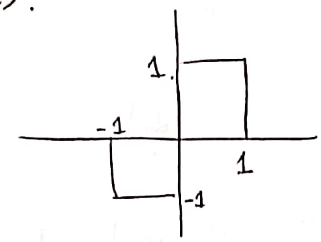
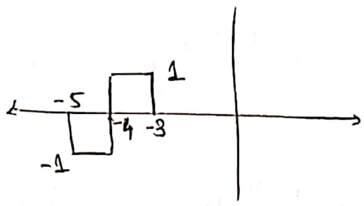
HW. 1.

1. $x(t)$.

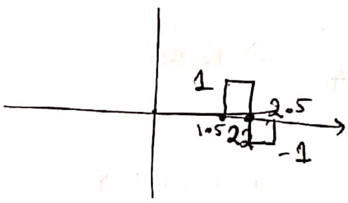


a) $x(-2t+4)$

$x_1(t) = x(t+4)$.

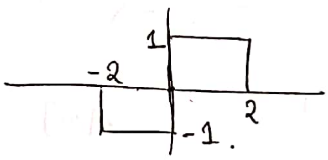


$x_2(t) = x_1(-2t) = x(-2t+4)$.

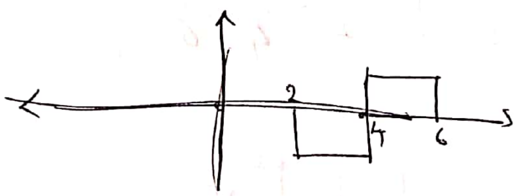


b) $x(t/2 - 2)$.

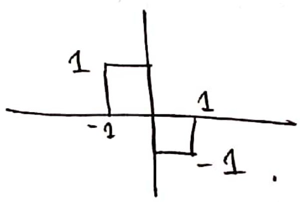
$x_1(t) = x(t/2)$.



$x_2(t) = x_1(t-4) = x\left(\frac{t-4}{2}\right) = x\left(\frac{t}{2} - 2\right)$.



c) $x(-t)$.



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Prob 2:

a) $\omega_0 = 11\pi$ $\qquad$ $\omega_0 = \frac{21}{2}\pi$.

$\omega_0 = 11\pi \equiv \omega_0 = \pi$ $\quad$ Period $= 2$. ✓ Higher frequency.

$\omega_0 = \frac{21}{2}\pi \equiv \omega_0 = -\frac{\pi}{2}$ $\quad$ Period $= 4$,

b). 4 different signals.

$\{1, e^{j\frac{\pi}{2}n}, e^{j\pi n}, e^{j\frac{3\pi}{2}n}\}$ as $e^{j2\pi n} = 1$.

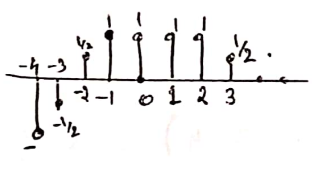
c) $x[n] = e^{j\frac{3\pi}{4}n} + e^{j5\pi/2\, n}$.

$\omega_0 = \frac{3\pi}{4}$ ~~$\Rightarrow$ Period $= 8$~~ $\qquad$ $\omega_0 = \frac{5\pi}{2}$

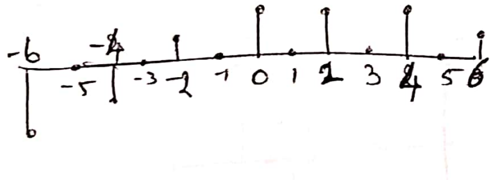
$\frac{\omega_0}{2\pi} = \frac{3}{8} \Rightarrow$ Period $= 8$ $\qquad$ $\frac{\omega_0}{2\pi} = \frac{5}{4} \Rightarrow$ Period $= 4$.

$LCM(8,4) = 8 \Rightarrow$ Period $= 8$.

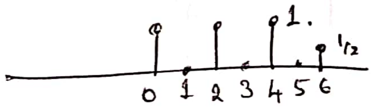
d) $x\left[\frac{n}{2}\right]u[n]$ $\qquad$ $x[n]$



$x\left[\frac{n}{2}\right]$.



$x\left[\frac{n}{2}\right]u[n]$



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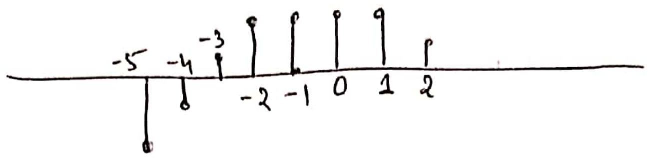
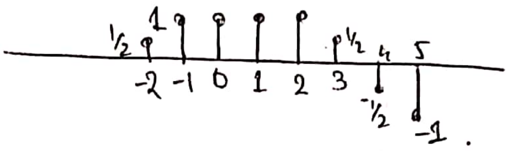
(ii) $x[-3n+1]$.

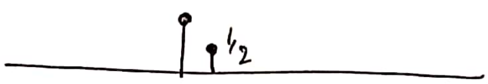
$x[n+1]$.



$x[-n+1]$.



$x[-3n+1]$.



~~Pro~~ 3.

a) $$x_e(t) = \frac{x(t) + x(-t)}{2}.$$

$$x_e(-t) = \frac{x(-t) + x(t)}{2} = x_e(t).$$

$\therefore$ $x_e(t)$ is even.

b) $$x_o(t) = \frac{x(t) - x(-t)}{2}.$$

$$x_o(-t) = \frac{x(-t) - x(t)}{2} = -x(+t).$$

$\therefore$ $x_o(t)$ is odd.

b) (i) n^{101} − odd.

$\sin \frac{\pi}{2} n$ − odd

$\Rightarrow x[n]$ is odd.

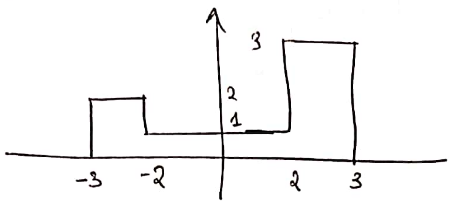
(ii) $x(t) = \cos(5\pi t) + t^4$

$\cos(5\pi t)$ ↓ Even

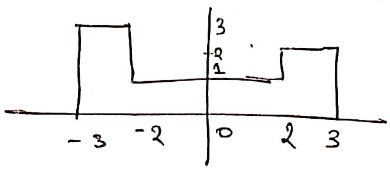
t^4 ↓ Even

$\therefore x(t)$ is even.

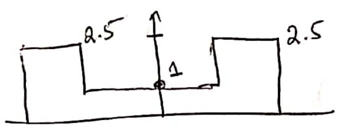
(iii). $x_1(t)$.



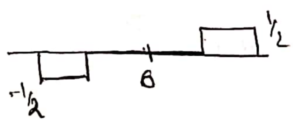
$x_1(-t)$.



$Even\{x_1(t)\}$ $\dfrac{x_1(t) + x_1(-t)}{2}$.



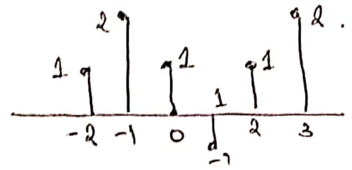
$Odd\{x_1(t)\}$ $\dfrac{x_1(t) - x_1(-t)}{2}$.



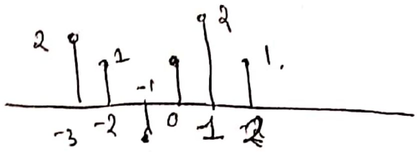
$x_2(t) = x_1(-t)$. $\Rightarrow$ $Even\{x_2(t)\} = Even\{x_1(t)\}$.

$Odd\{x_2(t)\} = x_{1,odd}(-t)$.

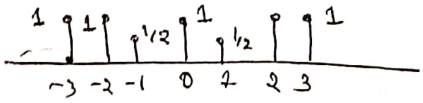
$x_1[n]$.



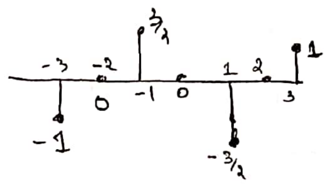
$x_1[-n]$

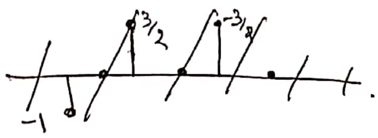


$$\text{Even}\{x_1[n]\} = \frac{x_1[n] + x_1[-n]}{2}$$



$$\text{Odd}\{x_1[n]\} = \frac{x_1[n] - x_1[-n]}{2}.$$





4. a) $E_\infty < \infty \quad P_\infty = 0.$

Possible.

$x[n] = \delta[n] \quad E_\infty = 1. \quad P_\infty = 0.$

b). $E_\infty = \infty \quad P_\infty = 0.$

Yes.

$x(t) = t^{-1/4} u(t). \quad E_\infty = \infty \quad P_\infty = 0.$

c). $E_\infty < \infty \quad P_\infty > 0.$

No. $E_\infty < \infty \Rightarrow \int_{-\infty}^{\infty} |x(t)|^2 dt \leq B$ B is finite

$\Rightarrow \int_{-T}^{T} |x(t)|^2 dt \leq B \quad \forall\, T \in \mathbb{R}^+.$

Consider $P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$

$$\leq \lim_{T\to\infty} \frac{1}{2T} B = 0$$

$P_\infty \leq 0 \Rightarrow P_\infty = 0$.

d. $E_\infty = \infty$ $\quad P_\infty > 0$. Possible.

$x(t) = 1$. $\quad E_\infty = \infty \quad P_\infty = 1$.

e). $E_\infty < \infty \quad P_\infty = \infty$

like to (c). Not possible.

f. $E_\infty = \infty \quad P_\infty = \infty$. Possible.

$x(t) = e^t$

5. a) $y(t) = x(t/7)$. $\quad y_{inv}(t) = x(7t)$.
Invertible.

b) $y(t) = \cos(\pi \cancel{\varepsilon} x(t))$.

$x_1(t) = 0 \quad \forall t$ $\quad$ For both $x_1(t)$ & $x_2(t)$

$x_2(t) = 2 \quad \forall t$. $\quad y(t) = 1. \quad \forall t$.

$\therefore$ Not invertible.

c). $y[n] = \tan\left(\frac{\pi}{28} x[n]\right)$.

$x_1[n] = 0$ $\quad$ Both give $y[n] = 0$.

$x_2[n] = 56$.

$\therefore$ Not invertible.

d) $\frac{d}{dt} x(t)$. $\quad x_1(t) = 1 \quad y_1(t) = 0$ $\quad \therefore$ Not invertible.

$x_2(t) = 2 \quad y_2(t) = 0$

e) $x_1[n] = 0 \;\forall n \in \mathbb{N},\; y_1[n] = 0$.

$x_2[n] = 1,\; \forall n \in \mathbb{N},\; y_2[n] = 0$. $\quad \therefore$ Not invertible.