HW - 7.

1. $2x[n] = y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2]$.

$\Updownarrow$ FT.

$$2X(e^{j\omega}) = Y(e^{j\omega}) - \frac{3}{4} e^{-j\omega} Y(e^{j\omega}) + \frac{1}{8} e^{-2j\omega} Y(e^{j\omega})$$

$$\Rightarrow \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-2j\omega}} = \frac{2}{(1-\frac{1}{4}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})}$$

~~Partial~~.

$$x[n] = \left(\frac{1}{4}\right)^n u[n] \xleftrightarrow{FT} \frac{1}{1-\frac{1}{4}e^{-j\omega}}.$$

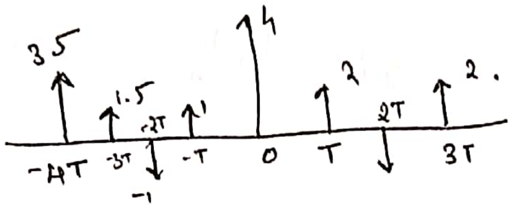
$$\Rightarrow Y(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega}) = \frac{2}{(1-\frac{1}{4}e^{-j\omega})^2 (1-\frac{1}{2}e^{-j\omega})}$$

$$= \frac{8}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}} - \frac{1}{(1-\frac{1}{4}e^{-j\omega})^2}$$

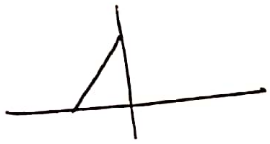
$\Updownarrow$ IFT

$$= 8 \cdot \left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n] - (n+1)\left(\frac{1}{4}\right)^n u[n].$$

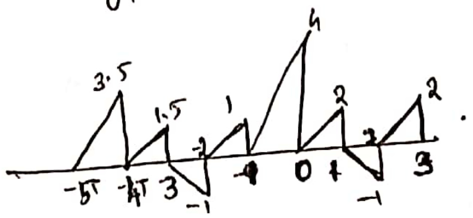
2.



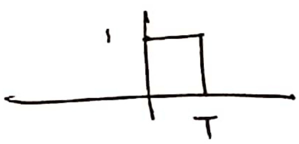
a) $h_1(t)$.



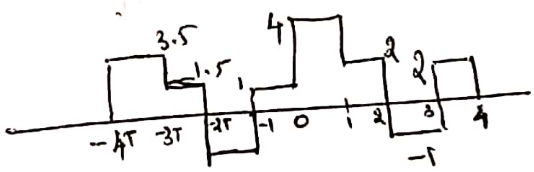
$y_1(t)$.



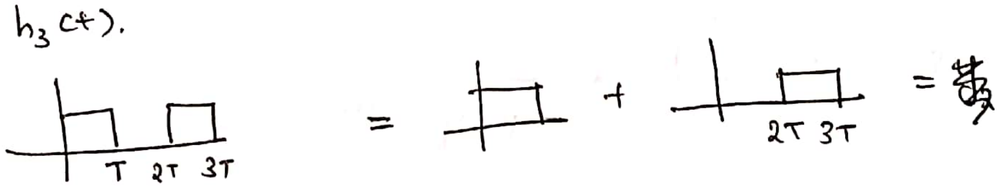
b) $h_2(t)$



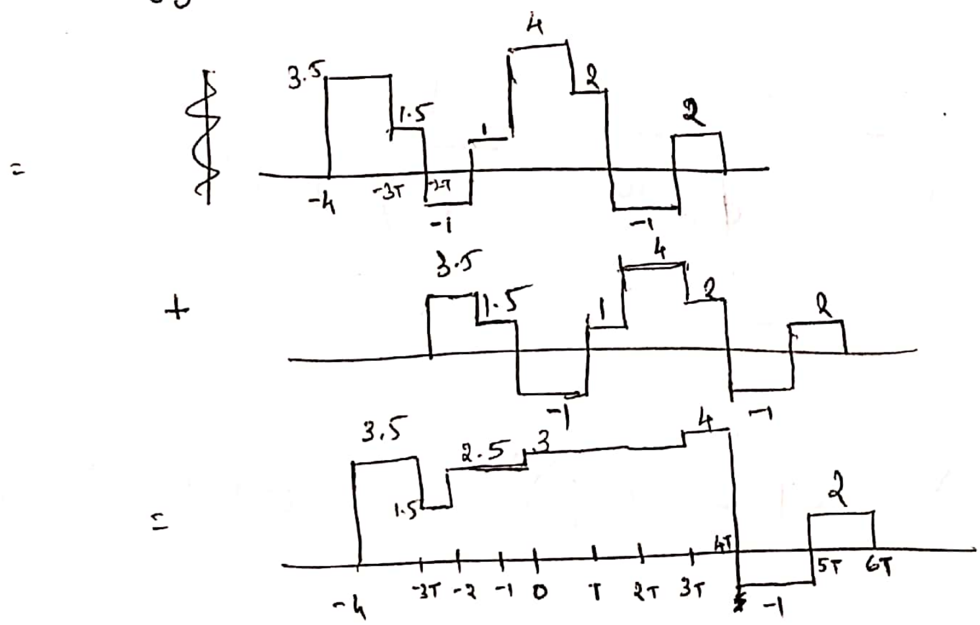
$y_2(t)$.



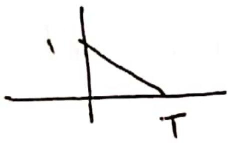
c) $h_3(t)$.



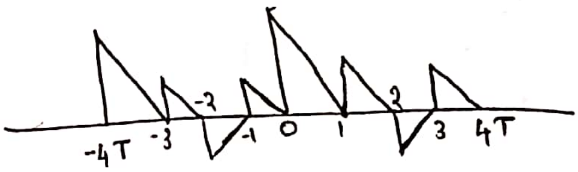
$$\Rightarrow \quad y_3(t) = y_2(t) + y_2(t-2T).$$



$h_4(t)$.



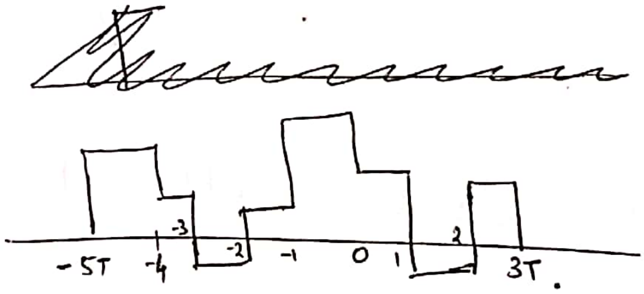
$y_4(t)$.



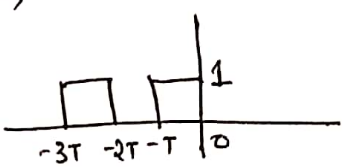
$h_5(t)$.

Notice $h_5(t) = h_2(t+T)$.

$\Rightarrow$ $y_5(t) = y_2(t+T)$.

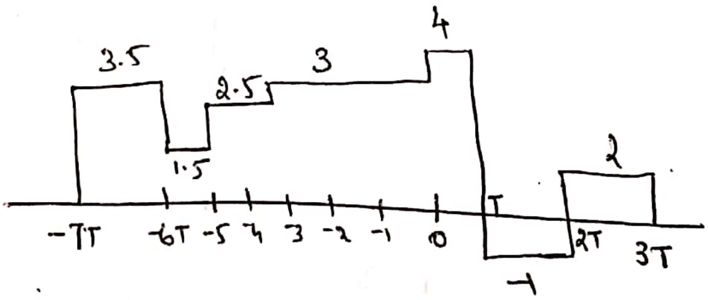


$h_6(t)$

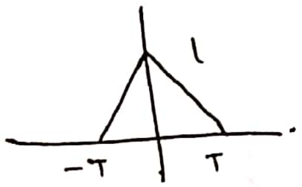


Notice. $h_6(t) = h_3(t+3T)$.

So. $y_6(t) = y_3(t+3T)$.



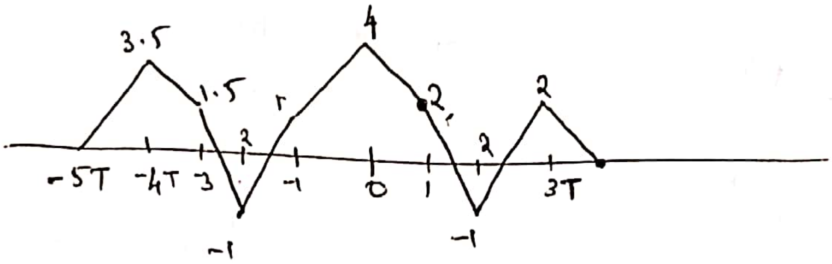
$h_7(t)$.



Notice $h_7(t) = h_1(t) + h_4(t)$.

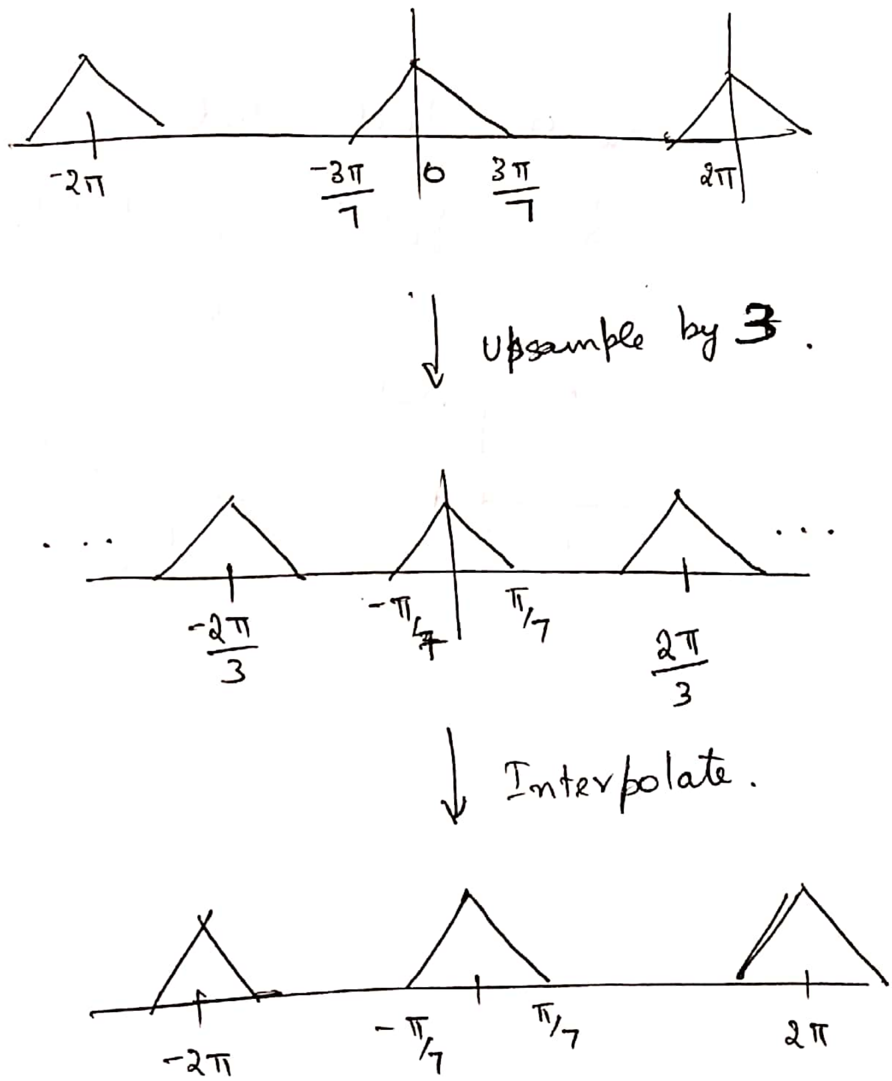
$\Rightarrow y_7(t) = y_1(t) + y_4(t)$.

[This is the interpolation operator].



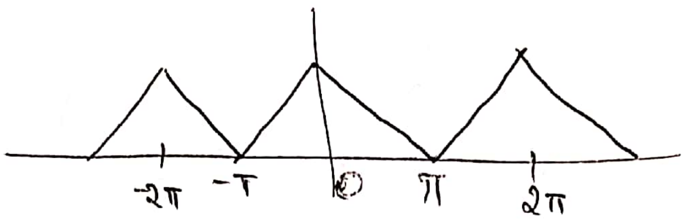
$h_8(t) \rightarrow$ Quiz 6. Problem 1.

3.

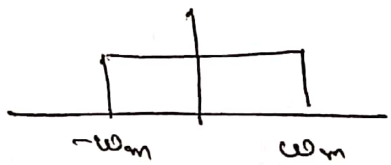


(PTO).

$\downarrow$ Downsampling by 7.

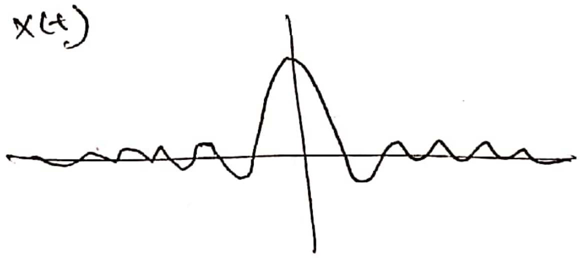


4. $X(j\omega)$



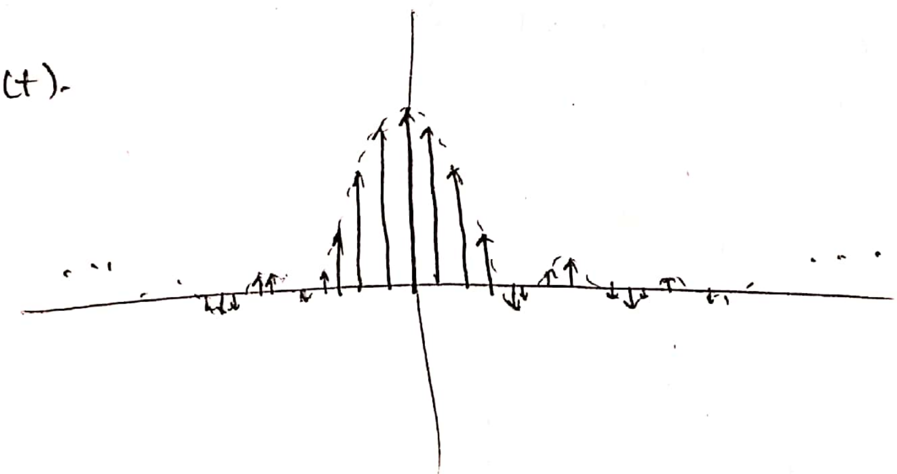
$$\overset{\text{IFT}}{\longleftrightarrow} \quad x(t) = \frac{\sin(\omega_c t)}{\pi t}.$$

$x(t)$



$p(t)$.

$\Rightarrow x_p(t)$.

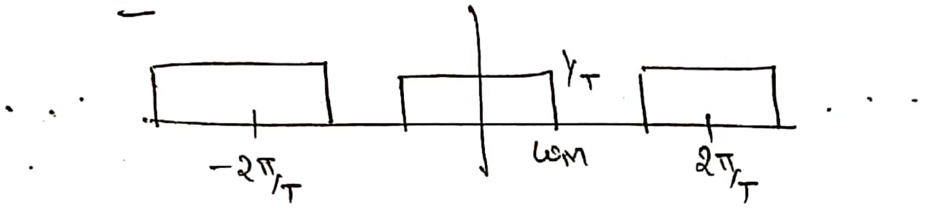


b) $X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$

$$P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s).$$

$$X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s)).$$

=



c) $$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\, \delta(t - nT).$$

$$X_p(j\omega) = \sum_{n=-\infty}^{\infty} x(nT)\, e^{-j\omega nT}$$

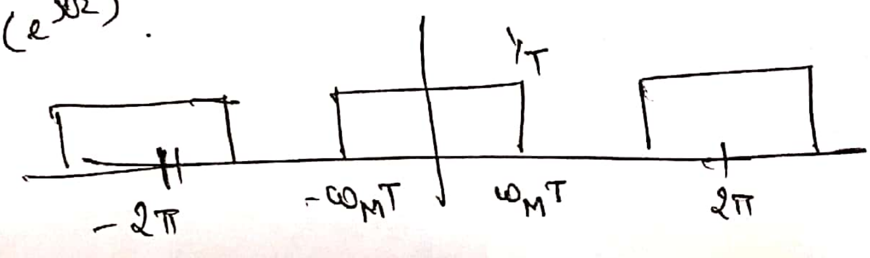
DTFT of $x_d[n]$

$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_d[n] e^{-j\Omega n}.$$

$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(nT)\, e^{-j\Omega n}$$

$$X_d(e^{j\Omega}) = X_p(j\Omega/T).$$

d). $X_d(e^{j\Omega})$.



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