tau$$

(For $t>4$)

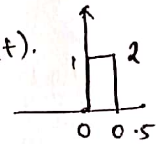
$$= \int_{0}^{t-4} e^{-a\tau}\, d\tau = \left[\frac{e^{-a\tau}}{-a}\right]_0^{t-4}$$

$$= \frac{e^{-a(t-4)} - 1}{-a} = \frac{1 - e^{-a(t-4)}}{a}.$$

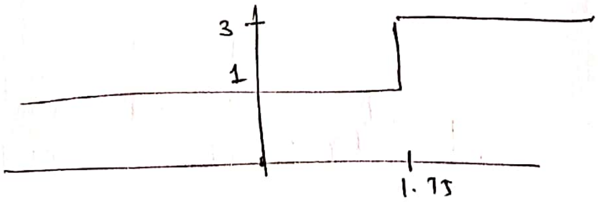
For $t \leq 4$ $y(t) = 0$.

$$\therefore y(t) = \frac{1 - e^{-a(t-4)}}{a} \cdot u(t-4).$$

6. a) $\delta_{0.5}(t)$.



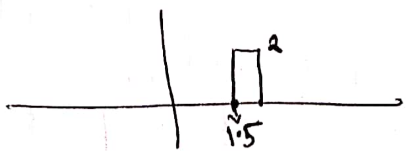
b).



$x(k\Delta)\ \delta_{\Delta}'(t-k\Delta)$ $k=3$ $\Delta=0.5$.

$x(1.5)\ \delta_{\Delta}(t-1.5)$.

$= 1 \cdot \delta_{\Delta}(t-1.5)$.



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