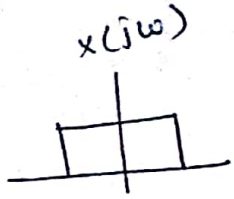
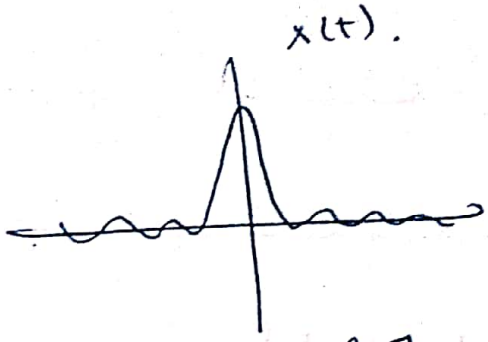
HW-5

1. a)



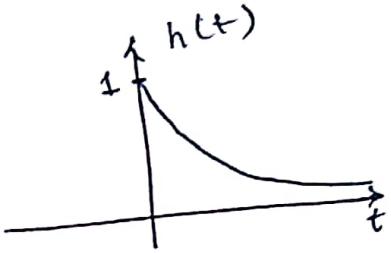
IFT



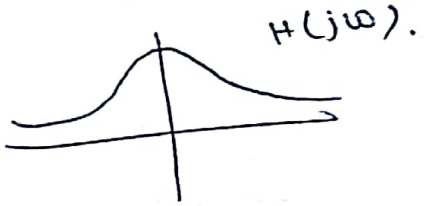
(b) * Non causal
* Oscillatory
] In time domain.

(c) Eg. of a practical low pass filter.

$e^{-at}\, u(t) \quad a>0$

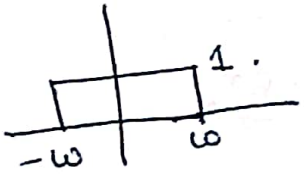


FT

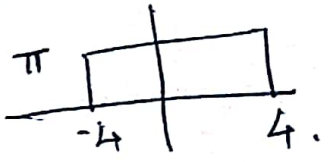


2. a) $\frac{\sin(4t)\,\sin(5t)}{t^2} = \frac{\sin 4t}{t} \cdot \frac{\sin 5t}{t}$.

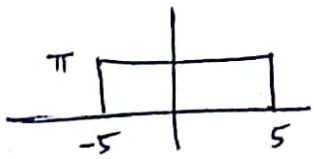
$\frac{\sin \omega t}{\pi t}$ $\xleftrightarrow{FT}$



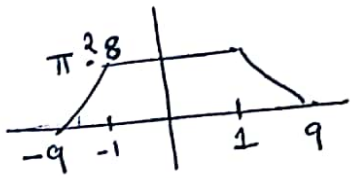
$\frac{\sin 4t}{t}$ $\xleftrightarrow{FT.}$



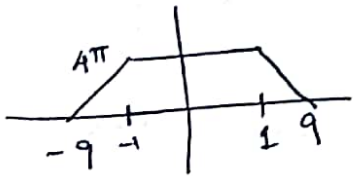
$$\frac{\sin 5t}{t} \longleftrightarrow$$



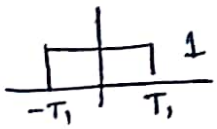
$$\frac{\sin 4t \cdot \sin 5t}{t^2} \longleftrightarrow \frac{1}{2\pi} \cdot$$



$=$

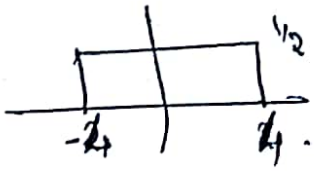


b). $\frac{\sin^2(4\omega)}{\omega^2}$

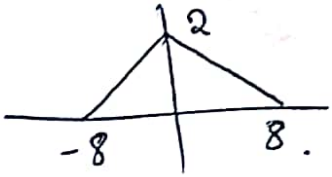


$\xleftrightarrow{FT.}$ $\frac{2\sin \omega T_1}{\omega}$.

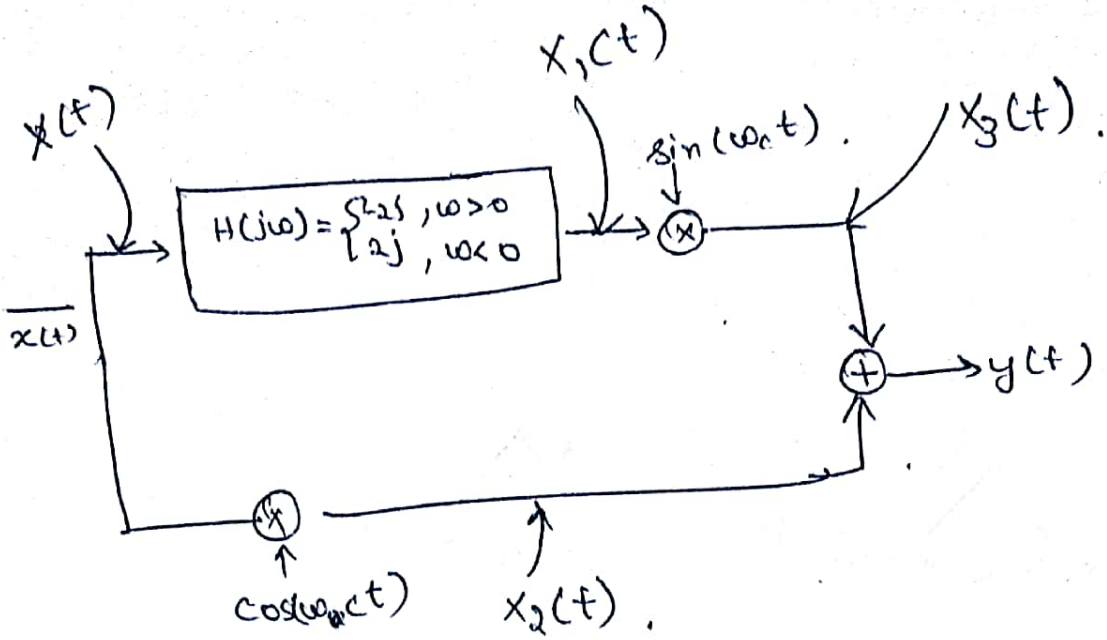
$\Rightarrow \frac{\sin 4\omega}{\omega} \xleftrightarrow{IFT}$



$\frac{\sin^2 4\omega}{\omega^2} \xleftrightarrow{IFT}$



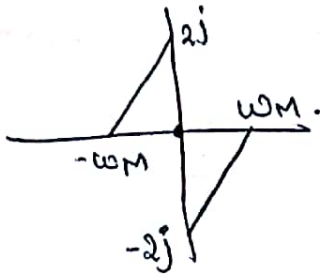
3.



$$x_1(j\omega) = x(j\omega)\, H(j\omega).$$

$x(j\omega)$

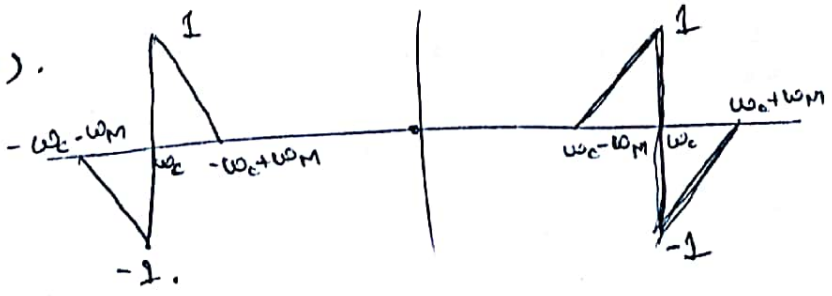
$\Rightarrow x_1(j\omega) =$



$$x_3(j\omega) = \frac{1}{2\pi}\left(\frac{\pi}{j}\left(\delta(\omega-\omega_c)\right) - \frac{\pi}{j}\left(\delta(\omega+\omega_c)\right) * x_1(j\omega)\right)$$

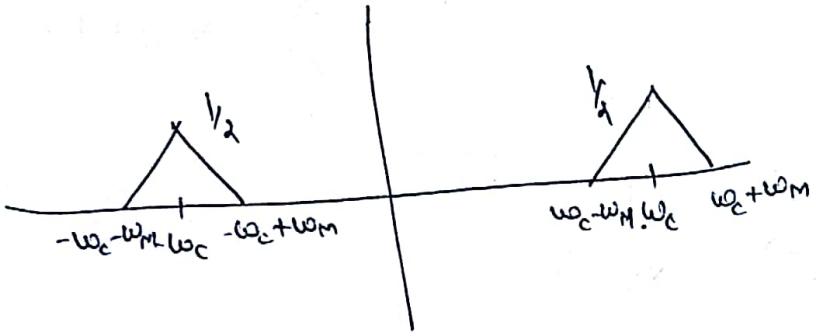
$$= \frac{1}{2j} x_1(j(\omega-\omega_c)) - \frac{1}{2j} x_1(j(\omega+\omega_c)).$$

$x_3(j\omega)$.

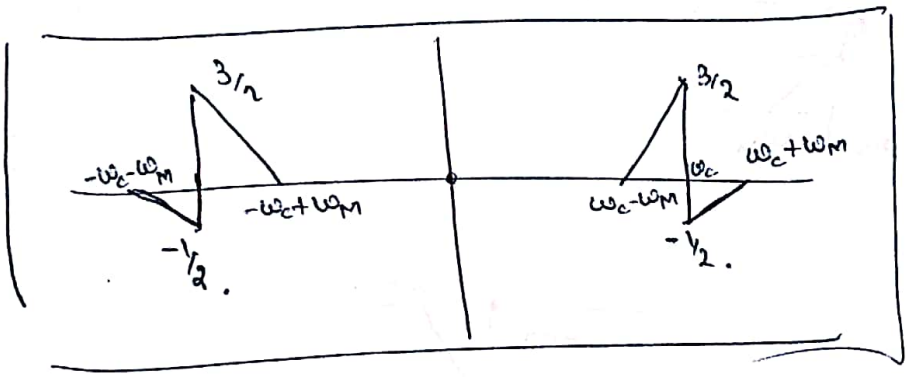


111 by

$X_2(j\omega)$.



$$y(j\omega) = X_1(j\omega) + X_2(j\omega).$$



4. Convolution property.

$$y(t) = x(t) * h(t).$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \right] e^{-j\omega t}\, dt.$$

$$= \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau)\, e^{-j\omega t}\, dt \right] d\tau$$

$$= \int_{-\infty}^{\infty} H(j\omega)\, x(\tau)\, e^{-j\omega\tau}\, d\tau = H(j\omega)\, X(j\omega).$$

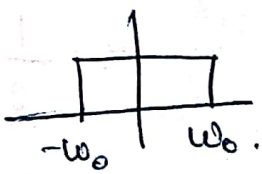
Problem 5.

a) $H_d(j\omega) = j\omega.$

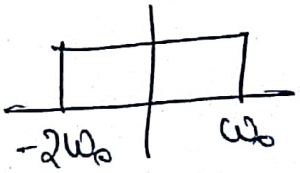
b) $H_i(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega).$

c). $\frac{\sin(\omega_0 t)}{\pi t} * \frac{\sin(2\omega_0 t)}{\pi t}.$

$\frac{\sin(\omega_0 t)}{\pi t}$ $\longleftrightarrow$

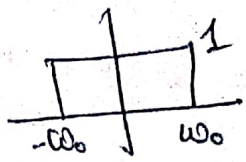


$\frac{\sin(2\omega_0 t)}{\pi t}$ $\longleftrightarrow$



Use convolution property.

$$\frac{\sin(\omega_0 t)}{\pi t} * \frac{\sin(2\omega_0 t)}{\pi t} \xleftrightarrow{FT}$$

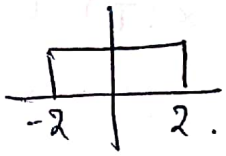


$\xleftrightarrow{IFT}$

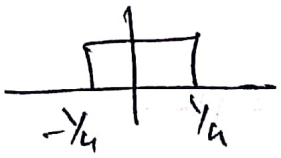
$$\frac{\sin \omega_0 t}{\pi t}$$

d. $\frac{(\pi)}{\pi} \cdot \frac{\sin(2t)}{\pi t} \cdot \frac{\sin(t/4)}{\pi t}$.

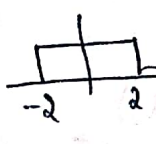
$$\frac{\sin(2t)}{\pi t} \longleftrightarrow$$



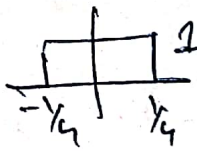
$$\frac{\sin(t/4)}{\pi t} \longleftrightarrow$$



$$\pi \cdot \frac{\sin(2t)}{\pi t} \cdot \frac{\sin(t/4)}{\pi t} \longleftrightarrow \frac{1}{2\pi} \cdot \pi \cdot$$



$*$



$$= \frac{1}{2}$$