P1. a) $h[n] = \delta[n-1]$.

1. Invertible ✓ $h^{-1}[n] = \delta[n+1]$
2. Stable ✓ $\sum_{n=-\infty}^{\infty} |h[n]| = 1 < \infty$
3. Causal ✓ $\rightarrow$ $h[n] = 0 \quad \forall\, n < 0$.

(b) $h(t) = u(t)\, u(t+1) = u(t)$.

1. Invertible ✓ $h^{-1}(t) = \frac{d}{dt}\delta(t)$.
2. Stable ✗ $\int_{-\infty}^{\infty} |h(t)|\,dt = \infty$
3. Causal ✓ $h(t) = 0 \quad \forall\; t < 0$.

P2.

P3. a) $x(t) = e^{j2\omega_0 t}$

$\omega_0^* = 2\omega_0$

By inspection $a_1 = 1$ $a_k = 0$ $k \neq 1$.

b) $x(t) = e^{j\omega_0 t}$

$\omega_0^* = \omega_0$

By inspection $a_1 = 1$ $a_k = 0$ $k \neq 1$.

c) $x(t) = e^{j2\omega_0 t} + e^{j\omega_0 t}$

$T_1 = \frac{2\pi}{2\omega_0}$ $T_2 = \frac{2\pi}{\omega_0}$.

$T = LCM\left(\frac{\pi}{\omega_0}, \frac{2\pi}{\omega_0}\right) = \frac{2\pi}{\omega_0}$. $\Rightarrow \omega_0^* = \frac{2\pi}{T} = \omega_0$.

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By inspection

$a_1 = 1 \quad a_2 = 1 \quad a_k = 0 \quad k \neq 1, 2.$

d) $x(t) = 1.$

$a_0 = 1 \quad a_k = 0 \quad k \neq 0.$

4. a) $\sum_{n=60}^{99} x[n] = 2 \cdot N \cdot a_0 = 2 \cdot 20 \times 1 = 40.$

b). Average power $= \frac{1}{N} \sum_{\langle N \rangle} |x[n]|^2 = \sum_{\langle N \rangle} a_k^2 = 1 + 4 \cdot 9 + (1) \cdot 5$

$= 1 + 36 + 5 = 42.$

c) $x[100] = x[0] = \sum_{\langle N \rangle} a_k = 1 + 2 \cdot 9 + 1 \cdot 5 = 24$

d) $x[101] = x[1] = \sum_{\langle N \rangle} a_k e^{jk \cdot \frac{2\pi}{20}}.$

$= 1 + 2 \sum_{k=1}^{9} \left(e^{jk \frac{2\pi}{20}} \right) - \sum_{k=15}^{19} e^{jk \frac{2\pi}{20}}.$

5. a)

Even$\{\cos t\}$ = $\cos t$ $\qquad \omega_0 = 1 \qquad a_k = 0 \quad k \neq +1, -1.$

$a_1 = 1/2 \quad a_{-1} = 1/2.$

$$X_T(j\omega) = 2\pi \sum_{n=-\infty}^{\infty} a_n \, \delta(\omega - n\omega_0)$$

$$= \pi\,\delta(\omega - 1) + \pi\,\delta(\omega + 1).$$

b) Odd$\{\cos t\}$ = 0 $\Rightarrow X(j\omega) = 0.$

c) Even$\{\sin t\} = 0$ $\Rightarrow X(j\omega) = 0.$

d) odd$(\sin t)$ = $\sin t$ $\qquad \omega_0 = 1 \quad a_k = 0 \quad k = +1, -1.$

$a_1 = 1/2j \quad a_{-1} = -1/2j$

$$X_T(j\omega) = \frac{\pi}{j}\,\delta(\omega - 1) - \frac{\pi}{j}\,\delta(\omega + 1).$$

e). $a_0 = \dfrac{2T_1}{T} \qquad a_k = \dfrac{\sin(k\omega_0 T_1)}{k\pi}.$

$T = 4 \quad T_1 = 0.25 \quad \omega_0 = \pi/2.$

$a_0 = 1/8 \qquad a_k = \dfrac{\sin(k\pi/8)}{k\pi}.$

$$X_T(j\omega) = \sum_{k=-\infty}^{\infty} a_k \, \delta(\omega - n\pi/2).$$

6. a) $e^{-2t}\, u(t)$.

$$\cancel{X(j\omega) = \int_{-\infty}^{\infty} e^{-2t}\, u(t)\, dt}$$

$$\cancel{= \int_{0}^{\infty} e^{-2t}\, dt = \left[\frac{e^{-2t}}{-2}\right]_0^{\infty} = \frac{0+1}{2}}$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-2t}\, u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-2t}\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-(2+j\omega)t}\, dt$$

$$= \left[\frac{e^{-(2+j\omega)t}}{-(2+j\omega)}\right]_0^{\infty}$$

$$= \frac{0+1}{2+j\omega} = \frac{1}{2+j\omega}$$

(b)

b) $e^{-2|t|}$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-2|t|} e^{-j\omega t} \, dt$$

$$= \int_{-\infty}^{0} e^{2t} e^{-j\omega t} \, dt + \int_{0}^{\infty} e^{-2t} e^{-j\omega t} \, dt$$

$$= \int_{-\infty}^{0} e^{(2-j\omega)t} \, dt + \int_{0}^{\infty} e^{-(2+j\omega)t} \, dt$$

$$= \left[\frac{e^{(2-j\omega)t}}{2-j\omega} \right]_{-\infty}^{0} + \left[\frac{e^{-(2+j\omega)t}}{-(2+j\omega)} \right]_{0}^{\infty}$$

$$= \frac{1}{2-j\omega} + \frac{1}{2+j\omega} = \frac{4}{4+\omega^2}.$$

c). $e^{-3|t|} + e^{-4t} u(t)$.

$\downarrow$

Similar to (b)

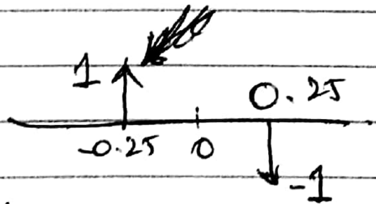
$$X_1(j\omega) = \frac{6}{9+\omega^2}$$

Similar to part (a).

$$X_2(j\omega) = \frac{1}{4+j\omega}.$$

$$X(j\omega) = X_1(j\omega) + X_2(j\omega).$$

(d). $\frac{d}{dt} x(t) =$



$$Y(j\omega) = e^{j(0.25)\omega} - e^{-j(0.25)\omega}$$

$$= 2j \sin((0.25)\omega).$$

$$= 2j \sin\left(\frac{\omega}{4}\right).$$

$$x(j\omega) = \frac{y(j\omega)}{j\omega} = \frac{2\sin\left(\frac{\omega}{4}\right)}{\omega}$$