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Signals and Systems

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What is a signal?

Function of one or more independent variables

We will usually call the independent variable "time"

$x(t)$ : signal

What is a system?

Transformation of input signals that result in output signals

What are the concepts?

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- Time and Frequency

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Description of Variations

Information  
Content of the  
signal

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- Linear and Time Invariant Systems

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(LTI Systems)

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# \* Linearity

## - Additivity

$$x_1(t) \rightarrow \boxed{S} \rightarrow y_1(t)$$

$$x_2(t) \rightarrow \boxed{S} \rightarrow y_2(t)$$

$$x_1(t) + x_2(t) \rightarrow \boxed{S} \rightarrow y_1(t) + y_2(t)$$

That has to apply for any choice of  $x_1(t)$  and  $x_2(t)$

## - Homogeneous system

$$x(t) \rightarrow \boxed{S} \rightarrow y(t)$$

any  
complex  
number

$$\xrightarrow{a} a x(t) \rightarrow \boxed{S} \rightarrow a y(t)$$

A system is linear if it is additive and homogeneous. (4)

## Time Invariance

A system is time invariant if any shift to the input signal results in the same shift in the output signal

$$y(t) = x(t-1) \quad \text{time invariant}$$

$$y(t) = t x(t) \quad \text{time variant}$$

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# Discrete-time and Continuous-time Signals (5)

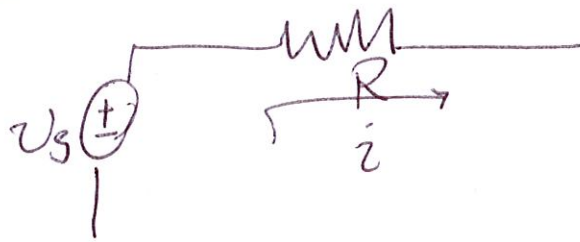
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$x[n]$  :  $n$  is an integer

$x(t)$  :  $t$  is a real number

## Energy and Power of Signals

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\* instantaneous power

$$p(t) = v(t) i(t) = \frac{1}{R} v^2(t)$$

Total energy over  $t_1 \leq t \leq t_2$

$$\int_{t_1}^{t_2} p(t) dt = \int_{t_1}^{t_2} \frac{1}{R} v^2(t) dt$$



(6)

Average Power over  $t_1 \leq t \leq t_2$ 

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{1}{R} v^2(t) dt$$

 $n_1 \leq n \leq n_2$ Total energy over  $t_1 \leq t \leq t_2$ 

$$\text{CT: } \int_{t_1}^{t_2} |x(t)|^2 dt$$

$$\text{DT: } \sum_{n=n_1}^{n_2} |x[n]|^2$$

 $n_1 \leq n \leq n_2$ 

Average Power over

 $t_1 \leq t \leq t_2$ 

$$\text{CT: } \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 dt$$

$$\text{DT: } \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

$$E_{\infty} = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad (7)$$

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$$\text{DT: } E_{\infty} = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P_{\infty} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$\text{DT: } \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2$$

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