Spring 2019 - Final Exam
ECE 301: Signals and Systems

Prof. Aly El Gamal

Last Name :

First Name :

PUID :

Instructions.

- Write your full name, PUID on this page
- This is a 120 minutes exam containing 8 questions.
- Please write clearly and legibly
- Your solutions must include detailed steps and/or explanations. Do not simply state the answer
- Three A-4 sized crib sheet front and back is allowed.
- There should be 9 pages including the cover page.

Question	Points
1	
2	
3	
4	
5	
6	
7	
8	
Total	

[50 points] Problem 1

1. [30 points] Consider x(t) given in Figure 1. Sketch

   a $x(-t)$

   b $x(-2t+4)$

   c $x(\frac{t}{2}-1)$

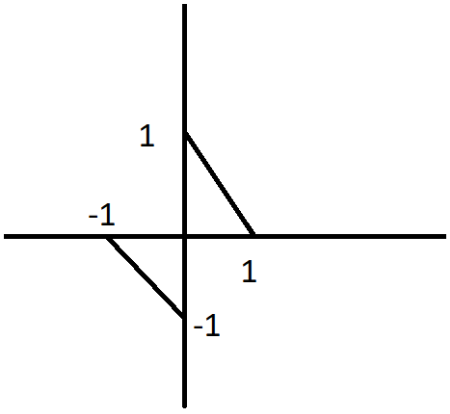

Figure 1: x(t)

2. [20 points] For each of the following, state if it is possible to have a signal satisfying the stated conditions. If yes, give an example. If no, justify your answer.

   a $E_\infty = \infty$, $P_\infty = 0$

   b $E_\infty < \infty$, $P_\infty > 0$

[30 points] Problem 2

1. [20 points] Consider a Discrete-Time LTI system, where the output at each time n is obtained by accumulating the values of the input from $n+3$ to $n+4$.

   a What is the impulse response?

   b What is y[n] when $x[n] = u[n+3] - u[n+4]$, where $u[n]$ indicates the unit step signal?

   c Is this system

      i Causal?

      ii Memoryless?

      iii Stable?

      iv Invertible? If so, what is the impulse response of the inverse system.

   d Repeat all the above when the output at time n is obtained by accumulating the values of the input from $n+3$ to ∞.

2. [10 points] For each of the following signals, comment if the signal is: Causal? Stable? Invertible? Memoryless? Justify your answer.

   a $h[n] = u[n]$

   b $h[n] = \delta[n]$

   c $h(t) = e^{-t}u(t)$

   d $h(t) = e^{t}u(-t)$

[40 points] Problem 3

1. [20 points]

   a What is Euler's observation on the movement of strings? What was the criticism of his observation?

   b Describe Fourier's discovery on the application of trigonometric signals.

   c If a periodic signal has finite energy, what does that imply on the convergence of its Fourier Series representation?

   d State Dirichlet conditions. Give examples of periodic signals violating each condition. If a periodic signal satisfies all three conditions, what does that imply?

2. [20 points] Derive the Fourier transform and inverse Fourier transform for continuous time finite duration signals from the Fourier series of continuous time periodic signals.

[50 points] Problem 4

1. [20 points]

   a What are the impulse response and the frequency response of the Ideal low pass filter?

   b What are the practical limitations of an ideal low pass filter?

   c Given an example of a low pass filter approximation. State both impulse and frequency responses.

2. [30 points] Consider the LTI system given in Figure 2. $H_{lp}(e^{j\omega})$ is given in Figure 3. Find the Frequency response of the system. Comment on the functionality of the system.

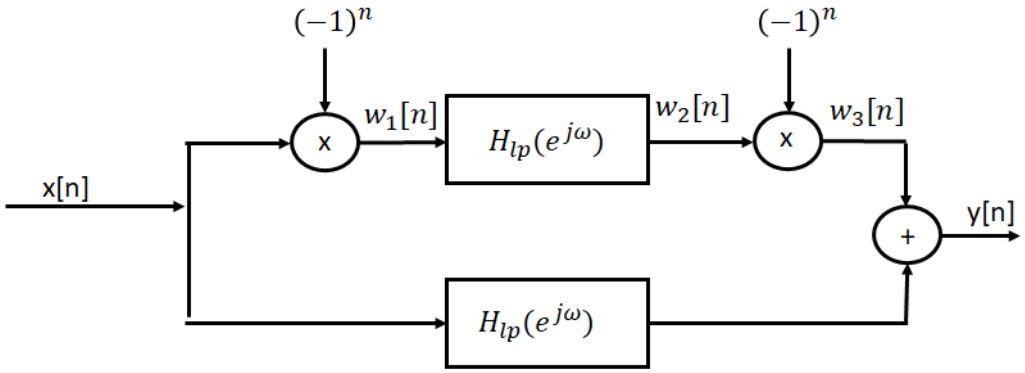

Figure 2

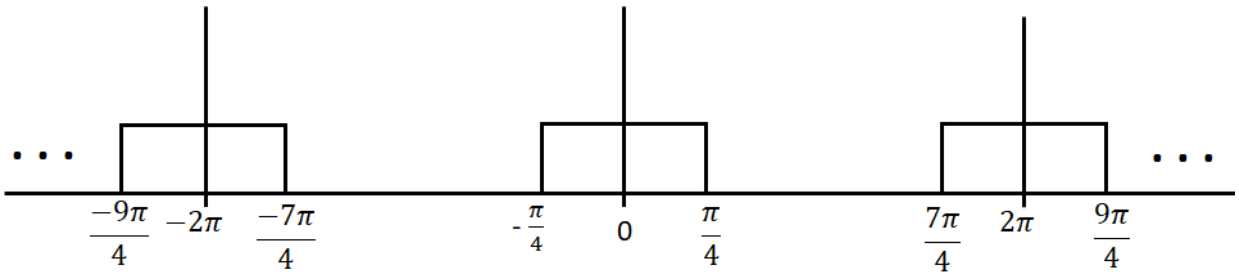

Figure 3: $H_{lp}(e^{j\omega})$

[40 points] Problem 5

1. [20 points] Find the DTFT of the periodic signal $x[n]$ given in Figure 4.

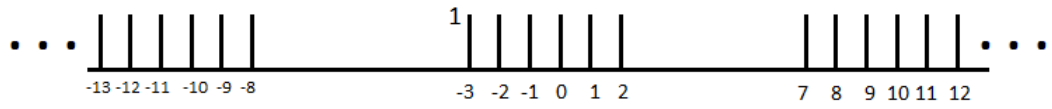

Figure 4: x[n]

2. [20 points] Find the DTFT of the periodic signal $y[n]$ given in Figure 5.

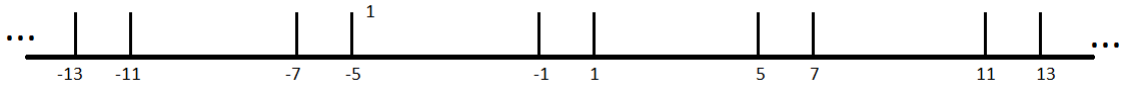

Figure 5: y[n]

[30 points] Problem 6

Given the DTFT $Y(e^{j\omega})$ of a signal $y[n]$, answer the following questions

$$Y(e^{j\omega}) = \cos(\omega)$$

a Is y[n] real?

b Is y[n] even?

c Evaluate $\sum_{n=-\infty}^{\infty} y[n]$

d Evaluate $\sum_{n=-\infty}^{\infty} (-1)^n y[n]$

[30 points] Problem 7

1. [20 points] Consider 2 LTI system with corresponding impulse responses in Figure 7. Find the output $y(t)$, for each of the systems when the input is $x(t)$ given in Figure 6.

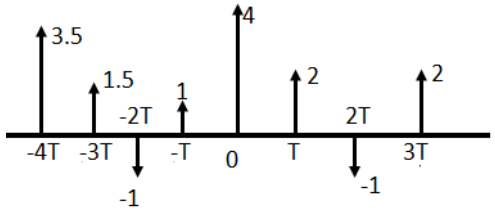

Figure 6: x(t)

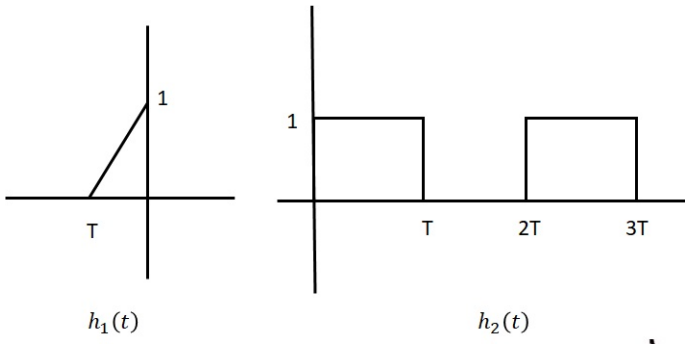

Figure 7: x(t)

2. [10 points] Find the most efficient downsampling and/or upsampling scheme for the signal $x[n]$ whose Fourier Transform $X(e^{j\omega})$ is as shown in Figure 8.

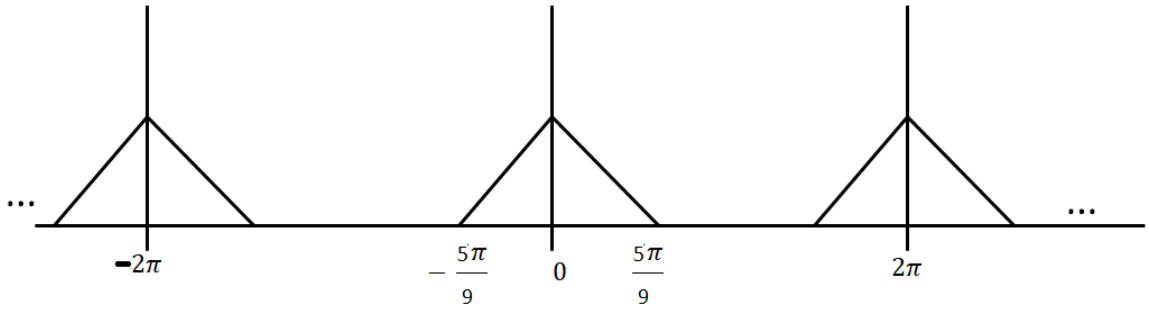

Figure 8: $X(e^{j\omega})$

[30 points] Problem 8

1.[20 points] Consider the signal $x(t) = \delta(t) - \frac{4}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$. Find the Laplace transform of $x(t)$ and its Region of Convergence.
2.[10 points] Does the Fourier transform exist for $x(t)$? Why?